Review (Outside) Money Taxation:

- Bonafide Taxes $\tau$
- Balanced Taxes $\tau$
- Bonafide $\tau \equiv$ Balanced $\tau$
- $l = 1$, $\mathcal{P}^m$ is interval $[0, \bar{P}^m)$
- $l > 1$, $\mathcal{P}^m$ is the union of intervals
- Multiple Currencies
- Problem Set 1
- Simple, finite static model
Enriching the model to include dynamics and uncertainty: Debreu’s isomorphism

- "Debreu" Isomorphism
  - Expand definition of commodity, $x_h^{i,s,t}$
  - Commodity type $i$, state of nature $s$, time $t$
  - Contingent claims
  - Futures Market

- Profit Maximization
  - Theorem
  - Not assumption
  - Diagrams
Futures Market

- $t = 1, 2$
- $l = 1$
- present prices, $(p^1, p^2) = (1, p^2)$

**CP:**

$$\max u_h(x^1_h, x^2_h)$$

e.g. $u_h(x^1_h, x^2_h) = \phi_h(x^1_h) + \beta_h \phi_h(x^2_h)$

s.t.

$$p^1(x^1_h - \omega^1_h) + p^2(x^2_h - \omega^2_h) = 0$$

**CE:** $(p^1, p^2)$ such that

$$\sum_h x^t_h = \sum_h \omega^t_h \text{ for } t = 1, 2$$

where $x^t_h$ solves CP for $t = 1, 2$. 
Criticism of Futures Market Interpretation:

- Do we really choose today all our future consumptions?
- Do ordinary people use futures market for personal choices over time?
- Everyone on a "meal plan" for everything?
- FM model is real, i.e., non-financial. More stable, but less realistic?
Inside Money Market for Dynamic Economy

- Spot market at each date, $t = 1, 2$
- Saving and dis-saving through ”money-market”
- Rational expectations about future spot prices
- Expectations play no role in FM model
Inside Money, continued

**CP:**

\[
\max u_h(x_h^1, x_h^2)
\]

\[
\text{s.t. } \begin{align*}
  p^1 x_h^1 + p^{m1} x_h^{m1} &= p^1 \omega_h^1 \\
  p^2 x_h^2 + p^{m2} x_h^{m2} &= p^2 \omega_h^2 \\
  x_h^{m1} + x_h^{m2} &= 0 \text{ or } x_h^{m2} = -x_h^{m1}
\end{align*}
\]

**CE:**

\[
(p^1, p^2; p^{m1}, p^{m2})
\]

\[
\text{s.t. } \begin{align*}
  \sum_h x^t_h &= \sum_h \omega_h^t \text{ for } t = 1, 2 \\
  \sum_h x_h^{m,t} &= 0 \text{ for } t = 1, 2
\end{align*}
\]

where \(x^t_h\) and \(x_h^{m,t}\) satisfy CP for \(t = 1, 2\).
Simplifying and substituting

\[ p^1(x_h^1 - \omega_h^1) = -p^{m1}x_{h}^{m1} \]
\[ p^2(x_h^2 - \omega_h^2) = p^{m2}x_{h}^{m1} \]
\[ x_{h}^{m1} \text{ is a slack variable permitting us to combine terms (discuss):} \]
\[ p^1(x_h^1 - \omega_h^1) + p^2(x_h^2 - \omega_h^2) = (p^{m2} - p^{m1})x_{h}^{m1} \]

\[ \text{In MM, buy low sell high, try to arbitrage the } (p^{m2} - p^{m1}) \text{ gap} \]
\[ \text{allowing unbounded consumptions denying CE} \]
\[ \text{therefore in CE: } p^{m1} = p^{m2} = p^{m} \geq 0 \]
Assume $p^m > 0$

- We have
  \[ p^1(x^1_h - \omega^1_h) + p^2(x^1_h - \omega^1_h) = 0 \]

- MM equilibrium allocation is identical to FM equilibrium allocation

- Irving Fisher (isomorphic to the Arrow article)

- Very important caveat
  - If $p^m = 0$, the money market is closed. No inter-temporal trades
  - This important outcome does not occur in the FM model
Uncertainty

- Two states, $s = \alpha, \beta$
- One commodity, $l = 1$
- Finite model, as before
- See Arrow RES article: History of article
- Contingent claims
  - "AD"
  - buy and sell contracts to deliver commodity contingent on the realization of $s$
- **CP:**

\[
\max \pi(\alpha)u_h(x_h(\alpha)) + \pi(\beta)u_h(x_h(\beta))
\]
\[
\pi(\alpha) + \pi(\beta) = 1
\]

s.t. \( p(\alpha)x_h(\alpha) + p(\beta)x_h(\beta) = p(\alpha)\omega_h(\alpha) + p(\beta)\omega_h(\beta) \)
CE for "AD" Economy

\[ p(\alpha), p(\beta) \]

such that

\[ \sum_h x_h(s) = \sum_h \omega_h(s) \text{ for } s = \alpha, \beta \]

where \( x_h(s) \) solves CP for \( s = \alpha, \beta \).
Arrow Securities

- Arrow money
- Buy and sell commodities on spot markets
- Buy and sell Arrow monies $b_h(s)$ for $s = \alpha, \beta$ before $s$ is realized

**CP:**

$$\max E_s u_h(s) = \pi(\alpha)u_h(x_h(s)) + \pi(\beta)u_h(x_h(s))$$

s.t. $x_h(s) = \omega_h(s) + b_h(s)$ for $s = \alpha, \beta$

and $p^b(\alpha)b_h(\alpha) + p^b(\beta)b_h(\beta) = 0$

Hidden assumption: 1 unit of $b_h(s)$ pays 1 unit of commodity in state $s$, 0 otherwise.
CE:

\[(p(\alpha), p(\beta); p^b(\alpha), p(\beta))\]

such that

\[
\sum_h x_h(s) = \sum_h \omega_h(s) \text{ for } s = \alpha, \beta
\]

where \(x_h(s)\) solves CP.
Results

- Every CE allocation in the AD economy can be decentralized as a CE allocation in the Arrow Securities economy.

- Every CE allocation in the AS economy in which we have \((p^b(\alpha), p^b(\beta)) \gg 0\) is also an equilibrium in the AD contingent claims economy.