Economics 4905: Lecture 6
Bank Runs, Deposit Insurance, and Liquidity

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• three periods: $T = 0, 1, 2$
• a single good
• a continuum of agents with measure 1
• Each agent is endowed with 1 unit of the good in period 0.
The Model: Asset Return

\[ T = 0 \quad T = 1 \quad T = 2 \]

\[
\begin{array}{c}
-1 \\
\{ \\
0 \\
1 \\
\}
\end{array}
\]

\[
R
\]

The Model: Preferences

- In period 0, all agents are identical.
- In period 1, some agents become “patient” and others become “impatient”. (private information)
  \[
  \begin{cases}
  u(c_1) & \text{if impatient} \\
  u(c_2) & \text{if patient}
  \end{cases}
  \]
- The probability of being impatient is $\lambda$ for each agent in period 0.
Autarky

- autarky:
  - utility of the impatient in period 1: $u(1)$
  - utility of the patient in period 2: $u(R)$
  - expected utility in period 0: $\lambda u(1) + (1 - \lambda) u(R)$

- $1 < R$
  - “insurance” against the liquidity shock is desirable.
Banks offers demand deposit contract \((d_1, d_2)\).

Agents

- make deposits in period 0.
- withdraw \(d_1\) in period 1.
- or withdraw \(d_2\) in period 2.

free-entry banking sector: \((d_1, d_2)\) maximizes the depositor’s expected utility.
optimal deposit contract

$$\max_{d_1,d_2} \lambda u(d_1) + (1 - \lambda) u(d_2)$$

s.t.

$$\underbrace{(1 - \lambda)d_2}_{\text{withdrawals in period 2}} \leq \underbrace{(1 - \lambda d_1) R}_{\text{resources in period 2}} \quad (RC)$$

$$d_1 \leq d_2 \quad (IC)$$
Optimal Deposit Contract:

\[(1 - \lambda)d_2 = (1 - \lambda d_1)R\]

\[slope = \frac{\lambda}{1 - \lambda}R\]
Optimal Deposit Contract:

\[
\lambda u(d_1) + (1 - \lambda) u(d_2) = \text{const}
\]

slope = \(-\frac{\lambda}{1 - \lambda} \frac{u'(d_1^*)}{u'(d_2^*)}\)

\[
(1 - \lambda)d_2 = (1 - \lambda d_1)R
\]

slope = \(-\frac{\lambda}{1 - \lambda} R\)

\[
\frac{\lambda}{1 - \lambda} \frac{u'(d_1^*)}{u'(d_2^*)} = \frac{\lambda}{1 - \lambda} R
\]

\[
\underbrace{\frac{\lambda}{1 - \lambda} \frac{u'(d_1^*)}{u'(d_2^*)}}_{MRS} = \underbrace{\frac{\lambda}{1 - \lambda} R}_{MRT}
\]
What do banks do?

- $u'(d_1^*) / u'(d_2^*) = R$
- $u'' < 0 \Rightarrow d_1^* < d_2^*$
- **CRRA:** $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$
  - $u'(c) = c^{-\gamma} \implies u'(d_1) / u'(d_2) = (d_2 / d_1)^\gamma$
  - if $\gamma = 1 \implies d_1^* = 1, d_2^* = R$
  - if $\gamma > 1 \implies 1 < d_1^* < d_2^* < R$
Why do bank runs occur?

- $\gamma > 1 \implies 1 < d_1^* < d_2^* < R$
- IC: $d_1 \leq d_2$
- Expectation: Only the impatient depositors withdraw in period 1.
- A patient depositor can
  \[
  \begin{cases} 
  \text{get } d_2^* & \text{if he withdraws in period 2} \\
  \text{get } d_1^* & \text{if he withdraws in period 1}
  \end{cases}
  \]
Why do bank runs occur?

- $\gamma > 1 \implies 1 < d_1^* < d_2^* < R$

- Expectation: All other depositors demand withdraw in period 1.

- A patient depositor can
  \[
  \begin{align*}
  &\text{get } \textit{nothing} \quad \text{if he withdraws in period 2} \\
  &\text{get } d_1^* \text{ w.p. } (1/d_1^*) \quad \text{if he withdraws in period 1}
  \end{align*}
  \]