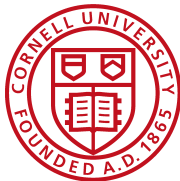


# The Transactions Demand for Cash: An Inventory Theoretic Approach

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# Introduction

- apply inventory control analysis to the theory of money
- analyze transactions demand for cash in a simple rational framework
- discuss real life implications of the model and its limitations

## Model Setup

- an individual will pay out  $T$  dollars in a steady stream in the course of a year
- cash is withdrawn in lots of  $C$  dollars *evenly* throughout the year
  - ▶ cash comes from either borrowing or withdrawing from an investment
  - ▶ the individual spends the cash in a steady stream, and withdraws the same amount as soon as it's gone
- there's a fixed “broker's fee” of  $b$  dollars for each withdrawal
- interest cost (opportunity cost of holding cash)  
=  $i$  percent/year

# Assumptions

- transactions are perfectly foreseen (i.e.  $T$ , the value of transactions, is predetermined) and occur in a steady stream
- the need for cash arises from transactional demand and there're no precautionary and speculative demands for cash
- **Optimality condition:** a rational individual only holds cash to satisfy the  $T$  payment at minimum cost

## Costs of Using Cash

- the individual makes  $\frac{T}{C}$  withdrawals in the course of the year
- the average cash holding is  $\frac{C}{2}$
- the total costs of using cash to meet the transaction are the sum of "brokers' fees" and interest cost:

$$\frac{bT}{C} + \frac{iC}{2} \quad (1)$$

## Cash Demand

- the individual chooses  $C$  to minimize the costs of using cash
- solve it by setting the derivative of (1) with respect to  $C$  equal to 0:

$$-\frac{bT}{C^2} + \frac{i}{2} = 0$$

- then

$$C = \sqrt{\frac{2bT}{i}}$$

(2)

## A More Complicated Case: Withholding Cash

- the individual receives cash from investment prior to expenditure and has the option of withholding some or all of the cash and simply keeping the cash until it is needed
- once the withheld cash is used up, the case is reduced to the previous simple one
- only need to concern about the  $T$  dollars of the cash, since excess cash above  $T$  will be invested

## A More Complicated Case: Withholding Cash

- let  $R$  be withheld cash, and  $I$  be dollar invested. Then  $R = T - I$ .
- $R$  can meet payments for  $\frac{T-I}{T}$  fraction of the year, with average cash holding equal  $\frac{T-I}{2}$  during the period
- let  $b_w + k_w C$  be "brokers' fee" for withdrawing cash and  $b_d + k_d I$  be "brokers' fee" for depositing cash
- total cost of withholding  $R$  dollars and investing the  $I$  dollars

$$\frac{T-I}{T} i \frac{T-I}{2} + b_d + k_d I$$

- total cost of obtaining cash for the rest  $\frac{I}{T}$  of the year is

$$\frac{I}{T} i \frac{C}{2} + (b_w + k_w C) \frac{I}{C}$$



## A More Complicated Case: Withholding Cash

- setting the derivative of the total costs with respect to  $l$  equal to 0:

$$-\frac{T-l}{T}i + k_d + \frac{Ci}{2T} + \frac{b_w}{C} + k_w = 0$$
$$\Rightarrow R = T - l = \frac{C}{2} + \frac{b_w T}{Ci} + \frac{T(k_d + k_w)}{i}$$

- use the result from (2) that  $C^2 = \frac{2Tb_w}{i}$  gives

$$R = C + T \left( \frac{k_w + k_d}{i} \right)$$

## Monetary Implications

- demand for cash becomes zero without “brokers’ fees”  $b$
- economies of scale in the use of cash: demand for cash,  $C$  or  $R$  rises less than in proportion with the value of transactions  $T$
- from (2), velocity of money  $\frac{T}{C} = \frac{i}{2b}C$ , so double stock of cash doubles velocity
- lends support to the Pigou effect:  
wage cuts  $\Rightarrow$  lower price level  $\Rightarrow$  higher purchasing power  
 $\Rightarrow$  higher demand for investments good which drives lower interest rate  $\Rightarrow$  higher employment

## Model & Reality

- demand for cash depends on  $b$  and  $i$  (problematic if both are zero)
- payments are spread out evenly through time, but cash might be employed more economically if payments are lumpy but foreseen (e.g. a single payment during the year)
- precautionary cash demand could increase if one expects everyone else to hold as little cash as possible