Let \( \tau = (5, 2, -2, 5) \) and \( \tau' = 2\tau \)
- \( \tau' = 2(5, 2, -2, -5) = (10, 4, -4, -10) \)
- \( x_1 = 20 - 10P^m > 0 \Rightarrow P^m < 2 \)
- \( x_2 = 15 - 4P^m > 0 \Rightarrow P^m < \frac{15}{4} \)
- \( 0 < P^m < 2 < \frac{15}{4} \)
- \( P^m \in [0, 2) \)
- \( \bar{P}^m = 2 \)
$P^m \in [0, 2), \mathcal{P}^m = [0, 2)$

- This is a statement about sets, not price levels
- If everyone believes QTM, then QTM is REE
- If not, not
Take-Away

- Indeterminacy of the price level
- Beliefs about $P^m$ and fundamentals $\omega$ jointly determine outcomes
- Beliefs matter
- The quantity theory of money is (too) subtle. Doubling $\tau$ will affect $P^m$ but not necessarily according to QTM.
Two Currencies, R and B:

- Bi-metalism in the US
- "Cross of Gold" speech
- Borrowers hurt by deflation
Two Currencies, R and B:

- $l = 1, \ n = 5, \ \omega = (25, 20, 15, 10, 5)$
- $\tau^B = (1, 1, 1, -1, -1), \ \tau^R = (1, 1, -1, -1, -1)$
- $\sum \tau^B_h = 1, \ \sum \tau^R_h = -1$
- $P^B \sum \tau^B_h + P^R \sum \tau^R_h = 0$
- $P^B - P^R = 0 \Rightarrow P^B = P^R$
Two Currencies, R and B:

- $x_1 = 25 - P^B - P^R = 25 - 2P^B > 0 \Rightarrow P^B < \frac{25}{2}$
- $x_2 = 20 - P^B - P^R \Rightarrow P^B < 10$
- $x_3 = 15 - P^B + P^R = 15$
- $0 \leq P^B < 10 < \frac{25}{2}$
\( \mathcal{P}^m = \{ P^B, P^R \mid P^B = P^R, P^B \in [0, 10) \} \)

\[ \{(x_1, x_2, x_3, x_4, x_5) \mid x_1 = 25 - 2P^B, x_2 = 20 - 2P^B, \]
\[ x_3 = 15, x_4 = 10 + 2P^B, \]
\[ x_5 = 5 + 2P^B, P^B \in [0, 10) \} \]

- The elements of \( x \) are not independent. They are constrained by \( \mathcal{P}^m \).
In General

- If \( \sum \tau_h^B \) and \( \sum \tau_h^R \) agree in sign, then \( P^B = P^R = 0 \).
- If \( \sum \tau_h^B \) and \( \sum \tau_h^R \) disagree in sign, then either the exchange rate is

\[
\frac{P^B}{P^R} = -\frac{\sum \tau_h^R}{\sum \tau_h^B}
\]

or

\( P^B = P^R = 0 \)
Why?

If \( \sum \tau^B_h = \sum \tau^R_h = 0 \), then \( \frac{P^B}{P^R} \) is indeterminate.

Why?
Some Take-aways:

- Surpluses in both "countries" lead to de-monetization. Deficits in both "countries" lead to de-monetization.
- In this simple economy, (real) fundamentals such as endowments do not affect exchange rates. They are purely financial.