Econ 7310: A Review of Bank Runs and Bailouts

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There are multiple episodes around the world (e.g. US 2008-2009) where government supplying funding to financial intermediaries and other firms was a component of the government’s response to a financial crisis.

Henry Thornton (1802) and Walter Bagehot (1877): it is good public policy for government to lend to firms in a financial crisis.

Bailouts are usually perceived to be a costly manifestation of time inconsistency on the part of the policymakers.
Recent Works

Theoretical:

- Green, "Bailouts", *Economic Quarterly-Volume 96, Number 1-First Quarter*, 2010
Recent Works

Empirical:

Recent Works

Other works:

Diamond-Dybvig Bank Runs Model with Costly Bailouts

The Model

- Three periods $t = 0, 1, 2$
- A continuum of investors indexed by $i \in [0, 1]$.
- Investors’ preferences are given by
  \[
  U(c_1, c_2, g; \omega_i) = u(c_1 + \omega_i c_2) + v(g)
  \]
- In $t = 0$, each investor is endowed with 1 unit of private good.
- In $t = 1$, each investor has probability $\pi$ of being impatient ($\omega_i = 0$), and probability $1 - \pi$ of being patient ($\omega_i = 1$).
- There is a constant returns to scale technology that yields either 1 in $t = 1$ or $R$ in $t = 2$. 
The withdrawals in $t = 1$ follows a sequential service

Investors arrive at a central location in the order based on a pre-determined index $i$

The payment made to an investor can depend only on the information received by the financial intermediaries up to that point
Financial Crises

- Investors condition their actions on a sunspot signal $s \in S$
- $S = \{\alpha, \beta\}$ is the set of possible states with corresponding probabilities $\{1 - q, q\}$
- Investor $i$ chooses a strategy based on her type $\omega_i$ and the state $s$
  \[ y_i(\omega_i, s) \in \{0, 1\} \]
- $y_i = 0$ corresponds to withdrawing early and $y_i = 1$ corresponds to waiting until $t = 2$
Potential Equilibria

- The model always has an equilibrium where

\[ y_i(\omega_i, s) = \omega_i \text{ for all } i \text{ and } s \]

- This is the "good" equilibrium that implements the first-best allocation of resources

- There might also exist other inferior equilibria in which some patient investors run by withdrawing early in some state \( s \)

- Without loss of generality, assume run occurs in state \( \beta \)

**Definition 1:** *The financial system is fragile if there exists an equilibrium strategy profit with \( y_i(1, \beta) = 0 \) for a positive measure of investors.*
Timeline

- Endowments deposited at time $t = 0$
- Investors observe $\omega_i, s$
- Fraction $\theta$ served
- Remaining $t = 1$ withdrawals
- Public good provided
- Taxes collected ($a$)
- Withdrawals begin ($b$)
- $s$ revealed; bailout payments (if any) made ($c$)
- Withdrawals end ($d$)
- Withdrawals at time $t = 2$

Figure 1
Timeline
Consider the following strategy profile for investors:

\[
y_i(\omega_i, \alpha) = \omega_i \text{ for all } i
\]

\[
y_i(\omega_i, \beta) = \begin{cases} 
0 & \text{for } i \leq \theta \\
\omega_i & \text{for } i > \theta 
\end{cases}
\]

Based on this strategy by the investors, the decisions of the policy maker and financial intermediaries can be solved using backward induction.
The Allocation of Remaining Private Consumption

- Let $\psi_s^j$ denote the quantity of resources intermediary $j$ has available for its remaining investors in state $s$ after $\theta$ investors have withdrawn.

- Based on the strategy profile, the intermediary can update the fraction $\hat{\pi}_s$ of the remaining investors who are impatient

$$\hat{\pi}_\alpha \equiv \frac{\pi - \theta}{1 - \theta} \quad \text{or} \quad \hat{\pi}_\beta \equiv \pi$$

- The payments to the remaining investors will be chosen to solve

$$V(\psi_s^j; \hat{\pi}_s) \equiv \max_{c_{1s}^j, c_{2s}^j} \left(1 - \theta\right) \left[\hat{\pi}_s u(c_{1s}^j) + (1 - \hat{\pi}_s) u(c_{2s}^j)\right]$$

$$\text{s.t.} \quad (1 - \theta) \left[\hat{\pi}_s c_{1s}^j + (1 - \hat{\pi}_s) \frac{c_{2s}^j}{R}\right] = \psi_s^j$$

- The first-order condition is

$$u'(c_{1s}^j) = Ru'(c_{2s}^j) = \mu_s^j$$
Bailout Policy

- In state $\beta$, the policy maker has to decide the optimal bailout package $\{b^j\}$

- Let $\sigma(j)$ denote the distribution of investors across intermediaries. The total size of the bailout package is given by

$$b \equiv \int b^j d\sigma(j)$$

- The policy maker will choose the bailout payments to solve

$$\max_{\{b^j\}} \int V(\psi^j_\beta; \hat{\pi}_\beta) d\sigma(j) + v(\tau - b)$$

subject to

$$\psi^j_\beta = 1 - \tau - \theta c^j_1 + b^j \text{ for all } j$$

- The first order condition requires

$$v'(\tau - b) = \mu^j_\beta$$
Bailout Policy

- The solution to this problem must equalize the marginal value of resources $\mu^j_\beta$ across all intermediaries.

- For a given size of the total bailout package $b$ per investor, this entails
  
  \[ b^j = b + \theta(c^j_1 - \bar{c}_j) \text{ for all } j \]

  where
  
  \[ \bar{c}_1 \equiv \int c^j_1 d\sigma(j) \]

- The remaining resources $\psi^j_\beta$ available to intermediary $j$ will only depend on aggregate conditions
  
  \[ \psi^j_\beta = 1 - \tau - \theta \bar{c}_1 + b \]
Distorted Incentives

- Intermediary $j$ will choose payment $c^j_1$ that solves

$$
\max_{c^j_1} \quad \theta u(c^j_1) + (1-q)V(1-\tau - \theta c^j_1; \hat{\pi}_\alpha) + qV(1-\tau - \theta \bar{c}_1 + b; \hat{\pi}_\beta)
$$

- The first-order condition for this problem is

$$
u'(c^j_1) = (1-q)V'(1-\tau - \theta c^j_1; \hat{\pi}_\alpha) = (1-q)\mu^j_\alpha
$$

- Notice that the solution to this problem only depends on $\tau$. The solution can then be written as $c_1(\tau)$.

- Also $c_1 < c_{2\alpha}$ is true as long as

$$
q < \frac{R - 1}{R}
$$
Choosing the Tax Rate

The policy maker will choose the tax rate $\tau$ to solve

$$
\max_{\tau} \quad \theta u(c_1(\tau)) + (1 - q)[V(\psi_\alpha; \hat{\pi}_\alpha)
+ v(g_\alpha)] + q[V(\psi_\beta; \hat{\pi}_\beta) + v(g_\beta)]
$$

s.t. \quad \psi_\alpha = 1 - \tau - \theta c_1(\tau)
\psi_\beta = 1 - \tau \theta ac_1(\tau) + b(\tau)
\quad g_\alpha = \tau
\quad g_\beta = \tau - b(\tau)

The first-order condition for this problem is

$$
\nu'(\tau) = \mu_\alpha + \frac{q}{1 - q} \mu_\beta \theta \frac{dc_1}{d\tau}
$$
Define an economy as \( e \equiv (R, \pi, u, v, \theta, q) \)

Let \( \Phi^B \) denote the set of economies that are fragile under the bailout regime.

**Proposition 1:** The financial system is fragile under the bailouts regime if and only if

\[
    c_1^B \geq c_2^B
\]

**Proposition 2:** For any \( e \in \Phi^B \), we have

\[
    (c_1^B, c_2^B, g_\beta^B) \ll (c_1^\alpha, c_2^\alpha, g_\alpha^B)
\]
Equilibrium and Fragility

Numerical Exercise with

\[ u(c) = \frac{c^{1-\gamma}}{1-\gamma} \quad \text{and} \quad v(g)\delta \frac{g^{1-\gamma}}{1-\gamma} \]

**Figure 2**
The fragile set \( \Phi^B \) under the bailouts regime
Equilibrium Under No-Bailouts Policy

- The decision for (d) is the same as in the policy with bailouts.
- Decision (c) is just

\[ b^j = 0 \text{ for all } j \]
Corrected Incentives

- Under a no-bailouts regime, each intermediary must use its own resources to provide consumption to all of its investors in both states.

- Intermediary $j$ will now choose $c_j^1$ to solve

$$\max_{c_j^1} \theta u(c_j^1) + (1 - q)V(1 - \tau - \tau c_j^1; \hat{\pi}_\alpha) + qV(1 - \tau - \theta c_j^1; \hat{\pi}_\beta)$$

- The first order condition for this problem is

$$u'(c_j^1) = (1 - q)\mu_\alpha + q\mu_\beta$$
Since there is no bailout, the entire amount of tax revenue will go into public good in both states

\[ g_\alpha = g_\beta = \tau \]

The first order condition for the tax problem is

\[ \nu'(\tau) = (1 - q)\mu_\alpha + q\mu_\beta \]
Proposition 3: The financial system is fragile under the no-bailouts regime if and only if $c_1^N \geq c_2^N \beta$ holds.

Proposition 4: $\rho^N < \rho^B$ holds for all $q > 0$, where

$$\rho \equiv \frac{\theta c_1}{1 - \tau}$$

Proposition 5: There exist economies in $\Phi^N$ that are not in $\Phi^B$ and vice versa.
Equilibrium and Fragility

Numerical Exercise with

\[ u(c) = \frac{c^{1-\gamma}}{1-\gamma} \quad \text{and} \quad v(g)\delta \frac{g^{1-\gamma}}{1-\gamma} \]

**Figure 3**
Comparing the sets \( \Phi^B \) and \( \Phi^N \)
Conclusion

- A strict no-bailouts policy cannot achieve an efficient allocation of resources.
- If bailouts is permitted, policy makers should use prudential policy measures to offset the resulting distortion in incentives (e.g. taxing short term liabilities).
Extensions:

- Keister and Mitkov (2017): Shocks on bank assets
- Keister and Narasiman (2016): Stochastic demand for liquidity
There is a continuum of banks indexed by $k \in [0, 1]$

At $t = 1$, $\sigma_k \in \Sigma \equiv \{0, \bar{\sigma}\}$ of the assets by bank $k$ will be revealed to be impaired.

A bank with $\sigma_k = 0$ is said to have sounds fundamental. A bank with $\sigma_k = \bar{\sigma}$ is said to have weak fundamental.
Aggregate Uncertainty

- There are two aggregate states of the economy: *good* and *bad*.
- In the good state, all banks have sound fundamentals.
- In the bad state, a fraction $n \in [0, 1]$ of banks have weak fundamentals. The total losses in the financial system are $n\bar{\sigma}$.
- The probability of the bad state is $q$.
- The ex-ante probability that a given bank’s fundamental will be weak is $qn$. 
Slight modification on sequential service:

- The banks are able to condition payments to all investors on the total demand for early withdrawal.
The Constrained Efficient Allocation

The constrained efficient allocation
\((c_{10}^*, c_{20}^*, c_{1S}^*, c_{2S}^*, c_{1W}^*, c_{2W}^*, b_S^*, b_W^*)\) is chosen to maximize

\[
(1 - q) \left[ \pi u(c_{10}) + (1 - \pi) u(c_{20}) + \nu(\tau) \right] \\
+ q \left[ (1 - n)(\pi u(c_{1S}) + (1 - \pi) u(c_{2S})) + n(\pi u(c_{1W}) + (1 - \pi) u(c_{2W})) \\
+ \nu(\tau - (1 - n)b_S - nb_W) \right]
\]

subject to feasibility constraints

\[
\pi c_{10} + (1 - \pi) \frac{c_{20}}{R} \leq 1 - \tau \\
\pi c_{1S} + (1 - \pi) \frac{c_{2S}}{R} \leq 1 - \tau + b_S \\
\pi c_{1W} + (1 - \pi) \frac{c_{2W}}{R} \leq 1 - \tau + b_W
\]

and restrictions on further taxation

\[
b_S \geq 0 \quad \text{and} \quad b_W \geq 0
\]
The Constrained Efficient Allocation

Proposition 1: The constrained efficient allocation satisfies

\[(c_{10}^*, c_{20}^*) = (c_{1S}^*, c_{2S}^*) \text{ and } b_S^* = 0\]

Proposition 2: The constrained efficient allocation satisfies

\[(c_{1S}^*, c_{2S}^*) \gg (c_{1W}^*, c_{2W}^*) \text{ and } b_W^* > 0\]
Similar to Keister (2016), the bailout amount $b^k_W$ given to a bank with weak fundamental is an increasing function of the payment $c^k_{1W}$ made by the bank.

When banks with weak fundamentals are expecting a bailout from the policy maker, they have an additional incentive to make higher payments $c^k_{1W}$.

This is referred to as ”bailouts crowding out bail-ins”.
Moral Hazard

**Proposition 8:** The equilibrium allocation of resources is never constrained efficient.
Macroprudential Policies

- Restricting early payments
- Increasing the tax rate
- Eliminating bailouts
Keister and Narasiman (2016):  
- The probability of each investor being impatient $\pi$ is stochastic.
  
  $$\pi = \begin{cases} 
  \pi_L & \text{in state } L \\
  \pi_H & \text{in state } H 
  \end{cases}$$

- There are four states $S = \{L_\alpha, L_\beta, H_\alpha, H_\beta\}$.
- The policy maker can monitor a fraction $\sigma \in [0, 1]$ of the payments in $t = 1$. 

Stochastic Demand for Liquidity
Fig. 1. Timeline of events.
Runs and Fragility

**Definition 1:** An economy is weakly fragile if there is an equilibrium in which depositors play strategy profile

\[ y_i(\omega_i, s) = \begin{cases} 
\omega_i & \text{for } s = L, H_\alpha \\
0 & \text{for } s = H_\beta 
\end{cases} \]

**Definition 2:** An economy is strongly fragile if the only equilibrium profile of withdrawal strategies is

\[ y_i(\omega_i, s) = \begin{cases} 
\omega_i & \text{for } s = L \\
0 & \text{for } s = H 
\end{cases} \]

**Definition 3:** An economy is not fragile if the only equilibrium profile of withdrawal strategies is the no-run profile

\[ y_i(\omega_i, s) = \omega_i \text{ for all } s \]
Utility Functions

\[ u(c) = \frac{c^{1-\gamma}}{1 - \gamma} \]

\[ v(g) = \delta \frac{g^{1-\gamma}}{1 - \gamma} \]
Proposition 6: For any $e$ with $\delta > 0$, there exists $\bar{\sigma} < 1$ such that allowing intervention strictly increases equilibrium welfare for all economies $(e, \sigma)$ with $\sigma > \bar{\sigma}$.

Proposition 7: For any economy with $\delta = 0$ and $\sigma < 1$, allowing intervention strictly decreases equilibrium welfare.
Numerical Exercises

An economy that is weakly fragile with no intervention:
\[ R = 1.05, \pi_L = 0.45, \pi_H = 0.55, q_{H\alpha} = q_{H\beta} = 0.02, \gamma = 4 \]
An economy that is strongly fragile with no intervention:
$R = 1.05$, $\pi_L = 0.45$, $\pi_H = 0.65$, $q_{H\alpha} = q_{H\beta} = 0.02$, $\gamma = 4$

![Diagram](image)

**Fig. 3.** An economy that is strongly fragile with no intervention.
Numerical Exercises

An economy that is not fragile with no intervention: 
\[ R = 1.05, \pi_L = 0.45, \pi_H = 0.55, q_{H\alpha} = q_{H\beta} = 0.02, \gamma = 2 \]

**Fig. 4.** An economy that is not fragile with no intervention.
Conclusion

- The model captures the fact that a bank run may be driven by expectations or fundamentals.
- Regardless of the cause of the bank run, there is no definite answer as to which policy regime works better.
- Intervention should be permitted only when prudential regulation and supervision are sufficiently effective.
- In particular, this is when the insurance benefit from bailouts outweighs the resulting incentive distortion.