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Diamond and Dybvig's Classic Theory of Financial Intermediation: What's Missing?

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History is replete with instances in which a seemingly healthy economy has plunged into difficulty, investors have become suddenly insistent on exercising contractual options to mitigate their individual risks, and financial intermediaries have consequently become unable to honor all their commitments. To design good policy to prevent or mitigate such crises, economic policymakers need to make a judgment about causality. Is financial intermediation's structure unstable and thus causing the broader economic difficulty? Or is the observed instability among financial intermediaries merely reflecting that broader economic difficulty? Douglas Diamond and Philip Dybvig (1983; reprinted in this issue of the *Quarterly Review*) have provided a classic theory to formalize the first possibility, the idea that financial intermediation's structure causes economic crises.[1] We demonstrate here that Diamond and Dybvig's theory is incomplete, so that the second possibility remains a live one. Further work is needed to determine what's missing in Diamond and Dybvig's theory and ultimately to provide policymakers with a better understanding of financial intermediation.

As Neil Wallace (1988, pp. 4, 8–9) points out, the environment Diamond and Dybvig consider has four key ingredients of an actual banking system: uncertainty about people's preferences for expenditure streams, which produces demand for liquid assets; privacy of information about these preferences after they have been realized (information about people's *types,* whether they are patient or impatient to consume); a *sequential service constraint,* or a rule that spending by different people must occur successively; and real investment projects that are costly to restart if they are interrupted. Diamond and Dybvig argue that welfare-maximizing agents in such an environment will select a banking arrangement that resembles a demand deposit contract which can implement the efficient allocation in an equilibrium. Moreover, unless the economy also has either deposit insurance or a suspension of payments contingency such as existed until the 1930s in the United States, financial intermediation via demand deposit contracts will have a bank run equilibrium. That is the sense in which Diamond and Dybvig think that financial intermediation's instability can be a cause, rather than merely a side effect, of broad economic crises.

Diamond and Dybvig make their fundamental point in a benchmark model which has no aggregate uncertainty about the number of agents who are impatient to consume (and thus want to withdraw their deposits early).

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[1]Diamond and Dybvig's work is one of the pivotal contributions to a large literature on banking contracts and bank runs. Other such contributions include the work of John Bryant (1980), Charles Jacklin (1987), and Neil Wallace (1988).

Diamond and Dybvig then show that a suspension of payments scheme can eliminate the bank run equilibrium in the benchmark model and that, with aggregate uncertainty, when the suspension of payments scheme does not work, a deposit insurance arrangement can do the trick.

The contractual arrangements considered by Diamond and Dybvig are limited to a space of feasible arrangements that, as they point out, is too narrow to implement an efficient allocation in the environment with aggregate uncertainty. Specifically, Diamond and Dybvig assume that the banking arrangement must give all depositors who demand early withdrawals the same amount of consumption, namely, the socially efficient amount calculated based on the true parameter (the fraction of impatient depositors), no matter how many depositors actually claim to be impatient. In the Diamond and Dybvig model, although the consumption given to individual depositors varies with their own claimed types, the amount for each type does not depend on the full information communicated to the bank by all depositors collectively. Let us call this approach *simple contracting.*

We suggest that Diamond and Dybvig's theory is incomplete in an important respect. In particular, we argue that the demand deposit contract considered by Diamond and Dybvig is only one of the feasible arrangements in the environment of their model. We show that a contractual arrangement exists that implements an efficient allocation in their environment with aggregate uncertainty, but without a bank run equilibrium.[2]

Instead of restricting attention to only simple contracting, we allow the bank to use more fully the information reported by all depositors regarding their preferences. The banking arrangement in our model specifies consumptions for each depositor, or *trader,* of each type under all possible configurations of types reported to the bank. In fact, for each vector of messages that the traders send to the bank, reporting their types, our arrangement assigns traders the efficient allocation computed for the entire reported economy. Moreover, under this arrangement, individual traders always find it in their best interest to truthfully report their types. Hence, the efficient allocation for the model's true economy prevails in the unique equilibrium. (Recall that, in Diamond and Dybvig's environment with aggregate uncertainty, the contractual arrangement that they study has multiple equilibria, and generally, all are inefficient.) Also, in our model, as in Diamond and Dybvig's, the banking arrangement contains elements of a demand deposit contract; traders have the freedom to choose either to consume early (by claiming to be impatient) or to wait to consume when their assets mature (by claiming to be patient). However, our arrangement will never involve bank runs. These properties hold even when the bank faces the sequential service constraint. In light of our findings, from a *mechanism design* approach, Diamond and Dybvig's bank run equilibrium appears to be an artifact of their simple contracting approach rather than a genuine feature of the economic environment that they have modeled.

Our analysis does not diminish the fundamental importance of Diamond and Dybvig's insight regarding financial instability, but we think it shows the need to synthesize that insight with further ideas in order to fully understand financial instability. In light of the strikingly opposite features of our model's results and the U.S. history of bank runs, we wonder, what might prevent rational agents in the Diamond and Dybvig environment from using efficient arrangements, or mechanisms, such as the one in our model? What would lead them instead to adopt the potentially destabilizing demand deposit contract considered by Diamond and Dybvig? Our purpose here is simply to raise such questions, not to answer them. Our results imply that environmental features from which Diamond and Dybvig's model abstracts are crucial to a full understanding of banking instability. At the end of the article, we reflect on our analysis to identify some promising candidates for further, complementary research.

The Model

The model we use is a finite-trader version of the Diamond and Dybvig model with aggregate risk.

Consider a population of I traders, each of whom is endowed with one unit of a divisible good. The good can be either consumed at date 0 or transformed into a consumption good available at date 1. For each unit of the good used as an input, the transformation technology produces $R > 1$ units of consumption good at date 1.

At the beginning of life (date 0), all traders are uncertain about their preferences over consumption streams. With probability p, a trader becomes *patient* (type 1) and values the sum of date 0 and date 1 consumption. With

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[2]Our mechanism design approach in general involves supposing that the formal model of the economic environment succeeds in capturing the significant relevant constraints on how contractual arrangements can be structured in the actual economy and then solving the optimization problem of designing an efficient arrangement subject to those constraints.

probability $1 - p$, a trader becomes *impatient* (type 0) and values date 0 consumption only. Here, as in Diamond and Dybvig's model, the utility function is $v(c_0)$ for an impatient trader and $v(c_0+c_1)$ for a patient trader, where c_t denotes consumption at date t.

For simplicity, we assume that the population is limited to three traders, each having the utility function $v(c) = c^{1-\gamma}/(1-\gamma)$ with a risk aversion parameter of $\gamma > 1$. Results derived in this simple setup, however, also hold in more general settings. The economic content of the assumption that $\gamma > 1$ is that traders' relative risk aversion is greater than 1 everywhere.

All traders learn which type they are (patient or impatient) at date 0. Types are private information. In autarky, a patient trader would have higher utility ex post than an impatient trader because patient traders have an opportunity to apply the intertemporal transformation technology to their endowments. Because traders are risk averse, they would like to enter ex ante into an arrangement to insure themselves (and so, one another) against preference shock risk.

Thus, to protect themselves against preference shocks, the traders at the beginning of date 0 (before anyone learns their types) pool their resources and set up a *bank*, which is actually a clublike arrangement among the traders. The bylaws of the bank specify a rule, according to which each trader will receive consumption that may depend on the trader's report, or *message*, to the bank about the trader's privately observed type. A message of 0 sent by a trader means that the trader is impatient, while a message of 1 means the trader is patient. The traders are assumed to specify the bylaws to maximize the ex ante total utility of all traders. The bylaws thus stipulate a *resource distribution rule*, which specifies the amount of consumption that each trader receives at each date for each possible configuration of reported trader types. We will show that this rule has an equal treatment property: In each state of nature, all type 0 traders should receive identical consumption c_0 at date 0, and all type 1 traders should receive identical consumption c_1 at date 1. (The date 0 consumption for type 1 traders should be zero because $R > 1$.)

So far, the only significant difference between our setting and Diamond and Dybvig's is the size of the population. Diamond and Dybvig consider an infinite population. In contrast, we have only three traders, and individual-level randomness implies that our model always has aggregate uncertainty.

Another aspect of our model differs from Diamond and Dybvig's: the sequential service constraint. Diamond and Dybvig discuss this constraint informally, but do not model it explicitly. We do. In our model, during date 0, traders arrive at the bank in random order. All traders observe their own arrival times. They also observe whether they, themselves, are the first, the second, or the third to arrive at the bank.[3] The resource distribution rule (stipulated in the bank bylaws) specifies that the ith trader to arrive at the bank sends a message $m_i \in \{0,1\}$ to the bank when approaching it, with $m_i = 0$ standing for being impatient and $m_i = 1$ for being patient. The bank then distributes the traders' pooled resources on the basis of the messages it has received.

Formally, we model sequential service in the following way. Let $x_i(m)$ denote the consumption given to the trader who is the ith to arrive at the bank, where $m \equiv (m_1,m_2,m_3)$.[4] The sequential service constraint requires that for any m, $x_1(m) = x_1$ if $m_1 = 0$ and $x_2(m) = x_2(m_1)$ if $m_2 = 0$. That is, the consumption given to the ith trader who reports being impatient must not depend on information from traders who arrive later, since those traders have not yet communicated their information to the bank. Since a patient trader does not consume until date 1, after all traders have sent their messages to the bank at date 0, the consumption given to a patient trader can be determined on the basis of all traders' messages.

Banking Without a Sequential Service Constraint

First, we consider the model environment without assuming a sequential service constraint. That is, we assume that each trader's consumption can be made to depend on the reports of all three traders. We characterize an optimal resource distribution rule, and we show that

[3]There are 6 (3!) possible orders of arrival of the three traders, and we assume that each occurs with probability 1/6. This formulation of sequential service is related to the night camping story told by Wallace (1988). However, in Wallace's formalization of sequential service, neither a trader's time of arrival nor a trader's place in line is in the trader's own information set. Under Wallace's assumption, the backward induction reasoning we use later may not necessarily work. In Green and Lin 1999, we relax the assumption that traders know their places in the order of arrival. However, we still assume richer information about arrival time than Wallace does.

[4]In principle, $x_i(m)$ should specify quantities to be consumed at both dates 0 and 1. For simplicity, we will suppose that a trader who sends a message of 0 (who reports being impatient) will be permitted to consume only at date 0 and that a trader who sends a message of 1 (who reports being patient) will be permitted to consume only at date 1. The following discussion should make it clear that the optimal resource distribution rule must have this feature, even if it were not imposed by assumption. We explicitly derive such a result in Green and Lin 1999.

this rule induces truth-telling as the unique reporting decision of rational, optimizing traders. If a bank run is interpreted as an inefficient equilibrium in which traders who are actually patient misrepresent themselves as being impatient, then no bank run can occur when this optimal rule is adopted.

We begin our study by abstracting from the sequential service constraint because this environment is precisely the finite-trader analog of the formal Diamond and Dybvig model environment. Thus, the result suggests that Diamond and Dybvig's ad hoc focus on a particular class of rules regarding demand deposit contracts, with or without suspension of payments, is crucial to their finding of a dilemma of having to choose between economic inefficiency and banking instability (the existence of a bank run equilibrium).

Another reason for beginning our study without the sequential service constraint is to exhibit, in as simple a setting as possible, the logic of our main argument. The argument has two parts. First, we imagine that traders' types are public information, and we characterize the optimal resource distribution rule that would use this information directly. Second, we take into account the fact that the resources must be distributed on the basis of traders' unverifiable and unfalsifiable reports, rather than on the basis of the true situation. Thus, the rule that we have characterized in the hypothetical environment with public information can only be used in the private-information environment if traders can be trusted to tell the truth voluntarily. That is, the rule is usable only if, whatever their types, traders can achieve higher utility by truth-telling than by lying. We show that an environment without the sequential service constraint has a very strong truth-telling incentive: each trader does best by telling the truth, regardless of whether or not other traders are truthful. (We will derive a result in the same spirit, although slightly weaker, when we take account of the sequential service constraint.) Clearly, with this unambiguous incentive, all traders will tell the truth, so there can be no bank run equilibrium.

To develop this argument, suppose that traders receive consumption after they all have reported their types to the bank. (That is, ignore the sequential service constraint.) Also, suppose that the true state of nature, ω, is known—or, equivalently, that the profile m of traders' messages to the bank is identical to ω. We will characterize the rule that maximizes the sum of traders' ex ante expected utility levels.

The trick to solving this maximization problem is to maximize the sum of traders' ex post utilities in each state of nature and note that the problems of maximizing ex ante and ex post utility have the same solution in this environment. Let $\theta(\omega)$ denote the number of patient traders in a given state of nature ω. Ex post efficiency requires that the endowment good's marginal utility to an impatient trader equal that to a patient trader in each state of nature,

$$(1) \qquad v'\big(c_0(\theta(\omega))\big) = Rv'\big(c_1(\theta(\omega))\big)$$

and that the following resource constraint be satisfied:[5]

$$(2) \qquad [I - \theta(\omega)]c_0\big(\theta(\omega)\big) + R^{-1}\theta(\omega)c_1\big(\theta(\omega)\big) = I.$$

Equations (1) and (2) determine the functions $c_0(\omega)$ and $c_1(\omega)$ completely.

For the assumed utility function $v(c) = c^{1-\gamma}/(1-\gamma)$, it is straightforward to solve these two equations:

$$(3) \qquad c_0(\theta) = I/[I + \theta(R^{(1/\gamma)-1}-1)]$$

and

$$(4) \qquad c_1(\theta) = IR^{1/\gamma}/[I + \theta(R^{(1/\gamma)-1}-1)].$$

This completes the first stage of our argument.

Now we must undertake the second stage, to show that traders would always choose to report their types truthfully if this rule (based on their reports rather than on their true states) were to determine their consumption. Since $R > 1$, we have that $c_1(\theta)$ is greater than $c_0(\theta)$ and that both increase with θ. The patient traders can take advantage of the transformation technology, so they each receive more consumption than do the impatient traders. Furthermore, in states of nature in which the number of impatient traders is smaller, more endowment gets to be transferred to date 1 consumption, enabling both types of traders to consume more.

At date 0, when traders learn their types, they send messages reporting their types to the bank. The bank calculates the value of $\theta(\omega)$ based on these reports and then distributes resources according to the consumptions derived above. Regardless of whether or not the calcu-

[5]These conditions also imply ex ante efficiency. See Green and Lin 1999.

lated value of θ is actually the true value, each trader has the incentive to truthfully report his or her own type, whatever messages other traders send.

To see this, consider separately each of the two possible types of traders. If a trader is type 0, then the trader receives $c_0(\theta)$ at date 0 if he or she sends a message of 0, but receives no consumption until date 1 if he or she sends a message of 1. Since an impatient trader values date 0 consumption only, this trader strictly prefers to tell the truth. Now consider a trader of type 1. Regardless of what messages other traders send, the patient trader receives $c_1(\theta)$ if he or she sends a message of 1 and $c_0(\theta-1)$ if he or she sends a message of 0. As we have explained above, $c_1(\theta) > c_1(\theta-1) > c_0(\theta-1)$. Thus, a patient trader prefers to send a truthful message as well. Therefore, the banking arrangement here has the property that truth-telling is the *strictly dominant strategy* for all traders. Roger Myerson (1991) has shown that a profile of strictly dominant strategies for a mechanism is the unique Bayesian Nash equilibrium of the mechanism. Therefore, here, no alternative, inefficient bank run equilibrium can exist.

Banking With a Sequential Service Constraint

The simple model of banking studied above abstracts from key features of an actual bank: usually, traders do not all contact the bank at the same time, and the bank must deal promptly with traders who contact it early. An actual bank is thus constrained from making its treatment of early traders contingent on information yet to be provided by later traders, especially if the early traders want to make withdrawals.[6] Now we modify the model to make it more realistic in this sense. We require traders to contact the bank sequentially during date 0, according to the sequential service constraint formalized above.

The general logic of our argument here is parallel to that just used. We first define the efficient allocation in this economy assuming that traders report their true types to the bank. Then we prove that under the specified rule of distributing resources, it is in the best interests of the traders to truthfully report their types to the bank and that the symmetric efficient allocation can be implemented as the perfect Bayesian Nash equilibrium of the mechanism.

The Bank's Planning Problem

An allocation in this three-trader economy with sequential service is a list of consumption bundles $x_i(m)$ for all i and m which the bank (here, the *social planner*) must choose. An allocation is feasible if, in each state of nature, the total amount of consumption the patient traders receive from the bank equals R times the amount of resources available after the bank gives consumption to impatient traders in that state of nature:

$$(5) \qquad \sum\nolimits_{i\in I, m_i=1} x_i(m) = R\left[I - \sum\nolimits_{i\in I, m_i=0} x_i(m)\right]$$

for all m. This is the economy's resource constraint.

The bank must choose an allocation to maximize the sum of expected utility of all traders. The efficient allocation is thus obtained by

$$\begin{aligned}(6) \qquad \max\ & (1-p)v(x_1(0)) + pE_{m_2,m_3}v(x_1(1,m_2,m_3)) \\ & + (1-p)E_{m_1}v(x_2(m_1,0)) \\ & + pE_{m_1,m_3}v(x_2(m_2,1,m_3)) \\ & + (1-p)E_{m_1,m_2}v(x_3(m_1,m_2,0)) \\ & + pE_{m_1,m_2}v(x_3(m_1,m_2,1))\end{aligned}$$

subject to the resource constraint (5).

The form of the above optimization problem (an additively separable objective function in terms of $v(x_i)$ and the resource constraint in distinct states of nature) implies that the problem can be solved using the usual dynamic programming techniques: maximizing total utility implies optimizing along each path of realizations of trader types. For instance, at the efficient allocation, $x_1(0)$ must be such that it maximizes the utility of the first trader to arrive at the bank, $v(x_1(0))$, plus the sum of the expected utilities of the second and third traders conditional on $m_1 = 0$. Therefore, in what follows, we will solve the bank's problem using the usual backward induction procedure. Specifically, we start by deriving the optimal consumption for the trader who is the last to arrive at the bank and then move on to optimization problems for traders who arrived earlier. Each problem is solved based on the information reported by the traders as they arrive at the bank. We then show that the traders have the incentive to truthfully reveal their types when they make decisions, so privacy of information is

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[6]This feature plays a key role in Diamond and Dybvig's intuitive discussion of their model, and it is formalized by Wallace (1988), who derives further consequences from it.

actually not a binding constraint in the bank's planning problem.

Consumption for the Last Trader

We start, again, with the last trader to arrive at the bank.

Let $y(m_1,m_2)$ denote the amount of endowment the bank has left when the last trader arrives. That is, $I - y$ has been given out to previously arriving traders, who have sent messages of m_1 and m_2. The bank's decision problem here is simple. If the last trader is patient ($m_3 = 1$), then that trader at date 1 will receive Ry/θ, the trader's share of the remaining endowment transformed by the R technology, where $\theta \equiv m_1 + m_2 + m_3$. If, instead, the last trader is impatient, then the bank needs to immediately assign that trader consumption at date 0, denoted by $x_3(m_1,m_2,0)$, by balancing the trader's marginal utility with that of the patient traders. Thus, $x_3(m_1,m_2,0)$ satisfies the following:[7]

$$(7) \qquad x_3(m_1,m_2,0) = \arg\max_{x_3} v(x_3) + (m_1+m_2)v\big(R(y-x_3)/(m_1+m_2)\big).$$

The first-order condition is thus $v'(x_3) = Rv'\big(R(y-x_3) \div (m_1+m_2)\big)$. Since $R > 1$, $v'(x_3) > v'\big(R(y-x_3)/(m_1+m_2)\big)$. Under the assumption that the traders' relative risk aversion is greater than 1, the first-order condition also implies that $v'(x_3) < v'\big((y-x_3)/(m_1+m_2)\big)$.[8] Since $v'' < 0$, we have the following result:

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The following bounds apply to $x_3(m_1,m_2,0)$:

$$(8) \qquad y/(m_1+m_2+1) < x_3(m_1,m_2,0) < Ry/(m_1+m_2+1).$$

An immediate implication of the lemma is that the trader who is the last to arrive at the bank never wants to lie about his or her type. If the trader is impatient, he or she surely does not want to claim to be patient because the trader does not value date 1 consumption. If the trader is patient, then he or she will receive more for telling the truth, $Ry/(m_1+m_2+1)$, than for lying, $x_3(m_1,m_2,0)$.

From the utility function $v(c) = c^{1-\gamma}/(1-\gamma)$, it is easily derived that

$$(9) \qquad x_3(m_1,m_2,0) = y(m_1,m_2)/[1 + (m_1+m_2)R^{(1/\gamma)-1}].$$

Consumption for the Second Trader

Now consider the trader who is second to arrive at the bank.

The amount of endowment the bank has available now is either I or $I - x_1(0)$, depending on whether or not the first trader is patient. If the second trader is patient, then at date 1 that trader will receive his or her share of R times the amount of the endowment not distributed to impatient traders, equally divided among all the patient traders. Otherwise, the bank must assign the trader date 0 consumption immediately. Below, we derive the optimal consumption for the second trader when that trader is of type 0, $x_2(m_1,0)$.

□ With a Patient First Trader

Suppose that the first-arriving trader claims to be patient ($m_1 = 1$). In determining the value of $x_2(1,0)$, the bank must take into consideration the possible type of the third trader, who is yet to arrive, as well as the first trader, who has been waiting to consume at date 1. Suppose that $x_2(1,0)$ is given to the second trader. With probability p, the third trader is patient; then that trader and the first each get $R[I - x_2(1,0)]/2$ at date 1. With probability $1 - p$, the third trader is impatient; then that trader receives at date 0 the amount $x_3(1,0,0)$, as determined optimally in equation (7), and the first trader receives the amount of $x_1(1,0,0) = R[I - x_2(1,0) - x_3(1,0,0)]$ at date 1. Given these probabilities, the bank chooses $x_2(1,0)$ to maximize the total utility of all three traders:

$$(10) \qquad \max_{x_2(1,0)} v(x_2(1,0)) + 2pv\big(R[I - x_2(1,0)]/2\big) + (1-p)\big[v(x_3(1,0,0)) + v(x_1(1,0,0))\big].$$

The first-order condition for this maximization problem, after the envelope theorem is applied, is

$$(11) \qquad v'(x_2(1,0)) = Rv'(x_1(1,0,0)) - pR\big(v'(x_1(1,0,0)) - v'(R[I - x_2(1,0)]/2)\big).$$

For $v(c) = c^{1-\gamma}/(1-\gamma)$, the solution of equation (11) can be found, via (9), to be

$$(12) \qquad x_2(1,0) = I/(1+A^{1/\gamma})$$

[7]If all traders claim to be impatient ($m_1 + m_2 + m_3 = 0$), then the third trader just consumes the endowment available, so $x_3(0,0,0) = I - y(0,0)$.

[8]To see this, let $\Phi(R) \equiv Rv'(Rc)$. Then $\Phi' = Rcv''(Rc) + v'(Rc)$, which is negative, since $cv''(c)/v'(c) \le -1$ for all c. Thus, $\Phi(R) < \Phi(1)$; that is, $Rv'(Rc) < v'(c)$ for all c and for $R > 1$.

where

$$(13) \quad A \equiv (1-p)(1 + R^{(1/\gamma)-1})^{\gamma} + p2^{\gamma}R^{1-\gamma}.$$

□ *With an Impatient First Trader*

Now suppose that the first trader to arrive at the bank is impatient, so that the amount of the endowment available to the bank when the second trader arrives is $I - x_1(0)$.

In deciding on $x_2(0,0)$, the consumption to be given to the second-arriving trader when that trader is type 0, the bank maximizes the sum of the expected utility of the second and third traders:

$$(14) \quad V_{00}(x_1(0)) \equiv \max_{x_2(0,0)} v(x_2(0,0)) + (1-p)v(x_3) + pv(Rx_3)$$

where $x_3 \equiv I - x_1(0) - x_2(0,0)$. With $x_1(0)$ given to the first trader and $x_2(0,0)$ to the second, the consumption for the third trader is $I - x_1(0) - x_2(0,0)$ if that trader is impatient and $R[I - x_1(0) - x_2(0,0)]$ if the trader is patient. The first-order condition for $x_2(0,0)$ is, thus,

$$(15) \quad v'(x_2) = (1-p)v'(I - x_1(0) - x_2) + pRv'(R[I - x_1(0) - x_2])$$

which, for $v(c) = c^{1-\gamma}/(1-\gamma)$, has the following solution:

$$(16) \quad x_2(0,0) = [I - x_1(0)]/(1+B)$$

where

$$(17) \quad B \equiv [1 - p + pR^{1-\gamma}]^{1/\gamma}.$$

What is the optimal consumption for the second-arriving trader if that trader is patient? With probability p, the third trader is patient; then both the second and third traders receive $R[I - x_1(0)]/2$ at date 1. With probability $1 - p$, the third trader is impatient; then the third trader's optimal date 0 consumption is $x_3(0,1,0)$ and the second trader receives $R[I - x_1(0) - x_3(0,1,0)]$ at date 1. Thus, the total utility of the second and third traders, conditional on the second being patient (and the first being impatient), is

$$(18) \quad V_{01}(x_1(0)) \equiv 2pv(R[I - x_1(0)]/2) + (1-p)\{v(x_3(0,1,0)) + v(R[I - x_1(0) - x_3(0,1,0)])\}.$$

Consumption for the First Trader

Now consider the trader who is the first to arrive at the bank.

If this trader is impatient, then the bank chooses $x_1(0)$ to maximize the sum of all three traders' expected utilities:

$$(19) \quad \max v(x_1(0)) + (1-p)V_{00}(x_1(0)) + pV_{01}(x_1(0)).$$

Use $v(c) = c^{1-\gamma}/(1-\gamma)$ and (9) to write the first-order condition for this optimization problem as

$$(20) \quad x_1(0) = (1-p)^2([I - x_1(0)]/(1+B^{-1}))^{-\gamma} + (1-p)pR(R[I - x_1(0)]/(1+B^{-1}))^{-\gamma} + p(1-p)R(R[I - x_1(0)]/(1+R^{1-(1/\gamma)}))^{-\gamma} + p^2R(R[I - x_1(0)]/2)^{-\gamma}.$$

This yields the following solution:

$$(21) \quad x_1(0) = I/\{1 + [pA + (1-p)(1+B)^{\gamma}]^{1/\gamma}\}.$$

Since both A and B decrease in p, $x_1(0)$ is an increasing function of p. The intuition for this is as follows. As p rises, the traders who arrive after the first trader are more likely to be patient. Since the consumption of these patient traders can be supported by the R transformation technology, $x_1(0)$ should increase accordingly in order to balance the marginal utility of the current impatient trader with that of these later arrivals. Similarly, the optimal consumption for the second trader derived earlier, $x_2(1,0)$, also increases with p. These properties will be used later, in the proof of Proposition 1.

The Symmetric Efficient Allocation

The optimal consumptions of traders in every state of nature have been derived. The banking arrangement (the *mechanism*) must distribute the resources according to these derived consumptions and the traders' reported types. Now we show that with the specified mechanism, traders in this model will truthfully report their types and that this truthful communication constitutes the unique perfect Bayesian Nash equilibrium of the mechanism. To prove this, we use standard backward induction reasoning.

First, consider the trader who arrives at the bank last. According to the lemma, this third trader always prefers to tell the truth regardless of the messages sent by previously arriving traders.

Next, consider the trader who arrives at the bank second. Since an impatient trader never claims to be patient, we only need to consider what happens when the second trader is patient. Before this trader's arrival, the available endowment is either $I - x_1(0)$ or I, depending on whether or not the first trader is impatient. Suppose the first trader is impatient. Can the second trader benefit from lying, claiming to be impatient too? If the trader tells the truth, he or she receives $x_2(0,1,m_3)$ at date 1, the value of which depends on the reported type of the third trader. If the second trader chooses to lie, the trader receives $x_2(0,0)$ right away, at the time of arrival. Thus, the trader will choose to tell the truth if and only if

$$(22) \quad v(x_2(0,0)) \leq E[v(x_2(0,1,m_3))]$$

where

$$(23) \quad E[v(x_2(0,1,m_3))] = (1-p)v\big(R[I - x_1(0) - x_3(0,1,0)]\big) + pv\big(R[I - x_1(0)]/2\big).$$

Similarly, if the first trader to arrive is patient, then the (patient) second trader will tell the truth if and only if

$$(24) \quad v(x_2(1,0)) \leq E[v(x_2(1,1,m_3))]$$

where

$$(25) \quad E[v(x_2(1,1,m_3))] = (1-p)v\big(R[I - x_3(1,1,0)]/2\big) + pv(R).$$

Note that, since the third trader never lies about his or her type, we can use the objective probability of becoming a patient trader, p, to evaluate the expected utilities in (22) and (25).

Now consider the first trader to arrive at the bank. This trader is patient and receives $x_1(0)$ for lying and $x_1(1,m_2,m_3)$ otherwise. Truth-telling thus requires that

$$(26) \quad v(x_1(0)) < E[v(x_1(1,m_2,m_3))]$$

where

$$(27) \quad E[v(x_1(1,m_2,m_3))] = (1-p)^2 v\big(R[I - x_2(1,0) - x_3(1,0,0)]\big) + p^2 v(R) + (1-p)p\Big(v\big(R[I - x_2(1,0)]/2\big) + v\big(R[I - x_3(1,1,0)]/2\big)\Big).$$

The following result, proved in the Appendix, shows that both the second and first traders strictly prefer to tell the truth about their types because they anticipate truthful communication by the third trader. Thus, truthful reporting by all traders is the only equilibrium outcome that results from backward induction. Hence, it is the unique perfect Bayesian Nash equilibrium of the mechanism.

PROPOSITION 1. *Assume that* $v(c) = c^{1-\gamma}/(1-\gamma)$ *for* $\gamma > 1$. *Then incentive conditions* (22), (25), *and* (26) *hold for all* $p \in [0,1]$ *and for* $R > 1$.

Finally, we present the following result, which corresponds to a partial suspension of payments scheme under the optimal banking arrangement in our model. (See Wallace 1990.)

PROPOSITION 2. *For all* $p \in [0,1]$, $R > 1$, *and* $\gamma > 1$, *there is* $x_1(0) > x_2(0,0) > x_3(0,0,0)$.

Proof. From the expressions of $x_1(0)$ and $x_2(0,0)$ derived earlier, we know that

$$(28) \quad x_2(0,0) = [I - x_1(0)]/(1+B) = \{[pA + (1-p)(1+B)^\gamma]^{1/\gamma}/(1+B)\}x_1(0).$$

Thus, $x_1(0) > x_2(0,0)$ is equivalent to $pA + (1-p)(1+B)^\gamma < (1+B)^\gamma$. The latter inequality always holds; it is straightforward to show that $A < (1+B)^\gamma$ for all p. Also, since $B < 1$, we know that $x_2(0,0) > [I - x_1(0)]/2$. Thus, $x_2(0,0) > I - x_1(0) - x_2(0,0)$; that is, $x_2(0,0) > x_3(0,0,0)$.

Q.E.D.

According to Proposition 2, if all traders demand early withdrawal in our model, then the traders who arrive at the bank earlier receive more consumption than those who arrive later. (This property also holds for other paths of realizations of types, such as those in which two of the three traders claim to be impatient.) Note that such a partial suspension of payments occurs with positive probability in our model, although a bank run never occurs.

Conclusion

In a finite-trader version of the model of Diamond and Dybvig (1983; reprinted in this issue), we have shown that the ex ante efficient allocation can be implemented as a unique equilibrium. In the mechanism we have considered, truth-telling is the strictly dominant strategy for all traders in the environment with as well as without sequential service. All traders prefer truth-telling even when there is a sequential service constraint because they expect (correctly) that those who arrive at the bank later will tell the truth. Therefore, in our model, unlike in Diamond and Dybvig's, there is no bank run equilibrium. (These results also hold in more general settings with I traders and general utility functions, as we have shown in Green and Lin 1999.)

Diamond and Dybvig interpret their model as an explanation of the numerous observed bank runs in U.S. history. We are not claiming, of course, that bank runs do not actually occur. Rather, we are simply trying to show that within the basic framework of Diamond and Dybvig—even with the sequential service constraint—an arrangement exists that implements the efficient allocation without leading to bank runs. In the model, rational agents have no reason to bypass this optimal arrangement and instead choose another that might produce bank runs in equilibrium. Therefore, we think that something essential has been neglected in the basic Diamond and Dybvig environment in order to have a theory that matches U.S. history.

Our model does not capture all of the key features of an actual banking system either. One obviously missing feature is the banking system's ongoing nature. If the population of an economy has an overlapping-generations structure, then no trader is the last to arrive at the bank, so our backward induction argument may not work. The same problem arises if the size of the population is not observable to individual traders, so that no one is certain whether he or she is the last one in line. When these features are present, the validity of our no bank run result needs to be reconsidered. Finally, also absent in our model are the incentive problems among bank officials to manage resources in a way that maximizes the utility of their depositors. Such incentive problems can result from incomplete information.[9] The banking contract in our model abstracts from these incentive problems, which might be a reason such a contract is not commonly observed. All of these missing features are worth investigating as we try to improve our understanding of banking instability.

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[9]These problems have been used by Douglas Diamond (1984) and Stephen Williamson (1987) to argue for the efficiency of standard debt contracts. A synthesis of their models with Diamond and Dybvig's might produce a bank run equilibrium under an efficient contract.

Appendix
Proof of Proposition 1

Here we prove the preceding text's Proposition 1, that both the first and second traders to arrive at the bank prefer to tell the truth about their types because they expect the third trader to do so.

PROPOSITION 1. *Assume that* $v(c) = c^{1-\gamma}/(1-\gamma)$ *for* $\gamma > 1$. *Then incentive conditions* (22), (25), *and* (26) *hold for all* $p \in [0,1]$ *and for* $R > 1$.

Proof. Note that for $v(c) = c^{1-\gamma}/(1-\gamma)$, $v'(c) = \Psi(v(c))$ for all c, where $\Psi(x) = [(1-\gamma)x]^{\gamma/(\gamma-1)}$ for $\Psi' < 0$ and $\Psi'' > 0$.

We first prove that (22) holds. By (16), $I - x_1(0) - x_2(0,0) = R[I - x_1(0)]/(1+B^{-1}) < R[I - x_1(0)]/2$ because $B < 1$. Yet the first-order condition for $x_3(0,1,0)$ implies that $x_3(0,1,0) = [I - x_1(0)]/(1+R^{(1/\gamma)-1})$. Thus,

$$\text{(A1)}\quad R[I - x_1(0) - x_3(0,1,0)] = R[I - x_1(0)]/(R^{1-(1/\gamma)}+1) > [I - x_1(0)]/2 > [I - x_1(0)]/(1+B^{-1}).$$

From these and the first-order condition for $x_2(0,0)$, equation (15), we get that

$$\text{(A2)}\quad v'(x_2(0,0)) > (1-p)v'(I - x_1(0) - x_2(0,0)) + pv'(R[I - x_1(0) - x_2(0,0)]).$$

Using $v'' < 0$ and the above derivations, we have that

$$\text{(A3)}\quad v'(x_2(0,0)) > (1-p)v'(R[I - x_1(0) - x_3(0,1,0)]) + pv'(R[I - x_1(0)]/2).$$

Rewriting the above inequality in terms of function Ψ yields that

$$\text{(A4)}\quad \Psi\big(v(x_2(0,0))\big) > (1-p)\Psi\big(v(R[I - x_1(0) - x_3(0,1,0)])\big) + p\Psi\big(v(R[I - x_1(0)]/2)\big).$$

Since Ψ is convex, we have that

$$\text{(A5)}\quad \Psi\big(v(x_2(0,0))\big) > \Psi\big((1-p)v(R[I - x_1(0) - x_3(0,1,0)]) + pv(R[I - x_1(0)]/2)\big)$$

which, along with $\Psi' < 0$, implies that

$$\text{(A6)}\quad v(x_2(0,0)) < (1-p)v(R[I - x_1(0) - x_3(0,1,0)]) + pv(R[I - x_1(0)]/2).$$

This proves that (22) holds.

By (12), we have that $x_2(1,0) = I/(2 + R^{(1/\gamma)-1}) > I/3$ at $p = 0$. Since $x_2(1,0)$ increases with p, we have that $R[I - x_2(1,0)]/2 < RI/3 = R$ for all p. Also note that $R^{(1/\gamma)-1} < 1 < R^{1-(1/\gamma)}$. Thus, $x_2(1,0) > I/(2 + R^{1-(1/\gamma)})$ for all $p > 0$. However, from the expressions for $x_3(1,0,0)$ and $x_3(1,1,0)$, we have that

$$\text{(A7)}\quad I - x_2(1,0) - x_3(1,0,0) = R^{(1/\gamma)-1}[I - x_2(1,0)]/(1 + R^{(1/\gamma)-1})$$

and

$$\text{(A8)}\quad R[I - x_3(1,1,0)]/2 = R^{(1/\gamma)-1}I/(1 + 2R^{(1/\gamma)-1}).$$

Since $x_2(1,0) > I/(2 + R^{1-(1/\gamma)})$, we know that

$$\text{(A9)}\quad I - x_2(1,0) - x_3(1,0,0) < I/(2 + R^{1-(1/\gamma)}) = R[I - x_3(1,1,0)]/2.$$

Using the first-order condition for $x_2(1,0)$, we find that

$$\text{(A10)}\quad v'(x_2(1,0)) > (1-p)v'(R[I - x_2(1,0) - x_3(1,0,0)]) + pv'(R[I - x_2(1,0)]/2).$$

The above derivations, along with $v'' < 0$, imply that

$$\text{(A11)}\quad v'(x_2(1,0)) > (1-p)v'(R[I - x_3(1,1,0)]/2) + pv'(R).$$

Hence,

$$\text{(A12)}\quad \Psi\big(v(x_2(1,0))\big) > (1-p)\Psi\big(v(R[I - x_3(1,1,0)]/2)\big) + p\Psi(v(R)) > \Psi\big((1-p)v(R[I - x_3(1,1,0)]/2) + pv(R)\big).$$

Since $\Psi' < 0$,

$$\text{(A13)}\quad (1-p)v(R[I - x_3(1,1,0)]/2) + pv(R) > v(x_2(1,0)).$$

Therefore, (25) holds.

The incentive compatibility condition (26) for the first trader to arrive at the bank can be similarly proved to hold by using the convexity of Ψ. The proof is lengthy and thus omitted here. Q.E.D.

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