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The I Theory of Money

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Overview

Motivation: A theory of money needs a place for **financial intermediaries** (inside money creation)

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So...

- They model money supply and demand and the role of intermediaries (I's)
  - Households (HH) manage projects subject to idiosyncratic risk (need for money as an insurance)
  - Intermediaries balance sheets (B/S) are subject to A/L mismatch
  - When I's suffers losses, they shrink their balance sheets (less inside money, fewer HH projects receive financing)

- Value of Money (VoM) = F(state of the financial system)
  1. I's undercapitalized / less inside money (short supply) / high money demand (Money has no idiosyncratic risk) / high VoM
  2. I's well capitalized / more inside money (high money multiplier) / lower money demand / low VoM

- Shocks to end borrowers (HH) hurt I's directly and indirectly
  - spirals / Paradox of Prudence (micro-prudent I's ➔ macro-imprudent effect)

- Normative implications
  - Monetary policy improves welfare but the combination of monetary and macro-prudential policy improves even further.

Related Literature

- Fundamentally different from the New Keynesian and Monetarist approaches
  - New Keynesian (int. rate channel): money as unit account and price/wage rigidities as the main friction (lower nominal rates ➔ lower real rates)
    - Christiano, Moto, Rostagno (2003)
  - I Theory: Money as a store of value and key frictions being financial
  - Monetarism: reduction in money multiplier ➔ disinflation (Friedman and Schwartz, 1963)
  - I Theory: Monetary intervention should recapitalize undercapitalized borrowers rather than simply increase money supply (inside and outside money are not perfect substitutes for the <u>whole</u> economy, just for *individual investors*)
    - By creating inside money I's diversify risks and foster economic growth.
    - I-Theory: money as a store of value (rather than the transaction role)
- Money view related to the "credit view"
  - Gurley and Shaw (1955), Patinkin (1965), Tobin (1969, 1970), Bernanke (1983), Bernanke and Blinder (1988) and Bernanke, Gertler and Gilchrist (1999)
- Macro literature (macro-shocks ➔ I's B/S ➔ Spirals)
  - Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999), Brunnermeier and Sannikov (2014)

Paper Outline

- Baseline Model (without policy intervention)
  - Describe technologies, preferences, financial constraints and asset returns (portfolios)
  - Define and derive the equilibrium
  - Evolution of the state variable that describes prices of capital and money
- Model without Financial Intermediaries (I's)
  - Value and Risk of Money
  - Welfare analysis
- Including Financial Intermediaries (I's)
  - Equilibrium
  - Inefficiencies and welfare analysis
- Monetary and Macro-prudential Policy
  - Introduction of Long-term nominal bonds
  - Removing amplification
  - Economy with perfect sharing of aggregate risk
  - Optimal macro-prudential policy

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Baseline Model

- Technologies:
  - All physical capital in the world allocated between two technologies *a* and *b*.
  - Combined they make $\big(A(\Psi)\big)K_t$, where Ψ is the fraction dedicated to produce good b (CES)

$$A(\psi) = \mathcal{A}\left(\frac{1}{2}\psi^{\frac{s-1}{s}} + \frac{1}{2}(1-\psi)^{\frac{s-1}{s}}\right)^{\frac{s}{s-1}}$$

  - Prices of a and b reflect their marginal contribution to the aggregate good

$$A^a(\psi) = -\psi A'(\psi) + A(\psi) \qquad A^b(\psi) = (1-\psi)A'(\psi) + A(\psi)$$

  - Physical capital is subject to shocks that depend on the technology employed

$$\frac{dk_t}{k_t} = (\underbrace{\Phi(\iota_t)}_{\text{Investment function}} - \delta)\, dt + \underbrace{\sigma^a\, dZ_t^a}_{\text{Sector-wide}} + \underbrace{\tilde{\sigma}^a\, d\tilde{Z}_t}_{\text{Specific}},$$

(Similar equation to technology b)

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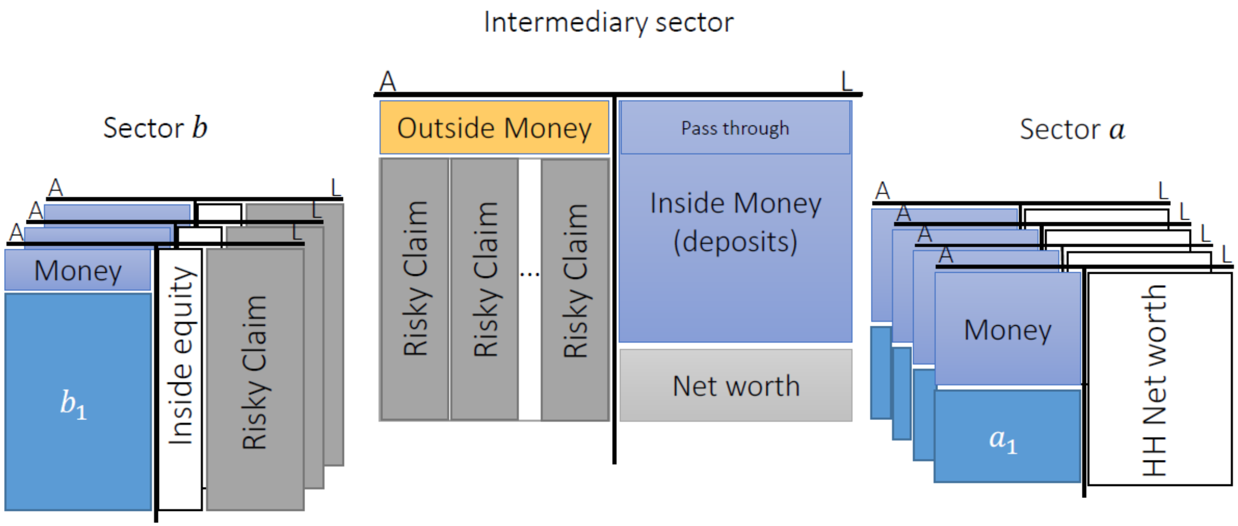
(Similar equation to technology *b*)

- Preferences:
  - All individuals have identical log-preferences with common discount rate ρ

$$E\left[\int_0^\infty e^{-\rho t} \log c_t \, dt\right]$$

Baseline Model

- Financing Constraints:
  - Each HH can invest in either technology *a* or *b*.
  - Risk offload to the intermediary follows $0 \leq \bar{\chi}^a < \bar{\chi}^b \leq 1.$ (Set $\bar{\chi}^a = 0,$ for simplicity)



Baseline Model

- Assets, Returns and Portfolios
  - q_t is the price of physical capital per unit of consumption good
  - $p_t K_t$ is the real value of outside money. p_t reflects how wealth distribution affects the value of money
  - Total wealth of all agents is given $(p_t + q_t)K_t$ - inside money doesn't not enter because it is a liability to the I's but an asset to HH's
  - q_t follows a Brownian process $\frac{dq_t}{q_t} = \mu_t^q \, dt + (\sigma_t^q)^T \, dZ_t,$
  - Return of an individual project in technology *a*:

$$dr_t^a = \frac{A^a(\psi_t) - \iota_t}{q_t} \, dt + \left(\Phi(\iota_t) - \delta + \mu_t^q + (\sigma_t^q)^T \sigma^a 1^a\right) dt + (\sigma_t^q + \sigma^a 1^a)^T \, dZ_t + \tilde{\sigma}^a \, d\tilde{Z}_t,$$

  - Return on technology *b* is split between HH's (inside equity) and I's (outside equity)

$$dr_t^b = (1 - \chi_t) \, dr_t^{bH} + \chi_t \, dr_t^{bI}$$

  - Return on money (Brownian process): $\frac{dp_t}{p_t} = \mu_t^p \, dt + (\sigma_t^p)^T \, dZ_t.$
  - Law of motion of Aggregate Capital: $\frac{dK_t}{K_t} = (\Phi(\iota_t) - \delta) \, dt + \underbrace{\psi_t \sigma^a \, dZ_t^a + (1 - \psi_t)\sigma^b \, dZ_t^b}_{(\sigma_t^K)^T \, dZ_t}.$
  - Real interest rate (capital gains rate; return on money):

$$dr_t^M = \frac{d(p_t K_t)}{p_t K_t} = \left(\Phi(\iota_t) - \delta + \mu_t^p + (\sigma_t^p)^T \sigma_t^K\right) dt + \underbrace{(\sigma_t^K + \sigma_t^p)^T \, dZ_t}_{(\sigma_t^M)^T \, dZ_t}.$$

Baseline Model

- Assets, Returns and Portfolios
  - Now one can define the net worth of HH's investing in projects a or b and the net worth of intermediaries

$$\frac{dn_t}{n_t} = x_t^a \, dr_t^a + (1 - x_t^a) \, dr_t^M - \zeta_t^a \, dt.$$

HH investing in *a*

$$\frac{dn_t}{n_t} = x_t^b \, dr_t^{bH} + (1 - x_t^b) \, dr_t^M - \zeta_t^b \, dt$$

HH investing in *b*

$$\frac{dn_t}{n_t} = x_t \, d\bar{r}_t^{bI} + (1 - x_t) \, dr_t^M - \zeta_t \, dt$$

I's (x_t can be >1 because I's use leverage)

Return on HH's outside equity

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$$\frac{dn_t}{n_t} = x_t \, d\bar{r}_t^{bI} + (1 - x_t) \, dr_t^M - \zeta_t \, dt$$

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Return on HH's outside equity

- Equilibrium definition
  - Agents start with endowment of capital and money
  - Over time they trade aiming to maximize utility subject to the budget constraints defined above
  - Net worth share of intermediaries $\eta_t = \dfrac{N_t}{(q_t + p_t)K_t}.$

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HH investing in *a*

$$\frac{dn_t}{n_t} = x_t^b\, dr_t^{bH} + (1 - x_t^b)\, dr_t^M - \zeta_t^b\, dt$$

HH investing in *b*

$$\frac{dn_t}{n_t} = x_t\, d\bar{r}_t^{bI} + (1 - x_t)\, dr_t^M - \zeta_t\, dt$$

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Return on HH's outside equity

- Equilibrium definition

Definition. *Given any initial allocation of capital and money among the agents, an equilibrium is a map from histories* $\{Z_s,\ s \in [0,t]\}$ *to prices* p_t *and* q_t*, return differential* $dr_t^{bH} - dr_t^{bI} \geq 0$*, the households' wealth allocation* α_t*, equity allocation* $\chi_t \leq \bar{\chi}$*, portfolio weights* (x_t^a, x_t^b, x) *and consumption propensities* $(\zeta_t^a, \zeta_t^b, \zeta_t)$*, such that*

(i) all markets, for capital, equity, money and consumption goods, clear,

(ii) all agents choose technologies, portfolios and consumption rates to maximize utility (households who produce good b *also choose* χ_t*).*

Baseline Model

- Deriving Equilibrium Conditions
  - Start with simplifying assumptions (analytical tractability): $\zeta_t = \zeta_t^a = \zeta_t^b = \rho$

    $\rho(q_t + p_t)K_t = (A(\psi_t) - \iota_t)K_t.$ (Market clearing for the consumption good)

  - Real aggregate risk of HH's investing in sectors *a* and *b*:

    $\sigma_t^{Na} = x_t^a \underbrace{\left(\sigma^a 1^a + \sigma_t^q - \sigma_t^M\right)}_{\nu_t^a} + \sigma_t^M$ (Measure of nominal risk)

    $\sigma_t^{Nb} = x_t^b \underbrace{\left(\sigma^b 1^b + \sigma_t^q - \sigma_t^M\right)}_{\nu_t^b} + \sigma_t^M$ (Measure of nominal risk)

  - In equilibrium HH's need to be indifferent between investing in *a* or *b*:

    $(x_t^a)^2(|\nu_t^a|^2 + (\tilde{\sigma}^a)^2) = (x_t^b)^2(|\nu_t^b|^2 + (\tilde{\sigma}^b)^2)$ (Proof is in the Appendix)

  - Fraction of world's wealth in the form of money $\vartheta_t = \dfrac{p_t}{q_t + p_t}$ (Remainder is in the form of capital)

  - Fraction of total HH wealth in sector *a* is $\alpha_t = \dfrac{(1-\psi_t)(1-\vartheta_t)}{x_t^a(1-\eta_t)}$

  - Equilibrium law of motion of the state variable that determines prices of capital and money

$$\frac{d\eta_t}{\eta_t} = \underbrace{(1-\eta_t)\left(x_t^2|\nu_t^b|^2 - (x_t^a)^2(|\nu_t^a|^2 + (\tilde{\sigma}^a)^2)\right)\, dt}_{\text{Relative earnings of I's and HH's}} + \underbrace{(x_t\nu_t^b + \sigma_t^\vartheta)^T(\sigma_t^\vartheta\, dt + dZ_t)}_{\text{Volatility due to imperfect risk sharing Between HH's and I's}}.$$

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Model without Intermediaries

- Objective:
  - Anticipate properties of a full equilibrium dynamics
  - Since intermediaries reduce the amount of idiosyncratic risk in the economy, the presence of a healthy intermediary sector is akin to a reduction in idiosyncratic risk in the model without intermediaries

- Assumptions and the Value and Risk of Money:
  - Assume that $\eta = 0$. and for simplicity $\sigma^a = \sigma^b = \sigma,\ \tilde{\sigma}^a = \tilde{\sigma}^b = \tilde{\sigma}$
  - Production function maximized at $\psi = 1/2$
  - Aggregate capital in the economy $\frac{dK_t}{K_t} = (\Phi(\iota_t) - \delta)\, dt + \frac{\sigma}{2}\, dZ_t^a + \frac{\sigma}{2}\, dZ_t^b.$
  - Volatility of the money $\bar{\sigma} \equiv \sqrt{\sigma^2/2}$
  - Incremental risk of a project in each sector $\hat{\sigma} \equiv \sqrt{\tilde{\sigma}^2 + \sigma^2/2},$
  - Economy is equivalent to a single-good economy with aggregate risk $\bar{\sigma}$ and project specific risk
  - Market clearing condition for the output: $\bar{A} - \iota(q) = \rho\, \underbrace{(p+q)}_{q/(1-\vartheta)}$
  - Each HH chooses a portfolio share of risky capital that equals the expected excess capital return over money
    - Special case of log-investment function $\Phi(\iota) = \log(\kappa\iota + 1)/\kappa$

$$q = \frac{\kappa\bar{A} + 1}{\kappa\sqrt{\rho}\hat{\sigma} + 1} \quad \text{and} \quad p = \frac{\hat{\sigma} - \sqrt{\rho}}{\sqrt{\rho}} q.$$

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- Conclusions
  - Results suggest what would happen with the value of money fluctuations in an economy with intermediaries
      - When η_t approaches 0, HH's face idiosyncratic risk in both sectors and this leads to a high VoM
      - When η_t is large enough, most of the idiosyncratic risk is concentrated in sector *a* as HH's pass the idiosyncratic risk of sector *b* to intermediaries
  - Impossibility of "As of" Representative Agent Economy
      - In any representative agent economy, absence of individual level idiosyncratic risk, capital returns strictly dominate money and hence money could never have some positive value.

Model without Intermediaries

- Welfare analysis:
  - General Result (Proof is in the Appendix)

Proposition 3. *Consider an agent who consumes at rate* ρn_t *where* n_t *follows*

$$\frac{dn_t}{n_t} = \mu_t^n \, dt + \sigma_t^n \, dZ_t$$

Then the agent's expected future utility at time t takes the form

$$E_t\left[\int_t^\infty e^{-\rho(s-t)} \log(\rho n_s)\, ds\right] = \frac{\log(\rho n_t)}{\rho} + \frac{1}{\rho} E_t\left[\int_t^\infty e^{-\rho(s-t)} \left(\mu_s^n - \frac{|\sigma_s^n|^2}{2}\right) ds\right].$$

  - Without intermediaries, drift and volatility of wealth for all HH's are time invariant
  - Special result (macro-prudential regulation):

Proposition 4. *Suppose* $\hat{\sigma}^2 > \rho$, *so that monetary equilibrium exists in the economy without intermediaries. Then in this equilibrium, the welfare of a household with initial wealth* $n_0 = 1$ *is*

$$U^H = \frac{\log(\rho)}{\rho} + \frac{\Phi(\iota(q)) - \delta - (\rho + \bar{\sigma}^2)/2}{\rho^2}.$$

Proposition 5. *Assume that* $\Phi(\iota) = \log(\kappa\iota + 1)/\kappa$. *Then if money can have positive equilibrium value, welfare in equilibrium with money is always greater than that in the moneyless equilibrium. Furthermore, relative to the value of* ϑ *in the equilibrium with money, optimal policy raises* ϑ *if and only if*

$$\hat{\sigma}(1 - \kappa\rho) < 2\sqrt{\rho}. \tag{3.9}$$

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  - Special result (macro-prudential regulation):

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Trade-off between money as an insurance and the distortionary effect of rising money value on investment

  - Returns of money are free of idiosyncratic risk
  - But in the money equilibrium, the price of capital is lower ➔ investment is lower ➔ overall growth is lower.

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Analysis including I's

- General Intuition:
  - Previous analysis = Extreme polar case where I's capitalization is 0
    - In the "money equilibrium": High VoM (insurance for HH's invested in either *a* or *b*)

      $\neq$

    - When the I sector is well functioning, those HH's who invested in *b* can offload some of their idiosyncratic risks ➔ less demand for insurance vehicles ➔ lower VoM
    - When η_t approaches 1, there is too much focus on sector *b* goods ➔ aggregate economic activity declines

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- The goal is to provide a full characterization of the equilibrium and conduct welfare analysis
  - Computational method without policy is done by using 7 equations (7 unknowns)
    - Shooting method to solve an ODE
    - Backward "iterative" method on a PDE with a terminal condition
  - Computational method with monetary policy is done by using 7 equations (7 unknowns)
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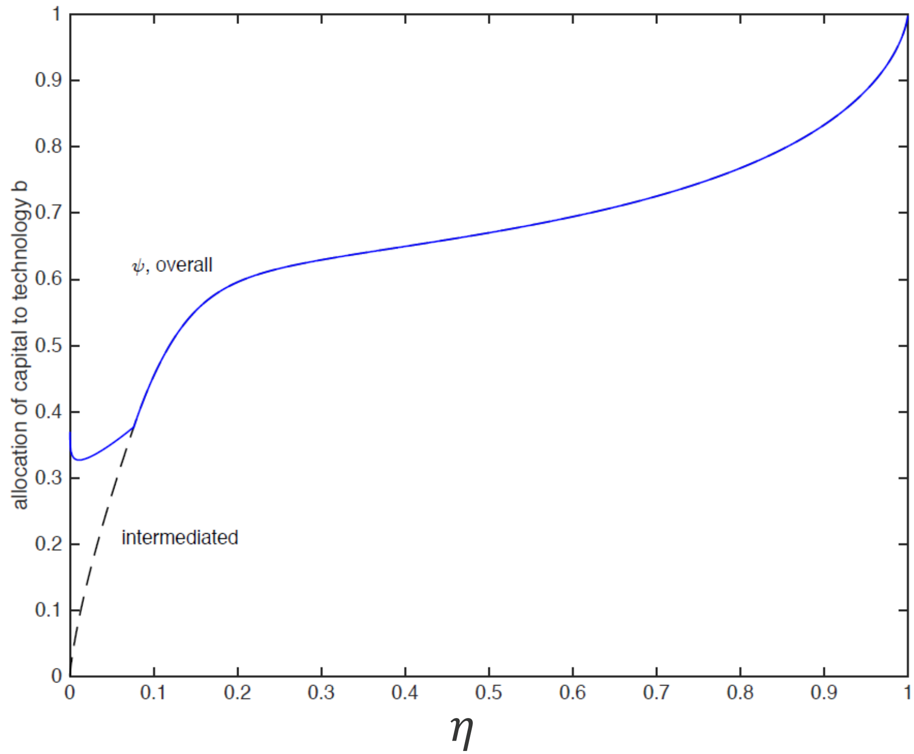
Analysis including I's

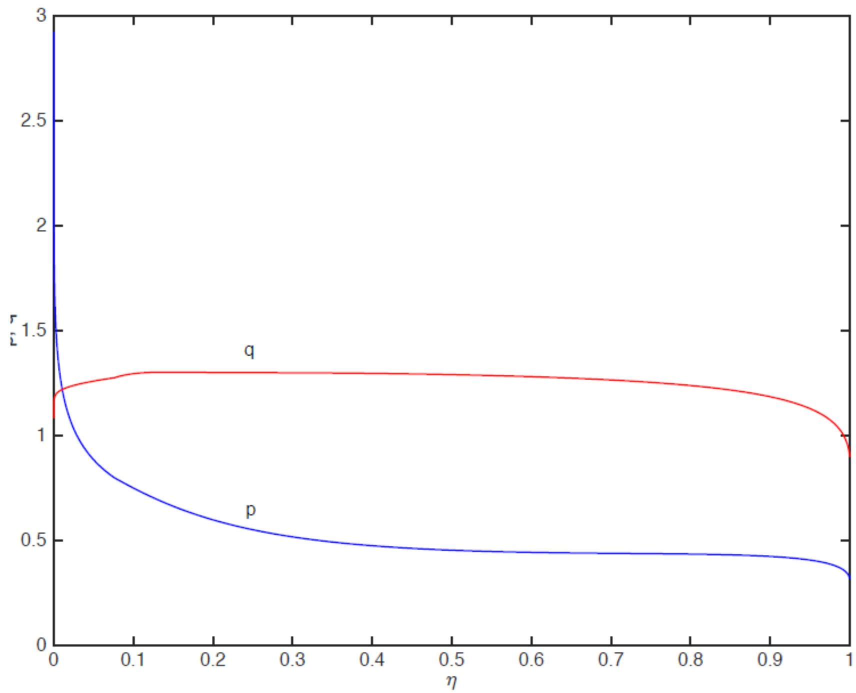
- Example:
  - Allocation to technology *b* with $\rho = 0.05,\ A = 0.5\ \sigma^a = \sigma^b = 0.1,$ $\tilde{\sigma}^a = 0.6,\ \tilde{\sigma}^b = 1.2,\ s = 0.8,\ \Phi(\iota) = \log(\kappa\iota + 1)/\kappa$ with $\kappa = 2$, and $\bar{\chi} \to 1$



  - When η drops the risk premia intermediaries demand for equity stakes in projects of HH's in sector *b* increase (HH's may be willing to sell less than the fraction allowed of outside equity)

Analysis including I's

- Example:
  - Prices of money and capital with $\rho = 0.05,\ A = 0.5\ \sigma^a = \sigma^b = 0.1,$
$\tilde{\sigma}^a = 0.6,\ \tilde{\sigma}^b = 1.2,\ s = 0.8,\ \Phi(\iota) = \log(\kappa\iota + 1)/\kappa$ with $\kappa = 2$, and $\bar{\chi} \to 1$



  - When η rises, price of capital rises and price of money drops
  - Money becomes less valuable as η rises because inside money (liabilities of banks) is a perfect substitute to outside money

Analysis including I's

- Volatility of η, Liquidity and Disinflationary Spirals
  - From the law of motion of η one can find $$\sigma_t^\eta = \underbrace{x_t(\sigma^b 1^b - \sigma_t^K)}_{\text{fundamental volatility}} + \underbrace{\sigma_t^\vartheta \left(1 - \frac{x_t}{1-\vartheta_t}\right)}_{\text{amplification}}$$
  - η has volatility for two reasons:
    - Mismatch between fundamental risk of assets held by I's and the overall fundamental risk of the economy
    - Amplification (from changes in the price of money relative to capital, $\upsilon(\eta_t)$), as long as the portfolio share of HH's equity x_t is greater than the world's capital share $1 - \upsilon_t$ and $\upsilon'(\eta) < 0$
      $$\sigma^\vartheta = (1 - \vartheta_t)(\sigma_t^p - \sigma_t^q)$$
  - Amplification arises from two spirals
    - Changes in the price of capital q_t - liquidity spiral
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- The paradox of prudence
  - For lower values of η_t (I's are undercapitalized) negative shocks are amplified in both LHS and RHS of I's B/S:
    - Price of capital q_t drops after a negative shock
    - I's respond to losses by shrinking their balance sheets (fire sales): lowering q_t and reducing inside money (increasing the value of liabilities, p_t)
    - As every I behaves in a micro-prudent way the overall effect is macro-imprudent as it decreases inside money and raises endogenous risk.

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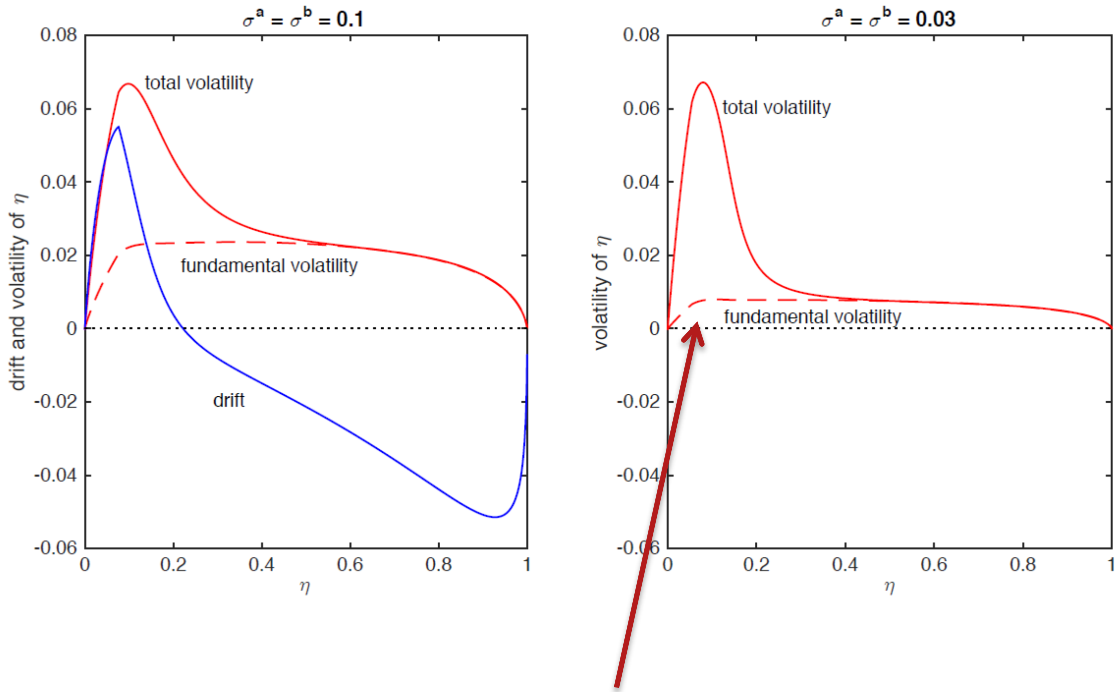
    - As every I behaves in a micro-prudent way the overall effect is macro-imprudent as it decreases inside money and raises endogenous risk.
- Drift of η

$$\mu_t^\eta \eta = \eta(1-\eta)\left(x_t^2|\nu_t^b|^2 - (x_t^a)^2(|\nu_t^a|^2 + (\tilde{\sigma}^a)^2)\right) + \eta_t(x_t\nu_t^b + \sigma_t^\vartheta)^T \sigma_t^\vartheta$$

Analysis including I’s

- Amplification Effect



- Endogenous risk persists doe to amplification even as fundamental risk declines.

- Stochastic steady-state of η_t is the point where the drift equals zero (earnings of I’s and HH’s balance each other out)

Analysis including I’s

- Sources of Inefficiencies
  - To prepare ground for policy recommendations, one needs to understand the sources of inefficiencies in the model

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Inefficient sharing of idiosyncratic risk

- mitigated through I's (diversification) → Avoid cycles of undercapitalized I's
- Mitigated through money (inside and outside) → avoid states of high VoM (lower price of capital and inefficient investment)

Inefficient sharing of aggregate risk

- When I's become undercapitalized, barriers to entry into the I sector help I's → price of good *b* rises with low η → help I's recapitalize
- Limited competition in the I sector creates a ToT hedge

Inefficient production

- When I's or HH's are undercapitalized, production may be inefficiently allocated towards good *a* or good *b*.

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<u>Inefficient production</u>

- When I’s or HH’s are undercapitalized, production may be inefficiently allocated towards good *a* or good *b*.

- To compute the overall effect of such inefficiencies one needs a proper welfare measure!

Analysis including I's

- Welfare Analysis
  - Following Proposition 3 (general wealth process), they create a "representative agent" who consumes a fixed portion of aggregate output.
  - Compute the welfare of I's, HH's (the focus of the analysis) and the "representative agent"

Proposition 6. *Welfare of a representative agent with net worth is given by* $\log(\rho n_t)/\rho + U^R(\eta_t)$*, where*

$$U^R(\eta_t) = -\frac{\log(p_t + q_t)}{\rho} + E_t\left[\int_t^\infty e^{-\rho(s-t)}\left(\log(p_s + q_s) + \frac{\Phi(\iota_s) - \delta}{\rho} - \frac{|\sigma_s^K|^2}{2\rho}\right) ds\right]. \quad (4.3)$$

Proposition 7. *The welfare of an intermediary with wealth* n_t^I *is* $\log(\rho n_t^I)/\rho + U^I(\eta_t)$*, where*

$$U^I(\eta_t) = U^R(\eta_t) - \frac{\log(\eta_t)}{\rho} + E_t\left[\int_t^\infty e^{-\rho(s-t)}\log(\eta_s)\, ds\right]. \quad (4.4)$$

The welfare of a household with net worth n_t^H *is* $\log(\rho n_t^H)/\rho + U^H(\eta)$*, where*

$$U^H(\eta_t) = U^R(\eta_t) + \quad (4.5)$$

$$\frac{1}{\rho}E_t\left[\int_t^\infty e^{-\rho(s-t)}\left(\eta_s\left((x_s^a)^2(|\nu_s^a|^2 + \tilde{\sigma}_a^2) - x_s^2|\nu_s^b|^2\right) + \frac{|\sigma_s^\vartheta|^2 - (x_s^a)^2(|\nu_s^a|^2 + \tilde{\sigma}_a^2)}{2}\right) ds\right].$$

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All quantities depend on η_t

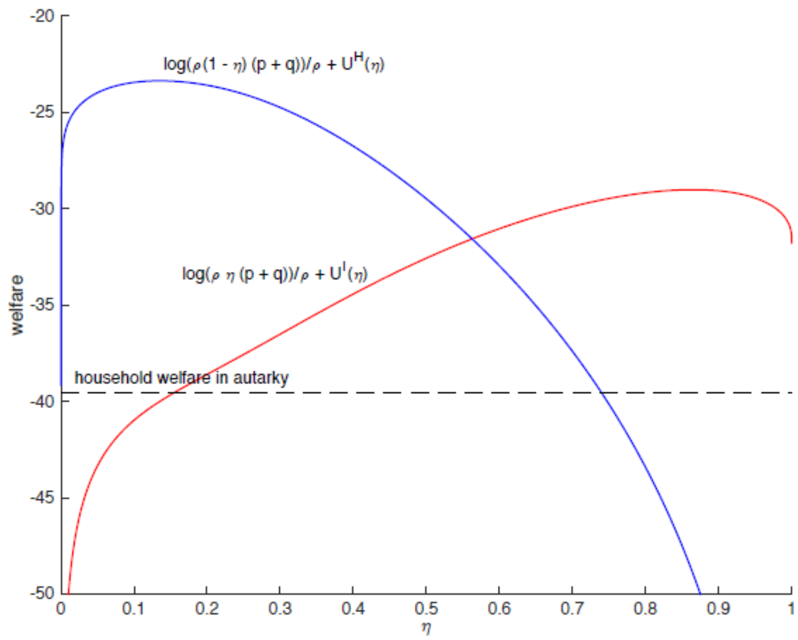
Welfare just requires solving an ODE

Analysis including I's

- Welfare Analysis - Example
  - Welfare of each type tends to increase in its wealth share (but only up to a certain point)
  - At the extreme, one agent type becomes so undercapitalized that productive inefficiencies make everyone worse off:
    - In such cases, redistributions towards the undercapitalized sector are Pareto improving.

$$\log(\rho n_0)/\rho + U^I(\eta_0) = \log(\rho\eta_0(p_0 + q_0))/\rho + U^I(\eta_0)$$ Representative I

$$\log(\rho(1-\eta_0)(p_0+q_0))/\rho + U^H(\eta_0)$$ Representative HH



Analysis including I's

- Preparing Ground for Policy (Focus on Monetary Policy)

  - VoM affects welfare (Positive fact)
    - High VoM helps hedge idiosyncratic risks but creates investment distortions
      - Monetary policy $\overrightarrow{endogenously}$ VoM
      - Macroprudential policy $\overrightarrow{directly}$ VoM

  - Inefficiencies related to aggregate risk sharing (Positive fact)
    - Production and investment distortions when one sector is undercapitalized
      - Monetary policy → Concentration of risk → Risk premia / earnings
      - Macroprudential policy → Risk premia / earnings (independently of risk taking)

Paper Outline

- Baseline Model (without policy intervention)
  - Describe technologies, preferences, financial constraints and asset returns (portfolios)
  - Define and derive the equilibrium
  - Evolution of the state variable that describes prices of capital and money
- Model without Financial Intermediaries (I’s)
  - Value and Risk of Money
  - Welfare analysis
- Including Financial Intermediaries (I’s)
  - Equilibrium
  - Inefficiencies and welfare analysis
- Monetary and Macro-prudential Policy
  - Introduction of Long-term nominal bonds
  - Removing amplification
  - Economy with perfect sharing of aggregate risk
  - Optimal macro-prudential policy

Monetary and Macro-prudential Policy

- Central Bank Controlling Money Supply

  - Start extending the baseline model to include a Central Bank controlling money supply (by setting up short-term interest rates)
    - Central Bank pays interest on reserves (outside money) held by I's
    - Funds these expenses by "printing money"

Proposition 8. (Super-Neutrality of Money) *If the central bank allows the nominal supply of outside money to grow at rate i_t by paying interest to holders of outside money, then the analysis of Section 4 is unaffected. That is, the law of motion of η_t, all real returns and asset allocation remain unchanged.*

  - Proof is straightforward. Call M_t the outstanding supply of outside money. We have $\frac{d(M_t)}{(M_t)} = i_t dt$ and since $p_t K_t = M_t$ we have $\frac{d(p_t K_t)}{(p_t K_t)} = i_t dt$. Inside money and outside money should earn the same return in equilibrium – so all equations characterizing the full equilibrium remain unchanged (no explicit dependence of i_t)

  - Interest rate policy does not have real effects but it does affect inflation. Given the Fisher equation ($d{r_t}^M = i_t dt - d\pi_t$), since $d{r_t}^M = 0$ (interest rate doesn't affect the return on money), we have $i_t dt = d\pi_t$

Monetary and Macro-prudential Policy

- Nominal Long-term Bonds
  - Introduce a monetary policy tool with redistributive effects
    - Nominal long-term perpetual bonds paying fixed interest i^B (revenue neutral interest or QE policies)

$$dM_t = i_t M_t \, dt + i^B L_t \, dt - (i^B B_t) \, dL_t.$$

Nominal quantity of money increases by printing to pay interest or issue more bonds and decreases when bonds are sold for money.

  - Call $p_t K_t$ the real value of all outstanding safe assets, outside money and perpetual bonds, and $b_t K_t$ the real value of all outstanding perpetual bonds

$$\frac{b_t}{p_t} = \frac{i^B B_t L_t}{i^B B_t L_t + M_t}$$

  - Central bank controls the pair (i_t, b_t). Given the nominal money supply M_t and the real value of money $(p_t - b_t)K_t$, price levels are

$$\frac{M_t}{(p_t - b_t)K_t} = \frac{i^B B_t L_t + M_t}{p_t K_t}$$

Return on capital expressions don't change but money earns the return that depends on policy.

  - Bond prices follow the endogenous equilibrium process $\dfrac{dB_t}{B_t} = \mu_t^B \, dt + (\sigma_t^B)^T \, dZ_t.$
  - Intermediaries hold bonds to hedge against net worth risk.

$$\frac{E_t[dr_t^B - dr_t^M]}{dt} = (\sigma_t^B)^T \sigma_t^N, \quad \sigma_t^N = \sigma_t^M + x_t \nu_t + x_t^B \sigma_t^B$$

Pricing expected returns on bonds (over money)

Monetary and Macro-prudential Policy

- Nominal Long-term Bonds
  - Return on the aggregate portfolio of bonds and money:

$$\frac{d(p_t K_t)}{p_t K_t} = \left(\Phi(\iota_t) - \delta + \mu_t^p + (\sigma_t^p)^T \sigma_t^K\right) dt + (\sigma_t^K + \sigma_t^p)^T \, dZ_t = \frac{b_t}{p_t} \, dr_t^B - \left(1 - \frac{b_t}{p_t}\right) dr_t^M,$$

  - Return and risk of money is altered $dr_t^M = \left(\Phi(\iota) - \delta + \mu_t^p + (\sigma_t^p)^T \sigma_t^K\right) dt - \frac{b_t}{p_t}(\sigma_t^B)^T \sigma_t^N \, dt + \underbrace{\left(\sigma_t^K + \sigma_t^p - \frac{b_t}{p_t}\sigma_t^B\right)}_{\sigma_t^M} dZ_t.$

Proposition 9. *The equilibrium law of motion of η_t is given by*

$$\frac{d\eta_t}{\eta_t} = (1 - \eta_t)\left(|x_t \nu_t^b + x_t^B \sigma_t^B|^2 - (x_t^a)^2(|\nu_t^a|^2 + (\tilde{\sigma}^a)^2)\right) \, dt +$$

$$\left(x_t \nu_t^b + \sigma_t^\vartheta + (1 - \eta_t) x_t^B \sigma_t^B\right)^T \left(dZ_t + \left(\sigma_t^\vartheta - \eta_t x_t^B \sigma_t^B\right) \, dt\right).$$

Proposition 10. *The real effect of monetary policy on equilibrium is fully summarized by the process $(b_t/p_t)\sigma_t^B$.*

And values of $(b_t/p_t){\sigma_t}^B$ depend on the policy (i_t, b_t)

  - While monetary policy can provide insurance, it cannot control risk from risk-raking and risk premia separately

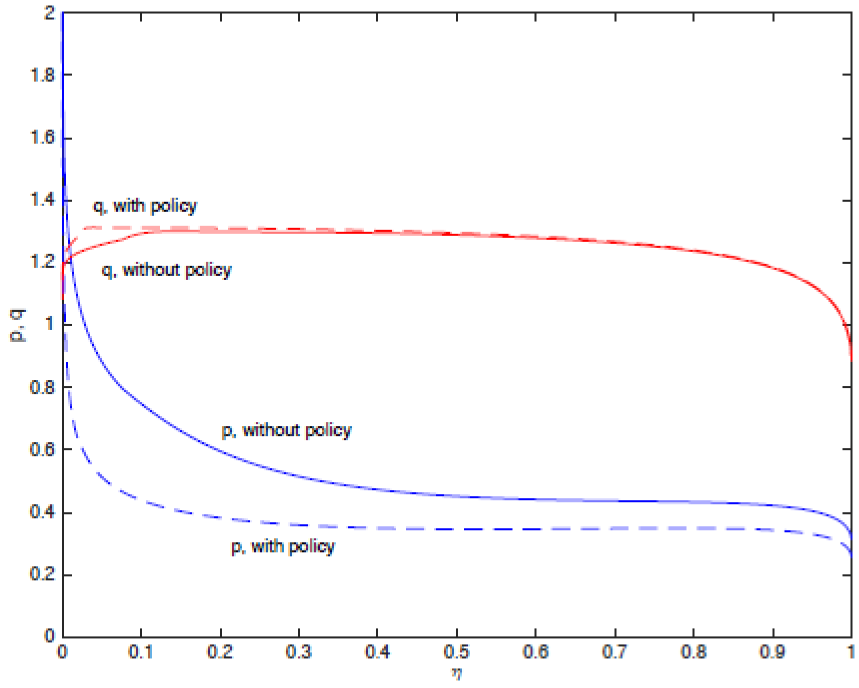
Monetary and Macro-prudential Policy

- Nominal Long-term Bonds
  - Imagine a policy that sets up short term interest rate and the level of b_t as a function of η_t (lowering i_t) when η_t drops.
  - Volatility of η_t can be re-written as $\sigma_t^\eta = \dfrac{x_t(\sigma^b 1^b - \sigma_t^K)}{1 + \underbrace{\frac{\vartheta'(\eta)}{\vartheta(\eta)}(\psi_t \chi_t - \eta_t)}_{\text{amplification spirals}} - \underbrace{\frac{b_t}{p_t}\frac{B'(\eta)}{B(\eta)}(x_t\eta_t + (1-\eta_t)\vartheta_t)}_{\text{mitigation}}}.$
    - When η_t falls, the mitigating effect $-(b_t/p_t)\, B'(\eta)/B(\eta)$ rises and the volatility declines
    - The one-dimensional function $\frac{b(\eta)}{p(\eta)}\frac{B'(\eta)}{B(\eta)}$ summarizes the effects of the policy tools (i_t , b_t)
    - A policy that sets up $b_t/p_t\sigma_t^B$ appropriately could completely eliminate the amplification in the law of motion of η_t

$$\sigma_t^\eta = x_t(\sigma^b 1^b - \sigma_t^K).$$

Monetary and Macro-prudential Policy

- Nominal Long-term Bonds
  - A policy that sets $b_t/p_t\sigma_t^B$ to remove amplification from the law of motion of η_t



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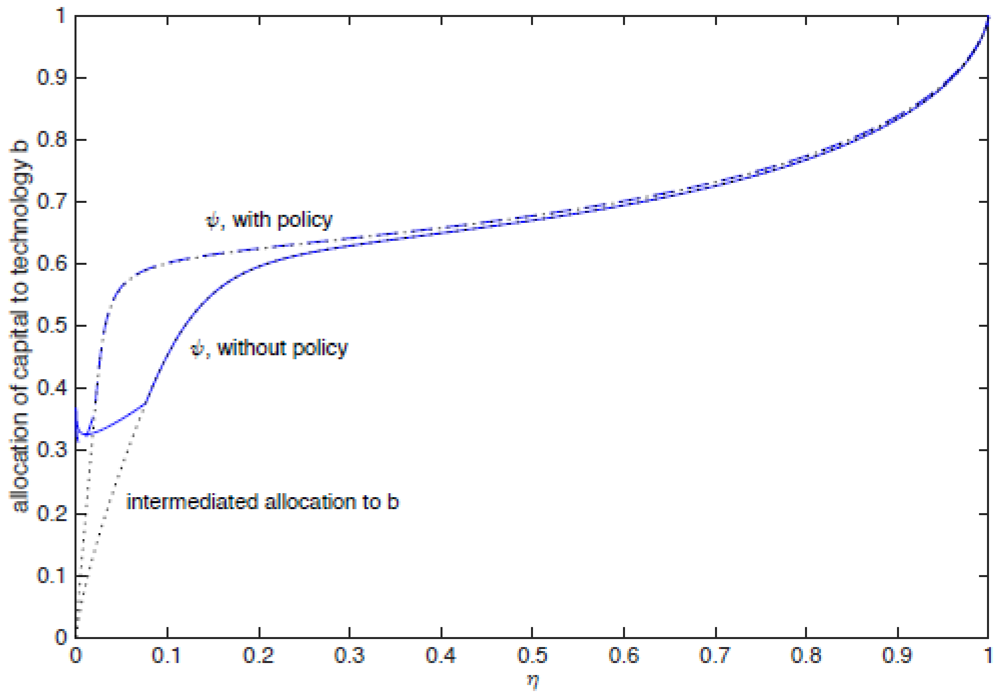

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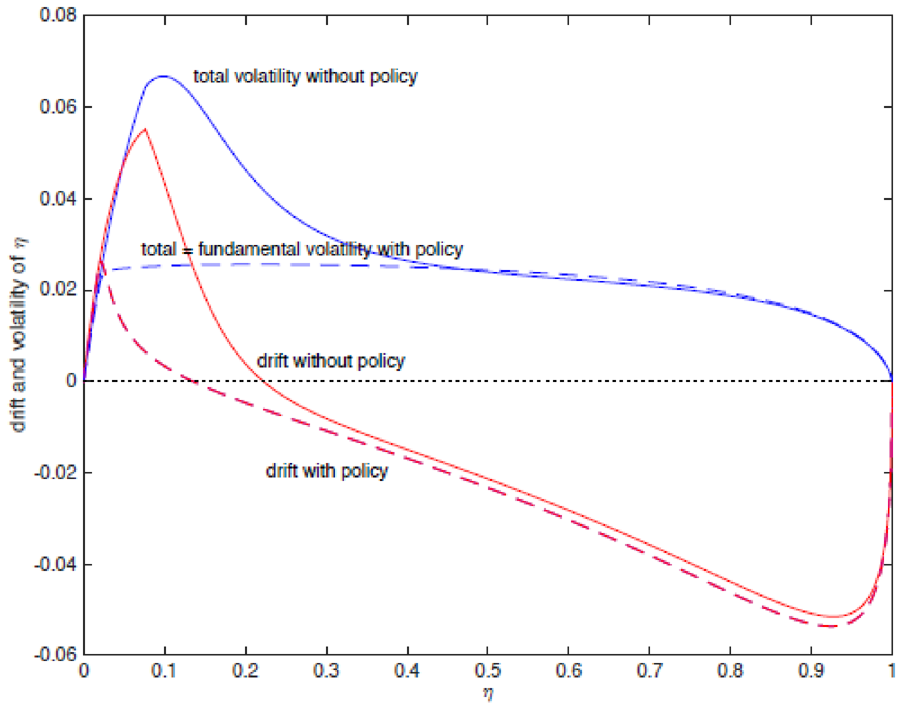

Monetary and Macro-prudential Policy

- Economy with Perfect Risk-sharing
  - If we have the mitigation term going to ∞, then we have ${\sigma_t}^\eta \to 0$ ⇔ Economy with perfect sharing of aggregate risk
  - This would be the outcome if HH's were able to trade contracts based on systemic risk

$$(\sigma^b 1^b - \sigma^a 1^a)^T \, dZ_t.$$

  - Aggregate risk exposures of HH's and I's would be proportional to ${\sigma_t}^K$
  - η_t and prices of money and capital would have no volatility
  - HH's in sector *b* would be able to issue maximal equity shares to intermediaries.

Proposition 11. *The function $\vartheta(\eta)$ satisfies the first-order differential equation*

$$\mu_t^\vartheta = \frac{\vartheta'(\eta)}{\vartheta(\eta)} \eta \mu_t^\eta,$$

where

$$\mu_t^\eta = -(1-\eta)(x_t^b)^2(\tilde{\sigma}^b)^2, \quad \mu_t^\vartheta = \rho + \mu_t^\eta,$$

and ψ_t, x_t^a, x_t^b and q_t satisfy

$$A(\psi_t) - \iota(q_t) = \frac{\rho q_t}{1-\vartheta_t}, \quad (1-\chi)\psi_t + (1-\psi_t)\frac{\tilde{\sigma}^a}{\tilde{\sigma}^b} = x_t^b \frac{1-\eta_t}{1-\vartheta_t}, \quad x_t^a \tilde{\sigma}^a = x_t^b \tilde{\sigma}^b \quad \text{and}$$

$$\frac{A^b(\psi_t) - A^a(\psi_t)}{q_t} = \psi_t(\sigma^b)^2 - (1-\psi_t)(\sigma^a)^2 + (1-\chi)\, x_t^b \tilde{\sigma}_b^2 - x_t^a \tilde{\sigma}_a^2.$$

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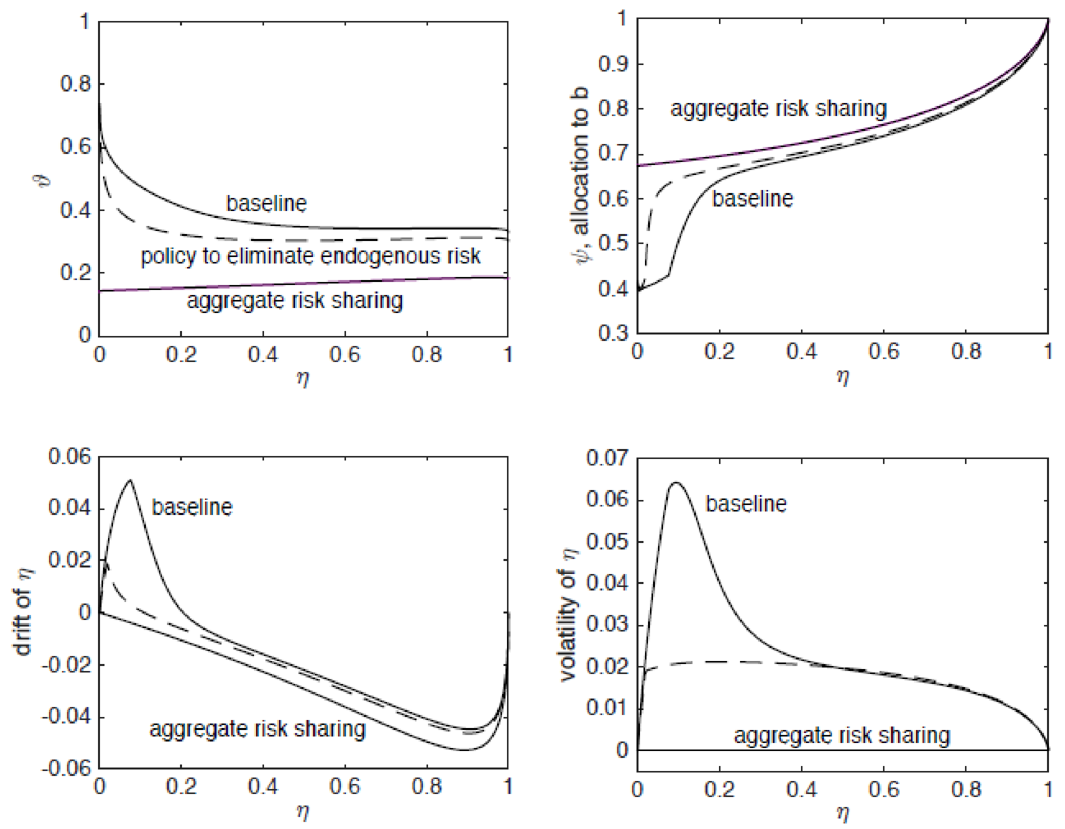

Monetary and Macro-prudential Policy

- Optimal Macro-prudential Policy
  - Macro-prudential policies can significantly improve economic welfare
    - Control quantities and affect allocation of resources independent of the allocation of risk
  - Objective: Study the theoretical limit that can be attained when markets for sharing aggregate risk are open and the policy maker can control asset allocation, portfolio and returns (but not consumption or investment)
    - “Force” some HH’s to specialize in either sector against their will?

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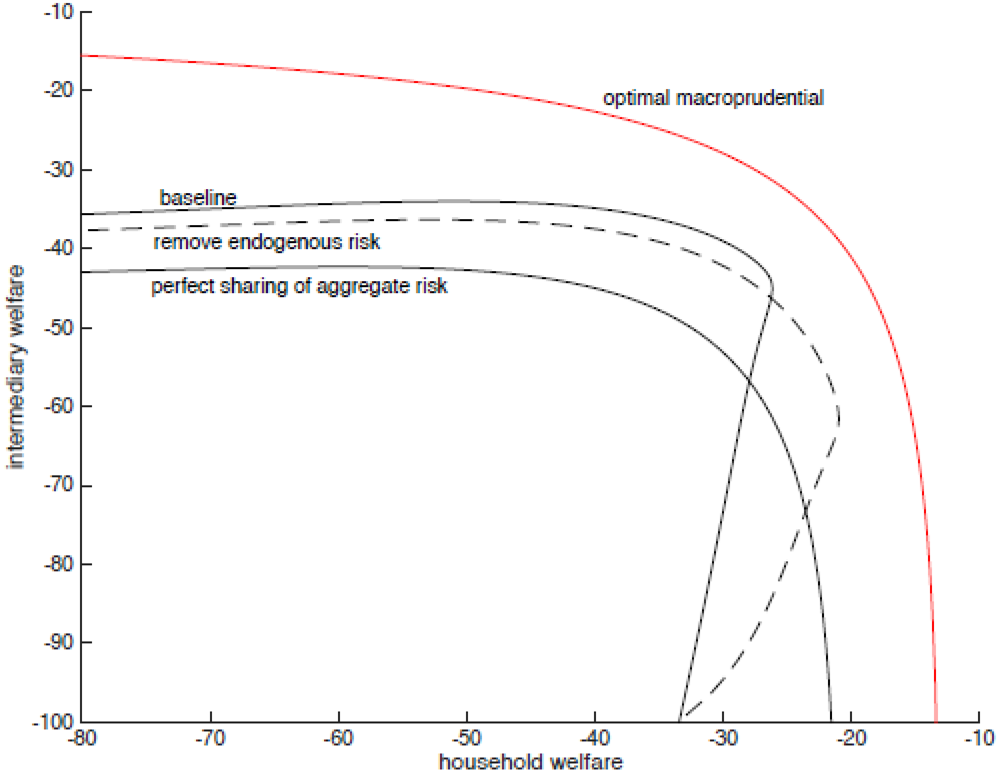
Proposition 13. *To maximize welfare, the policy maker must expose households in sectors a and b to the same amounts of idiosyncratic risk. It is also welfare-maximizing for households in the two sectors to earn the same expected returns, and with this, households are indifferent between specializing in sectors a and b.*

  - This policy (Proposition 13) can be implemented by imposing portfolio weight constraints on HHs, plus taxes/subsidies on goods *a* and *b* to achieve an appropriate ψ_t
    - Regulator does not need to control HH's choices between sectors *a* and *b* or the market for aggregate risk.

Monetary and Macro-prudential Policy

- Optimal Macro-prudential Policy



Monetary and Macro-prudential Policy

- Optimal Macro-prudential Policy

  - Monetary policy alone can change the risk profile of assets and provide natural hedges in incomplete markets but cannot control risk raking / risk premia separately from itself (endogenous)
    - Improves the sharing of aggregate risk, stimulates the price of capital relative to money so HH's are over-exposed to idiosyncratic risk.
    - I's become less likely to be undercapitalized → I's provide insurance to HH's that offsets some of the idiosyncratic risk,
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Macro-prudential policy limiting HH’s weights on capital is welfare enhancing as it reduces their exposures to idiosyncratic risk.

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Normative argument for macro-prudential tools controlling HH's portfolio choices, such as loan-to-value ratios for HH's borrowing against some assets.