The Transactions Demand for Cash: An Inventory Theoretic Approach

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Introduction

• apply inventory control analysis to the theory of money
• analyze transactions demand for cash in a simple rational framework
• discuss real life implications of the model and its limitations
Model Setup

- an individual will pay out $T$ dollars in a steady stream in the course of a year
- cash is withdrawn in lots of $C$ dollars *evenly* throughout the year
  - cash comes from either borrowing or withdrawing from an investment
  - the individual spends the cash in a steady stream, and withdraws the same amount as soon as it’s gone
- there’s a fixed “broker’s fee” of $b$ dollars for each withdrawal
- interest cost (opportunity cost of holding cash) $= i$ percent/year
Assumptions

• transactions are perfectly foreseen (i.e. $T$, the value of transactions, is predetermined) and occur in a steady stream
• the need for cash arises from transactional demand and there’re no precautionary and speculative demands for cash
• **Optimality condition**: a rational individual only holds cash to satisfy the $T$ payment at minimum cost
Costs of Using Cash

- the individual makes $\frac{T}{C}$ withdrawals in the course of the year
- the average cash holding is $\frac{C}{2}$
- the total costs of using cash to meet the transaction are the sum of "brokers' fees" and interest cost:

$$\frac{bT}{C} + \frac{iC}{2}$$  \hspace{1cm} (1)
Cash Demand

- the individual chooses $C$ to minimize the costs of using cash
- solve it by setting the derivative of (1) with respect to $C$ equal to 0:

$$-\frac{bT}{C^2} + \frac{i}{2} = 0$$

- then

$$C = \sqrt{\frac{2bT}{i}} \quad (2)$$
A More Complicated Case: Withholding Cash

- the individual receives cash from investment prior to expenditure and has the option of withholding some or all of the cash and simply keeping the cash until it is needed.
- once the withheld cash is used up, the case is reduced to the previous simple one.
- only need to concern about the \( T \) dollars of the cash, since excess cash above \( T \) will be invested.
A More Complicated Case: Withholding Cash

- let $R$ be withheld cash, and $I$ be dollar invested. Then $R = T - I$.
- $R$ can meet payments for $\frac{T - I}{T}$ fraction of the year, with average cash holding equal $\frac{T - I}{2}$ during the period.
- let $b_w + k_w C$ be ”brokers’ fee” for withdrawing cash and $b_d + k_d I$ be ”brokers’ fee” for depositing cash.
- total cost of withholding $R$ dollars and investing the $I$ dollars

$$\frac{T - I}{T} i \frac{T - I}{2} + b_d + k_d I$$

- total cost of obtaining cash for the rest $\frac{I}{T}$ of the year is

$$\frac{I}{T} i \frac{C}{2} + (b_w + k_w C) \frac{I}{C}$$
A More Complicated Case: Withholding Cash

- setting the derivative of the total costs with respect to \( I \) equal to 0:

\[
\frac{T - I}{T}i + k_d + \frac{Ci}{2T} + \frac{b_w}{C} + k_w = 0
\]

\[\Rightarrow R = T - I = \frac{C}{2} + \frac{b_w T}{Ci} + \frac{T(k_d + k_w)}{i}\]

- use the result from (2) that \( C^2 = \frac{2Tb_w}{i} \) gives

\[R = C + T \left( \frac{k_w + k_d}{i} \right)\]
Monetary Implications

- demand for cash becomes zero without “brokers’ fees” $b$
- economies of scale in the use of cash: demand for cash, $C$ or $R$ rises less than in proportion with the value of transactions $T$
- from (2), velocity of money $\frac{T}{C} = \frac{i}{2b} C$, so double stock of cash doubles velocity
- lends support to the Pigou effect:
  wage cuts $\Rightarrow$ lower price level $\Rightarrow$ higher purchasing power $\Rightarrow$ higher demand for investments good which drives lower interest rate $\Rightarrow$ higher employment
Model & Reality

- Demand for cash depends on $b$ and $i$ (problematic if both are zero).
- Payments are spread out evenly through time, but cash might be employed more economically if payments are lumpy but foreseen (e.g., a single payment during the year).
- Precautionary cash demand could increase if one expects everyone else to hold as little cash as possible.