The Economics of Uncertainty

- The state of nature: $s$
- Realizations: $s_1, s_2, \ldots$
- Intrinsic Uncertainty
  - Random fundamentals
  - Examples
    
    $s_1 = \text{rain}$  $s_2 = \text{drought}$
    
    $s_1 = \text{hot}$  $s_2 = \text{cold}$

- Extrinsic Uncertainty
  - Randomness that does not affect the fundamentals, but does affect outcomes.
  - Examples
    
    $s_1 = \text{no run}$  $s_2 = \text{run}$
    
    $s_1 = \text{sunspots}$  $s_2 = \text{no sunspots}$
Expected Utility

- von Neumann and Morgenstern
  - Expected Utility
    - \( V = \pi(s_1)u(x(s_1)) + \pi(s_2)u(x(s_2)) \)
    - \( \pi(s_1) = 1 - \pi(s_2) \)
    - \( V = \int u(x(s))\pi(s)ds \)
  - Risk aversion:
    - \( u(x) \)
    - \( u'(x) > 0 \)
    - \( u''(x) < 0 \) Risk-averse
  - Risk-neutral
    - \( u''(x) = 0 \)
  - Risk-loving
    - \( u''(x) > 0 \)
Arrow-Debreu

- Isomorphism
  - Contingent-claims
  - Securities
$F$ – Fire, $N$ – no fire, $E$ – expected value

$\pi(F)u(F) + \pi(N)u(N)$

$u(x)$
CRRA

- Kenneth Arrow
- John Pratt
- $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$
  - For $\gamma = 1$, define $u(c) = \log(c)$
- $u'(c) = \left(\frac{1-\gamma}{1-\gamma}\right)c^{-\gamma} = c^{-\gamma} > 0$
- $u''(c) = -\gamma c^{-\gamma-1} < 0$
- Risk-aversion:

$$-\frac{cu''(c)}{u'(c)} = \frac{\gamma c^{-\gamma-1}c}{c^{-\gamma}} = \frac{\gamma c^{-\gamma}}{c^{-\gamma}} = \gamma$$
Profit Maximization

- **NOT** an axiom
- Theorem, requiring assumptions
- Perfect markets
Robinson Crusoe

Graph showing the Production Possibility Frontier (PPF) between Fish and Coconuts.
Robinson Crusoe
- Produces to market. Profit max
  - In order to max utility
- Dynamic extension
  - Max PDV
  - If borrowing and lending are perfect
- Uncertainty extension
  - Max contingent-claim profit
  - If insurance is perfect
- Present prices: $p(t)$ and $p(t + 1)$
- $p(t) = R(t)p(t + 1)$, where $R(t)$ is the interest factor
- $r(t) = R(t) - 1$ is the interest rate
- Profit max becomes PDV max:

\[
PDV = p(t)y(t) + p(t + 1)y(t + 1)
= p(t)\left[y(t) + \frac{y(t + 1)}{1 + r(t)}\right]
\]