The Transactions Demand for Cash: An Inventory Theoretic Approach

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September 5, 2017

Introduction

- apply inventory control analysis to the theory of money
- analyze transactions demand for cash in a simple rational framework
- discuss real life implications of the model and its limitations

Model Setup

- an individual will pay out T dollars in a steady stream in the course of a year
- cash is withdrawn in lots of C dollars *evenly* throughout the year
  - cash comes from either borrowing or withdrawing from an investment
  - the individual spends the cash in a steady stream, and withdraws the same amount as soon as it's gone
- there's a fixed "broker's fee" of b dollars for each withdrawal
- interest cost (opportunity cost of holding cash) $= i$ percent/year

Assumptions

- transactions are perfectly foreseen (i.e. T, the value of transactions, is predetermined) and occur in a steady stream
- the need for cash arises from transactional demand and there're no precautionary and speculative demands for cash
- **Optimality condition**: a rational individual only holds cash to satisfy the T payment at minimum cost

Costs of Using Cash

- the individual makes $\frac{T}{C}$ withdrawls in the course of the year
- the average cash holding is $\frac{C}{2}$
- the total costs of using cash to meet the transaction are the sum of "brokers' fees" and interest cost:

$$\frac{bT}{C} + \frac{iC}{2} \tag{1}$$

Cash Demand

- the individual chooses C to minimize the costs of using cash
- solve it by setting the derivative of (1) with respect to C equal to 0:

$$-\frac{bT}{C^2} + \frac{i}{2} = 0$$

- then

$$\boxed{C = \sqrt{\frac{2bT}{i}}} \tag{2}$$

◂ Withholding Cash

A More Complicated Case: Withholding Cash

- the individual receives cash from investment prior to expenditure and has the option of withholding some or all of the cash and simply keeping the cash until it is needed
- once the withheld cash is used up, the case is reduced to the previous simple one
- only need to concern about the T dollars of the cash, since excess cash above T will be invested

A More Complicated Case: Withholding Cash

- let R be withheld cash, and I be dollar invested. Then $R = T - I$.
- R can meet payments for $\frac{T-I}{T}$ fraction of the year, with average cash holding equal $\frac{T-I}{2}$ during the period
- let $b_w + k_w C$ be "brokers' fee" for withdrawing cash and $b_d + k_d I$ be "brokers' fee" for depositing cash
- total cost of withholding R dollars and investing the I dollars

$$\frac{T-I}{T} i \frac{T-I}{2} + b_d + k_d I$$

- total cost of obtaining cash for the rest $\frac{I}{T}$ of the year is

$$\frac{I}{T} i \frac{C}{2} + (b_w + k_w C)\frac{I}{C}$$

A More Complicated Case: Withholding Cash

- setting the derivative of the total costs with respect to I equal to 0:

$$
\begin{aligned}
-\frac{T-I}{T} i + k_d + \frac{Ci}{2T} + \frac{b_w}{C} + k_w &= 0 \\
\Rightarrow R = T - I &= \frac{C}{2} + \frac{b_w T}{Ci} + \frac{T(k_d + k_w)}{i}
\end{aligned}
$$

- use the result from (2) that $C^2 = \frac{2Tb_w}{i}$ gives

$$
\boxed{R = C + T\left(\frac{k_w + k_d}{i}\right)}
$$

Cash demand

Monetary Implications

- demand for cash becomes zero without "brokers' fees" b
- economies of scale in the use of cash: demand for cash, C or R rises less than in proportion with the value of transactions T
- from (2), velocity of money $\frac{T}{C} = \frac{i}{2b}C$, so double stock of cash doubles velocity
- lends support to the Pigou effect:
  wage cuts $\Rightarrow$ lower price level $\Rightarrow$ higher purchasing power
  $\Rightarrow$ higher demand for investments good which drives lower interest rate $\Rightarrow$ higher employment

Model & Reality

- demand for cash depends on b and i (problematic if both are zero)
- payments are spread out evenly through time, but cash might be employed more economically if payments are lumpy but foreseen (e.g. a single payment during the year)
- precautionary cash demand could increase if one expects everyone else to hold as little cash as possible