Economics 4905: Lecture 5 Continued

Karl Shell

Cornell University

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Review (Outside) Money Taxation:

- Bonafide Taxes τ
- Balanced Taxes τ
- Bonafide $\tau \equiv$ Balanced τ
- $I = 1$, $\mathcal{P}^m$ is interval $[0, \bar{P}^m)$
- $I > 1$, $\mathcal{P}^m$ is the union of intervals
- Multiple Currencies
- Problem Set 1
- Simple, *finite* static model

Enriching the model to include dynamics and uncertainty: Debreu's isomorphism

- "Debreu" Isomorphism
  - Expand definition of commodity, $x_h^{i,s,t}$
  - Commodity type i, state of nature s, time t
  - Contingent claims
  - Futures Market
- Profit Maximization
  - Theorem
  - Not assumption
  - Diagrams

Futures Market

- $t = 1, 2$
- $l = 1$
- present prices, $(p^1, p^2) = (1, p^2)$

CP:

$$\max u_h(x_h^1, x_h^2)$$

$$\text{e.g.} \quad u_h(x_h^1, x_h^2) = \phi_h(x_h^1) + \beta_h \phi_h(x_h^2)$$

$$\text{s.t.}$$

$$p^1(x_h^1 - \omega_h^1) + p^2(x_h^2 - \omega_h^2) = 0$$

CE: (p^1, p^2) such that

$$\sum_h x_h^t = \sum_h \omega_h^t \text{ for } t = 1, 2$$

where x_h^t solves CP for $t = 1, 2$.

Criticism of Futures Market Interpretation:

- Do we really choose today all our future consumptions?
- Do ordinary people use futures market for personal choices over time?
- Everyone on a "meal plan" for everything?
- FM model is real, i.e., non-financial. More stable, but less realistic?

Inside Money Market for Dynamic Economy

- Spot market at each date, $t = 1, 2$
- Saving and dis-saving through "money-market"
- Rational expectations about future spot prices
- Expectations play no role in FM model

Inside Money, continued

CP:

$$\max u_h(x_h^1, x_h^2)$$

$$\begin{aligned} \text{s.t.} \quad & p^1 x_h^1 + p^{m1} x_h^{m1} = p^1 \omega_h^1 \\ & p^2 x_h^2 + p^{m2} x_h^{m2} = p^2 \omega_h^2 \\ & x_h^{m1} + x_h^{m2} = 0 \text{ or } x_h^{m2} = -x_h^{m1} \end{aligned}$$

CE:

$$(p^1, p^2; p^{m1}, p^{m2})$$

$$\begin{aligned} \text{s.t.} \quad & \sum_h x_h^t = \sum_h \omega_h^t \text{ for } t = 1, 2 \\ & \sum_h x_h^{m,t} = 0 \text{ for } t = 1, 2 \end{aligned}$$

where x_h^t and $x_h^{m,t}$ satisfy CP for $t = 1, 2$.

Simplifying and substituting

- $p^1(x_h^1 - \omega_h^1) = -p^{m1}x_h^{m1}$
- $p^2(x_h^2 - \omega_h^2) = p^{m2}x_h^{m1}$
- x_h^{m1} is a slack variable permitting us to combine terms (discuss):

$$p^1(x_h^1 - \omega_h^1) + p^2(x_h^2 - \omega_h^2) = (p^{m2} - p^{m1})x_h^{m1}$$

- In MM, buy low sell high, try to arbitrage the $(p^{m2} - p^{m1})$ gap
- allowing unbounded consumptions denying CE
- therefore in CE: $p^{m1} = p^{m2} = p^m \geq 0$

Assume $p^m > 0$

- We have
$$p^1(x_h^1 - \omega_h^1) + p^2(x_h^1 - \omega_h^1) = 0$$
- MM equilibrium allocation is identical to FM equilibrium allocation
- Irving Fisher (isomorphic to the Arrow article)
- Very important caveat
  - If $p^m = 0$, the money market is closed. No inter-temporal trades
  - This important outcome does not occur in the FM model

Uncertainty

- Two states, $s = \alpha, \beta$
- One commodity, $l = 1$
- Finite model, as before
- See Arrow RES article: History of article
- Contingent claims
  - "AD"
  - buy and sell contracts to deliver commodity contingent on the realization of s
- **CP:**

$$\max \pi(\alpha)u_h(x_h(\alpha)) + \pi(\beta)u_h(x_h(\beta))$$

$$\pi(\alpha) + \pi(\beta) = 1$$

$$\text{s.t.} \quad p(\alpha)x_h(\alpha) + p(\beta)x_h(\beta) = p(\alpha)\omega_h(\alpha) + p(\beta)\omega_h(\beta)$$

CE for "AD" Economy

$$p(\alpha), p(\beta)$$

such that

$$\sum_h x_h(s) = \sum_h \omega_h(s) \text{ for } s = \alpha, \beta$$

where $x_h(s)$ solves CP for $s = \alpha, \beta$.

Arrow Securities

- Arrow money
- Buy and sell commodities on spot markets
- Buy and sell Arrow monies $b_h(s)$ for $s = \alpha, \beta$ before s is realized

CP:

$$\max E_s u_h(s) = \pi(\alpha) u_h(x_h(s)) + \pi(\beta) u_h(x_h(s))$$

$$\begin{aligned} \text{s.t.} \quad & x_h(s) = \omega_h(s) + b_h(s) \text{ for } s = \alpha, \beta \\ \text{and} \quad & p^b(\alpha) b_h(\alpha) + p^b(\beta) b_h(\beta) = 0 \end{aligned}$$

Hidden assumption: 1 unit of $b_h(s)$ pays 1 unit of commodity in state s, 0 otherwise.

CE:

$$(p(\alpha), p(\beta); p^b(\alpha), p(\beta))$$

such that

$$\sum_h x_h(s) = \sum_h \omega_h(s) \text{ for } s = \alpha, \beta$$

where $x_h(s)$ solves CP.

Results

- Every CE allocation in the AD economy can be decentralized as a CE allocation in the Arrow Securities economy.
- Every CE allocation in the AS economy in which we have $(p^b(\alpha), p^b(\beta)) >> 0$ is also an equilibrium in the AD contingent claims economy.