Bank Runs: The Pre-Deposit Game

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Introduction to Bank Runs

- Bryant (1980) and Diamond and Dybvig (1983): “bank runs” in the *post-deposit* game
  - multiple equilibria in the *post-deposit* game
- One cannot understand bank runs or the optimal contract without the full *pre-deposit* game
- Peck and Shell (2003): A *sunspot-driven* run can be an equilibrium in the *pre-deposit* game for sufficiently small run probability.
- We show how sunspot-driven run risk affects the optimal contract depending on the parameters.
The Model: Consumers

- 2 ex-ante identical vNM consumers and 3 periods: 0, 1 and 2.
- Endowments: $y$
- Preferences: $u(c^1)$ and $v(c^1 + c^2)$:
  - impatient: $u(x) = A \frac{(x)^{1-b}}{1-b}$, where $A > 0$ and $b > 1$.
  - patient: $v(x) = \frac{(x)^{1-b}}{1-b}$.
- Types are uncorrelated (so we have aggregate uncertainty): $p$
The Model: Technology

- Storage:

\[
\begin{array}{ccc}
  t = 0 & t = 1 & t = 2 \\
  -1 & 1 & \\
  -1 & 1 \\
\end{array}
\]

- More Productive

\[
\begin{array}{ccc}
  t = 0 & t = 1 & t = 2 \\
  -1 & 0 & R \\
\end{array}
\]
The Model

- Sequential service constraint (Wallace (1988))
- Suspension of convertibility.
- A depositor visits the bank only when he makes withdrawals.
- When a depositor makes his withdrawal decision, he does not know his position in the bank queue.
- If more than one depositor chooses to withdraw, a depositor’s position in the queue is random. Positions in the queue are equally probable.
- Aggregate uncertainty
Post-Dropit Game: Notation

- $c \in [0, 2y]$ is any feasible banking contract
- $\hat{c} \in [0, 2y]$ is the unconstrained optimal banking contract
- $c^* \in [0, 2y]$ is the constrained optimal banking contract
- Smaller $c$ is conservative; larger $c$ is fragile
Post-Deposit Game: \(c^{\text{early}}\)

- A patient depositor chooses early withdrawal when he expects the other depositor to also choose early withdrawal.

\[
\frac{v(c) + v(2y - c)}{2} > v[(2y - c)R]
\]

- Let \(c^{\text{early}}\) be the value of \(c\) such that the above inequality holds as an equality.
Post-Deposit Game: $c^{\text{wait}}$

- A patient depositor chooses late withdrawal when he expects the other depositor, if patient, to also choose late withdrawal. (ICC)

$$p \nu[(2y - c)R] + (1 - p) \nu(yR) \geq p[\nu(c) + \nu(2y - c)]/2 + (1 - p) \nu(c).$$

- Let $c^{\text{wait}}$ be the value of $c$ such that the above inequality holds as an equality.
Post-Deposit Game: “usual” values of the parameters

- $c^{early} < c^{wait}$ if and only if

$$b < \min\{2, 1 + \ln 2 / \ln R\}$$

The post-deposit game has two equilibria: one run and one non-run.

- Only the non-run equilibrium exists.
- Only the run equilibrium exists.
Post-Deposit Game: “usual” values of the parameters

- We call these values of $b$ and $R$ “usual” since the set of DSIC contracts (i.e, $[0, c^{wait}]$) is a strict subset of BIC contracts (i.e, $[0, c^{early}]$).
- The interval $(c^{early}, c^{wait}]$ is the region of $c$ for which the patient depositors’ withdrawal decisions exhibit \textit{strategic complementarity}. 
Post-Deposit Game: “unusual” values of the parameters

- The values of $b$ and $R$ are “unusual” when the set of DSIC contracts is the same as the set of BIC contracts.
- According to the Revelation Principle, when we search for the optimal contract we only have to focus on the BIC contracts.
- Hence, for the “unusual” parameters, the optimal contract must be DSIC and the bank runs are not relevant.
Post-Deposit Game: “unusual” values of the parameters

- The “unusual” values of $b$ and $R$ can cause $c^{early} \geq c^{wait}$.
- $(c^{wait}, c^{early}]$ is the region of $c$ for which the patient depositors’ withdrawal decisions exhibit strategic substitutability.

Figure 8. Equilibrium in the Post-Detention Game

strategic substitutability:
A patient depositor withdraws late if and only if he expects that the other patient depositor withdraws early.

Only the non-run equilibrium exists.

For the optimal contract, the only relevant region is $[0, c^{wait}]$ (i.e., BIC contracts).
For the rest of the presentation, we focus on the "usual" values of $b$ and $R$.

Whether bank runs occur in the pre-deposit game depends on whether the optimal contract $c^*$ belongs to the region of strategic complementarity (i.e., $c \in (c^{\text{early}}, c^{\text{wait}}]$).

To characterize the optimal contract, we divide the problem into three cases depending on $\hat{c}$, the contract supporting the unconstrained efficient allocation.

- $\hat{c} \leq c^{\text{early}}$ (Case 1)
- $\hat{c} \in (c^{\text{early}}, c^{\text{wait}}]$ (Case 2)
- $\hat{c} > c^{\text{wait}}$ (Case 3)
Impulse parameter $A$ and the 3 cases

- $\hat{c}$ is the $c$ in $[0, 2y]$ that maximizes

\[
\hat{W}(c) = \left\{ p^2 [u(c) + u(2y - c)] + 2p(1 - p)[u(c) + \nu((2y - c)R)] + 2(1 - p)^2 \nu(yR) \right\}.
\]

- $\hat{c} = \frac{2y}{\{p/(2 - p) + 2(1 - p)/[(2 - p)AR^{b-1}]\}^{1/b} + 1}$.

- $\hat{c}(A)$ is an increasing function of $A$. 
Parameter A and the 3 Cases

- Neither $c^{early}$ nor $c^{wait}$ depends on $A$
Example

- The parameters are

\[ b = 1.01; \ p = 0.5; \ y = 3; \ R = 1.5 \]

- We see that \( b \) and \( R \) satisfy the condition which makes the set of contracts permitting strategic complementarity non-empty. We have that \( c_{\text{early}} = 4.155955 \) and \( c_{\text{wait}} = 4.280878 \).

- \( A_{\text{early}} = 6.217686 \) and \( A_{\text{wait}} = 10.27799 \).

- If \( A \leq A_{\text{early}} \), we are in Case 1; If \( A_{\text{early}} < A \leq A_{\text{wait}} \), we are in Case 2; If \( A > A_{\text{wait}} \), we are in Case 3.
The Optimal Contract: Case 1

- Case 1: The *unconstrained efficient allocation* is DSIC, i.e., \( \hat{c} \leq c^{early} \).

- It is straightforward to see that the optimal contract for the *pre-deposit* game supports the *unconstrained efficient allocation*  

\[
c^*(s) = \hat{c}.
\]

and that the optimal contract doesn’t tolerate runs.
Case 2: The *unconstrained efficient allocation* is BIC but not DSIC, i.e., $c^{\text{early}} < \hat{c} \leq c^{\text{wait}}$.

The optimal contract $c^*(s)$ satisfies: (1) if $s$ is larger than the threshold probability $s_0$, the optimal contract is run-proof and $c^*(s) = c^{\text{early}}$. (2) if $s$ is smaller than $s_0$, the optimal contract $c^*(s)$ tolerates runs and it is a strictly decreasing function of $s$. 

The Optimal Contract: Case 2
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- Using the same parameters as the previous example. Let $A = 8$. (We have seen that we are in Case 2 if $6.217686 < A \leq 10.27799$.)
- $c^*$ switches to the best run-proof contract (i.e. $c^{early}$) when $s > s_0 = 1.382358 \times 10^{-3}$.

![Figure 3. $c^*$ for A=8](image-url)
The Optimal Contract: Case 3

- Case 3: The *unconstrained efficient allocation* is not BIC, i.e., $c^{wait} < \hat{c}$.

- The optimal contract $c^*(s)$ satisfies: (1) If $s$ is larger than the threshold probability $s_1$, we have $c^*(s) = c^{early}$ and the optimal contract is run-proof. (2) If $s$ is smaller than $s_1$, the optimal contract $c^*(s)$ tolerates runs and it is a weakly decreasing function of $s$. Furthermore, we have $c^*(s) = c^{wait}$ for at least part of the run tolerating range of $s$. 

The Optimal Contract: Case 3

- Using the same parameters as in the previous example. Let $A = 10.4$. (We have seen that we are in Case 2 if $A > 10.27799$.)
- $c^*$ switches to the best run-proof (i.e. $c^{early}$) when $s > 4.524181 \times 10^{-3}$.
- ICC becomes non-binding when $s \geq s_2 = 1.719643 \times 10^{-3}$.
Let $A = 11$. (PS case)

c* switches to the best run-proof (i.e. $c^{early}$) when $s > s_1 = 5.281242 \times 10^{-3}$. 

![Graph showing $c^*$ and c-No-ICC for A=11](image)
The Optimal Contract

- $c^*$ versus $s$ and $A$
Probability of Impatience: \( p \)

- \( b = 1.01, A = 10, y = 3, R = 1.5. \) If \( p \geq 0.548823 \), the optimal contract does not tolerate runs, \( c^*(s) = \hat{c} \). If \( p \in [0.497423, 0.548823) \), then \( c^* \) is strictly decreasing in \( s \) until it levels off to \( c^{early} = 4.155955 \). If \( p < 0.497423 \), then the ICC binds when \( s \) is small.
Return $R$

- $b = 1.01$, $A = 10$, $y = 3$, $p = 0.5$. If $R \geq 1.572948$, the optimal contract does not tolerate runs, $c^*(s) = \widehat{c}$. If $R \in [1.497374, 1.572948)$, $c^*(s)$ is strictly decreasing in $s$ until it levels off to $c^{early}$. $c^{early}(R)$ is increasing in $R$. If $R < 1.497374$, then the ICC binds when $s$ is small.
Risk-aversion $b$

- $A = 10$, $y = 3$, $p = 0.5$, $R = 1.5$. If $b \geq 1.112528$, the optimal contract does not tolerate runs, $c^*(s) = \hat{c}$. $\hat{c}$ depends on $b$. If $b \in [1.00524, 1.112528)$, then $c^*(s)$ is strictly decreasing in $s$ until it levels off to $c^{early}$. If $b < 1.00524$, then the ICC binds when $s$ is small.
Summary and Concluding Remark

- The general form of the optimal contract to the *pre-deposit* game is analyzed.
- The *unconstrained efficient allocation* falls into one of the three cases:
  - (1) DSIC
  - (2) BIC but not DSIC
  - (3) not BIC.
Summary and Concluding Remark

- In Cases 2 and 3, the optimal contract tolerates runs when the run probability is sufficiently small:
- In Case 2, the optimal contract adjusts continuously and becomes strictly more conservative as the run probabilities increases.
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- In Case 2, the optimal contract adjusts continuously and becomes strictly more conservative as the run probabilities increases.
  - The optimal allocation is never a mere randomization over the unconstrained efficient allocation and the corresponding run allocation from the post-deposit game. Hence this is also a contribution to the sunspots literature: another case in which SSE allocations are not mere randomizations over certainty allocations.
Summary and Concluding Remark

- In Case 3, the ICC binds for small run-probabilities, which forces the contract to be more conservative than it would have been without the ICC. Hence, for Case 3, the optimal contract does not change with $s$ until the ICC no longer binds.
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- For small $s$, the optimal allocation is a randomization over the constrained efficient allocation and the corresponding run allocation from the post-deposit game.