Bank Runs: The Pre-Deposit Game

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Introduction to Bank Runs

- Bryant (1980) and Diamond and Dybvig (1983): "bank runs" in the *post-deposit* game
  - multiple equilibria in the *post-deposit* game
- One cannot understand bank runs or the optimal contract without the full *pre-deposit* game
- Peck and Shell (2003): A *sunspot-driven* run can be an equilibrium in the *pre-deposit* game for sufficiently small run probability.
- We show *how* sunspot-driven run risk affects the optimal contract depending on the parameters.

The Model: Consumers

- 2 ex-ante identical vNM consumers and 3 periods: 0, 1 and 2.
- Endowments: y
- Preferences: $u(c^1)$ and $v(c^1 + c^2)$:
  - impatient: $u(x) = A\frac{(x)^{1-b}}{1-b}$, where $A > 0$ and $b > 1$.
  - patient: $v(x) = \frac{(x)^{1-b}}{1-b}$.
- Types are uncorrelated (so we have aggregate uncertainty.): p

The Model: Technology

- Storage:

$t=0$	$t=1$	$t=2$
-1	1	
	-1	1

- More Productive

$t=0$	$t=1$	$t=2$
-1	0	R

The Model

- Sequential service constraint (Wallace (1988))
- Suspension of convertibility.
- A depositor visits the bank only when he makes withdrawals.
- When a depositor makes his withdrawal decision, he does not know his position in the bank queue.
- If more than one depositor chooses to withdraw, a depositor's position in the queue is random. Positions in the queue are equally probable.
- Aggregate uncertainty

Post-Deposit Game: Notation

- $c \in [0, 2y]$ is any feasible banking contract
- $\widehat{c} \in [0, 2y]$ is the unconstrained optimal banking contract
- $c^* \in [0, 2y]$ is the constrained optimal banking contract
- Smaller c is conservative; larger c is fragile

Post-Deposit Game: c^{early}

- A patient depositor chooses early withdrawal when he expects the other depositor to also choose early withdrawal.

$$[v(c) + v(2y - c)]/2 > v[(2y - c)R]$$

- Let c^{early} be the value of c such that the above inequality holds as an equality.

Post-Deposit Game: c^{wait}

- A patient depositor chooses late withdrawal when he expects the other depositor, if patient, to also choose late withdrawal. (ICC)

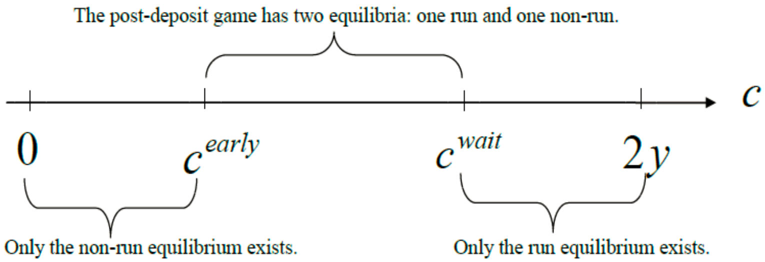
$$pv[(2y-c)R]+(1-p)v(yR) \geq p[v(c)+v(2y-c)]/2+(1-p)v(c).$$

- Let c^{wait} be the value of c such that the above inequality holds as an equality.

Post-Deposit Game: "usual" values of the parameters

- $c^{early} < c^{wait}$ if and only if

$$b < \min\{2, 1 + \ln 2 / \ln R\}$$



Post-Deposit Game: “usual” values of the parameters

- We call these values of b and R “usual” since the set of DSIC contracts (i.e, $[0, c^{wait}]$) is a strict subset of BIC contracts (i.e, $[0, c^{early}]$).
- The interval $(c^{early}, c^{wait}]$ is the region of c for which the patient depositors' withdrawal decisions exhibit *strategic complementarity*.

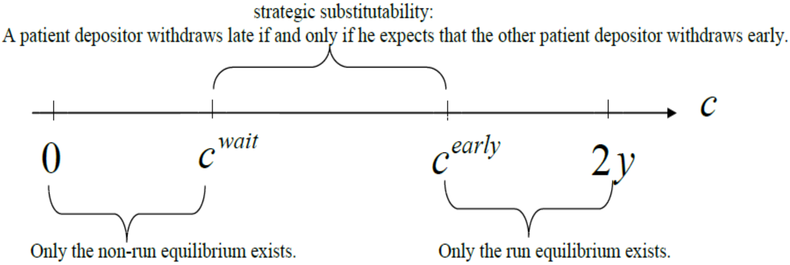
Post-Deposit Game: “unusual” values of the parameters

- The values of b and R are “unusual” when the set of DSIC contracts is the same as the set of BIC contracts.
- According to the Revelation Principle, when we search for the optimal contract we only have to focus on the BIC contracts.
- Hence, for the “unusual” parameters, the optimal contract must be DSIC and the bank runs are not relevant.

Post-Deposit Game: "unusual" values of the parameters

- The "unusual" values of b and R can cause $c^{early} \geq c^{wait}$.
- $(c^{wait}, c^{early}]$ is the region of c for which the patient depositors' withdrawal decisions exhibit *strategic substitutability.*

Figure 8. Equilibrium in the Post-Deposit Game



- For the optimal contract, the only relevant region is $[0, c^{wait}]$ (i.e., BIC contracts).

Pre-Deposit Game

- For the rest of the presentation, we focus on the "usual" values of b and R.
- Whether bank runs occur in the *pre-deposit* game depends on whether the optimal contract c^* belongs to the region of *strategic complementarity (i.e., $c \in (c^{early}, c^{wait}]$)*.
- To characterize the optimal contract, we divide the problem into three cases depending on $\widehat{c}$, the contract supporting the *unconstrained efficient allocation*.
  - $\widehat{c} \leq c^{early}$ (Case 1)
  - $\widehat{c} \in (c^{early}, c^{wait}]$ (Case 2)
  - $\widehat{c} > c^{wait}$ (Case 3)

Impulse parameter A and the 3 cases

- $\widehat{c}$ is the c in $[0, 2y]$ that maximizes

$$\widehat{W}(c) = \{p^2[u(c) + u(2y - c)] + 2p(1 - p)[u(c) + v((2y - c)R)] + 2(1 - p)^2 v(yR)\}.$$

-

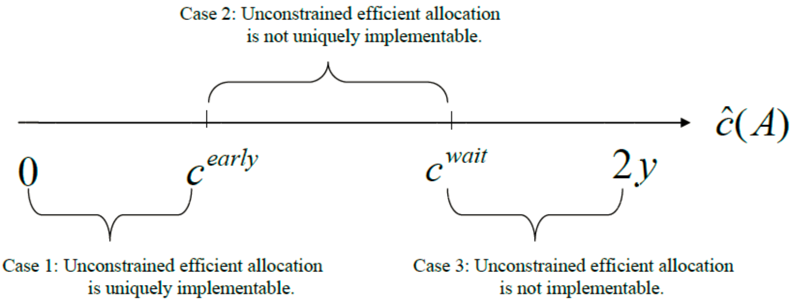
$$\widehat{c} = \frac{2y}{\{p/(2 - p) + 2(1 - p)/[(2 - p)AR^{b-1}]\}^{1/b} + 1}.$$

- $\widehat{c}(A)$ is an increasing function of A.

Parameter A and the 3 Cases

- Neither c^{early} nor c^{wait} depends on A

Figure 2. Three Cases



Example

- The parameters are

$$b = 1.01; p = 0.5; y = 3; R = 1.5$$

- We see that b and R satisfy the condition which makes the set of contracts permiting strategic complementarity non-empty. We have that $c^{early} = 4.155955$ and $c^{wait} = 4.280878$.
- $A^{early} = 6.217686$ and $A^{wait} = 10.27799$.
- If $A \leq A^{early}$, we are in Case 1; If $A^{early} < A \leq A^{wait}$, we are in Case 2; If $A > A^{wait}$, we are in Case 3.

The Optimal Contract: Case 1

- Case 1: The *unconstrained efficient allocation* is DSIC, i.e., $\widehat{c} \leq c^{early}$.
- It is straightforward to see that the optimal contract for the *pre-deposit* game supports the *unconstrained efficient allocation*

$$c^*(s) = \widehat{c}.$$

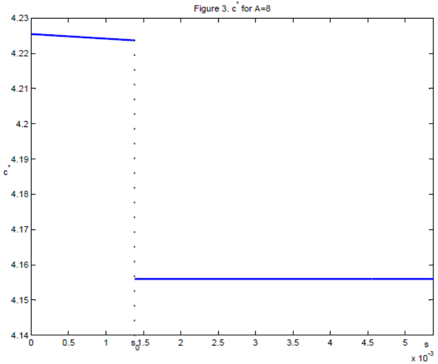
and that the optimal contract doesn't tolerate runs.

The Optimal Contract: Case 2

- Case 2: The *unconstrained efficient allocation* is BIC but not DSIC, i.e., $c^{early} < \widehat{c} \leq c^{wait}$.
- The optimal contract $c^*(s)$ satisfies: (1) if s is larger than the threshold probability s_0, the optimal contract is run-proof and $c^*(s) = c^{early}$. (2) if s is smaller than s_0, the optimal contract $c^*(s)$ tolerates runs and it is a strictly decreasing function of s.

The Optimal Contract: Case 2

- Using the same parameters as the previous example. Let $A = 8$. (We have seen that we are in Case 2 if $6.217686 < A \leq 10.27799$.)
- c^* switches to the best run-proof contract (i.e. c^{early}) when $s > s_0 = 1.382358 \times 10^{-3}$.

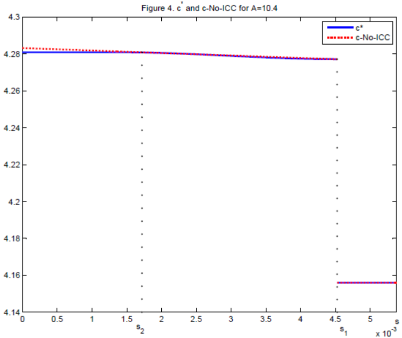


The Optimal Contract: Case 3

- Case 3: The *unconstrained efficient allocation* is not BIC, i.e., $c^{wait} < \widehat{c}$.
- The optimal contract $c^*(s)$ satisfies: (1) If s is larger than the threshold probability s_1, we have $c^*(s) = c^{early}$ and the optimal contract is run-proof. (2) If s is smaller than s_1, the optimal contract $c^*(s)$ tolerates runs and it is a weakly decreasing function of s. Furthermore, we have $c^*(s) = c^{wait}$ for at least part of the run tolerating range of s.

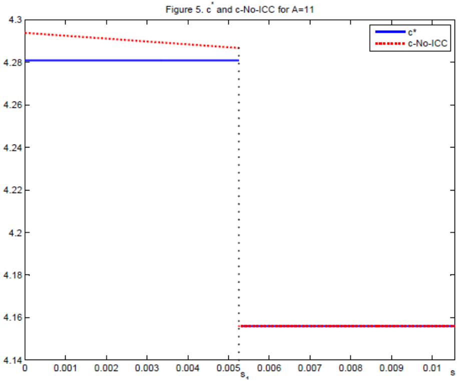
The Optimal Contract: Case 3

- Using the same parameters as in the previous example. Let $A = 10.4$. (We have seen that we are in Case 2 if $A > 10.27799$.)
- c^* switches to the best run-proof (i.e. c^{early}) when $s > 4.524181 \times 10^{-3}$.
- ICC becomes non-binding when $s \geq s_2 = 1.719643 \times 10^{-3}$.



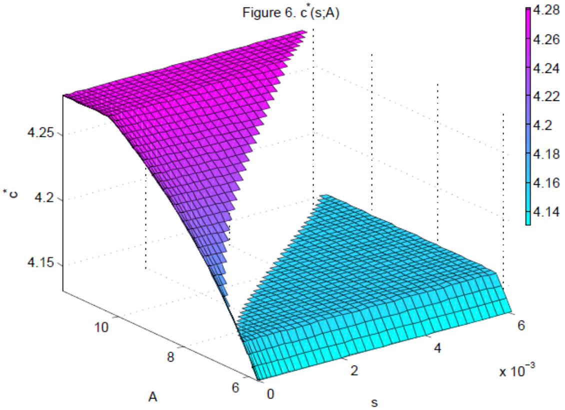
The Optimal Contract: Case 3

- Let $A = 11$. (PS case)
- c^* switches to the best run-proof (i.e. c^{early}) when $s > s_1 = 5.281242 \times 10^{-3}$.



The Optimal Contract

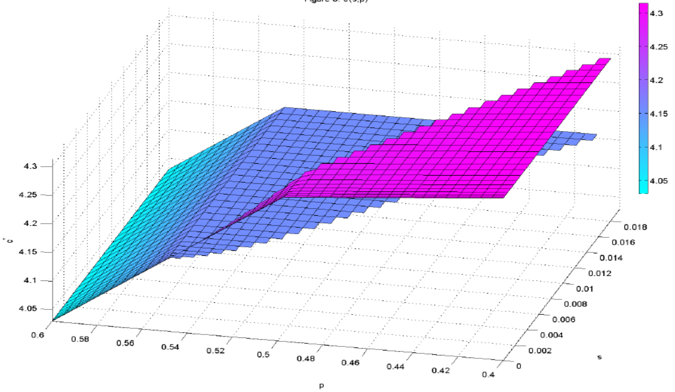
- c^* versus s and A



Probability of Impatience: p

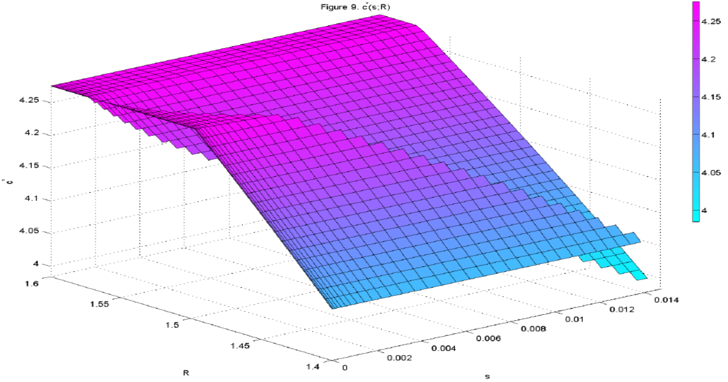
- $b = 1.01$, $A = 10$, $y = 3$, $R = 1.5$. If $p \geq 0.548823$, the optimal contract does not tolerate runs, $c^*(s) = \widehat{c}$. If $p \in [0.497423, 0.548823)$, then c^* is strictly decreasing in s until it levels off to $c^{early} = 4.155955$. If $p < 0.497423$, then the ICC binds when s is small.

Figure 8. $c^*(s;p)$



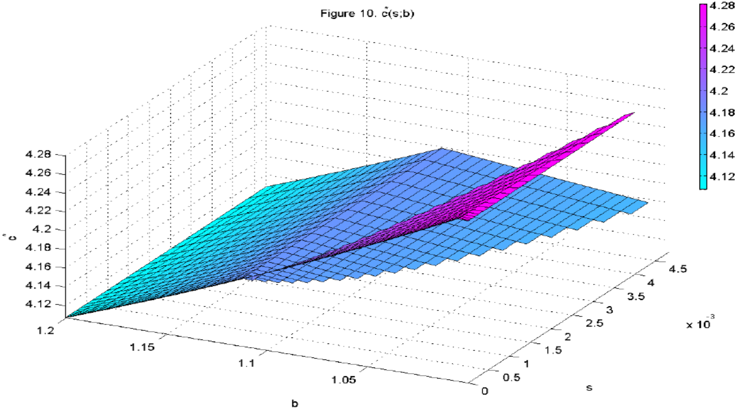
Return R

- $b = 1.01$, $A = 10$, $y = 3$, $p = 0.5$. If $R \geq 1.572948$, the optimal contract does not tolerate runs, $c^*(s) = \widehat{c}$. If $R \in [1.497374, 1.572948)$, $c^*(s)$ is strictly decreasing in s until it levels off to c^{early}. $c^{early}(R)$ is increasing in R. If $R < 1.497374$, then the ICC binds when s is small.



Risk-aversion b

- $A = 10$, $y = 3$, $p = 0.5$, $R = 1.5$. If $b \geq 1.112528$, the optimal contract does not tolerate runs, $c^*(s) = \widehat{c}$. $\widehat{c}$ depends on b. If $b \in [1.00524, 1.112528)$, then $c^*(s)$ is strictly decreasing in s until it levels off to c^{early}. If $b < 1.00524$, then the ICC binds when s is small.



Summary and Concluding Remark

- The general form of the optimal contract to the *pre-deposit* game is analyzed.
- The *unconstrained efficient allocation* falls into one of the three cases:
  - (1) DSIC
  - (2) BIC but not DSIC
  - (3) not BIC.

Summary and Concluding Remark

- In Cases 2 and 3, the optimal contract tolerates runs when the run probability is sufficiently small:
- In Case 2, the optimal contract adjusts continuously and becomes strictly more conservative as the run probabilities increases.

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    - The optimal allocation is never a mere randomization over the *unconstrained efficient allocation* and the corresponding run allocation from the *post-deposit* game. Hence this is also a contribution to the sunspots literature: another case in which SSE allocations are not mere randomizations over certainty allocations.

Summary and Concluding Remark

- In Case 3, the ICC binds for small run-probabilities, which forces the contract to be more conservative than it would have been without the ICC. Hence, for Case 3, the optimal contract does not change with s until the ICC no longer binds.

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    - For small s, the optimal allocation is a randomization over the *constrained efficient allocation* and the corresponding run allocation from the *post-deposit* game.