Economics 4905: Lecture 5
The Economics of Uncertainty

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The Economics of Uncertainty

- The state of nature: s
- Realizations: $s_1, s_2, \ldots$
- Intrinsic Uncertainty
  - Random fundamentals
  - Examples

$$s_1 = \textit{rain} \quad s_2 = \textit{drought}$$
$$s_1 = \textit{hot} \quad s_2 = \textit{cold}$$

- Extrinsic Uncertainty
  - Randomness that does not affect the fundamentals, but does affect outcomes.
  - Examples

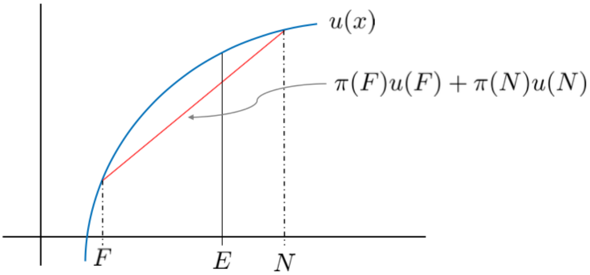
$$s_1 = \textit{no run} \quad s_2 = \textit{run}$$
$$s_1 = \textit{sunspots} \quad s_2 = \textit{no sunspots}$$

Expected Utility

- von Neumann and Morgenstern
  - Expected Utility
  - $V = \pi(s_1)u(x(s_1)) + \pi(s_2)u(x(s_2))$
  - $\pi(s_1) = 1 - \pi(s_2)$
  - $V = \int u(x(s))\pi(s)ds$
- Risk aversion:
  - $u(x)$
  - $u'(x) > 0$
  - $u''(x) < 0$ Risk-averse
- Risk-neutral
  - $u''(x) = 0$
- Risk-loving
  - $u''(x) > 0$

Arrow-Debreu

- Isomorphism
  - Contingent-claims
  - Securities



F – Fire, N – no fire, E – expected value

CRRA

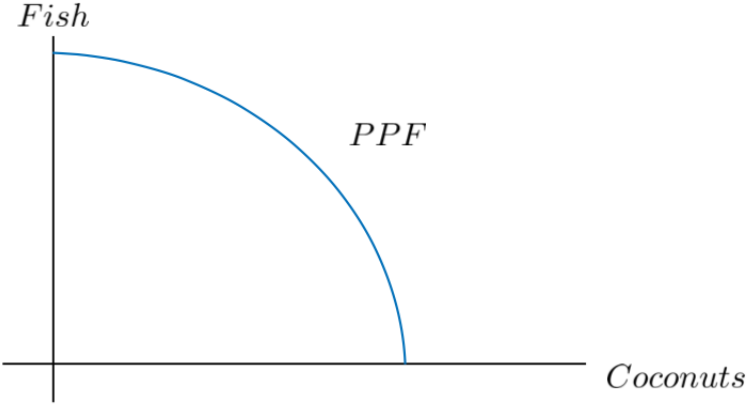
- Kenneth Arrow
- John Pratt
- $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$
    - For $\gamma = 1$, define $u(c) = \log(c)$
- $u'(c) = \left(\frac{1-\gamma}{1-\gamma}\right) c^{-\gamma} = c^{-\gamma} > 0$
- $u''(c) = -\gamma c^{-\gamma-1} < 0$
- Risk-aversion:

$$-\frac{cu''(c)}{u'(c)} = \frac{\gamma c^{-\gamma-1} c}{c^{-\gamma}} = \frac{\gamma c^{-\gamma}}{c^{-\gamma}} = \gamma$$

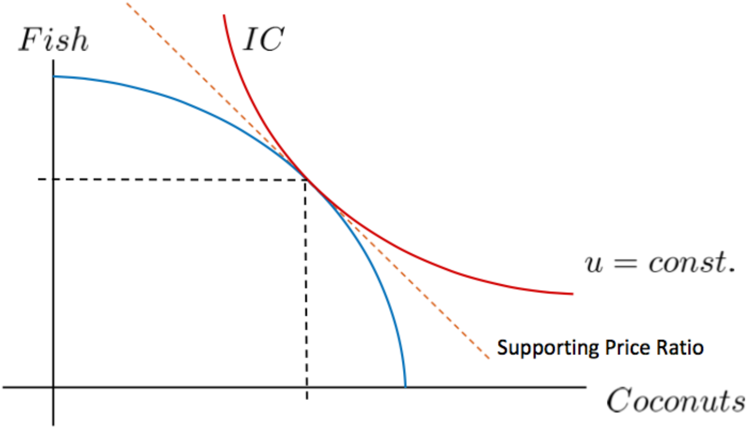
Profit Maximization

- **NOT** an axiom
- Theorem, requiring assumptions
- Perfect markets

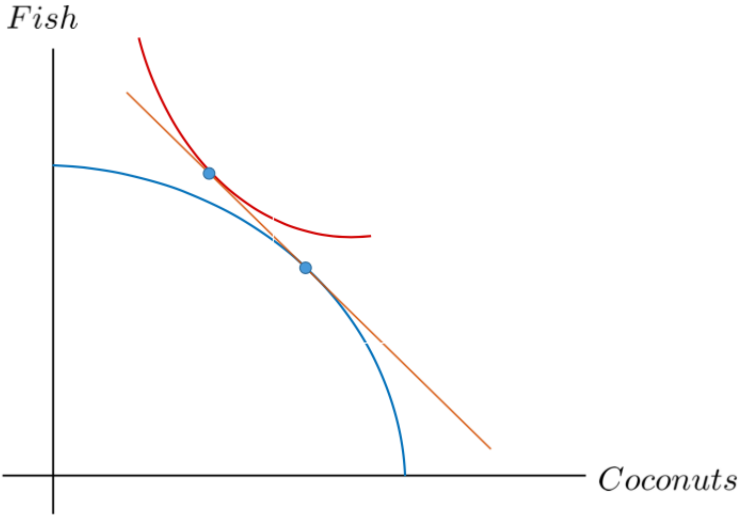
Robinson Crusoe



Robinson Crusoe



Robinson Trades



- Produces to market. Profit max
  - In order to max utility
- Dynamic extension
  - Max PDV
  - If borrowing and lending are perfect
- Uncertainty extension
  - Max contingent-claim profit
  - If insurance is perfect

Intertemporal

- Present prices: $p(t)$ and $p(t+1)$
- $p(t) = R(t)p(t+1)$, where $R(t)$ is the interest factor
- $r(t) = R(t) - 1$ is the interest rate
- Profit max becomes PDV max:

$$\begin{aligned} PDV &= p(t)y(t) + p(t+1)y(t+1) \\ &= p(t)\left[y(t) + \frac{y(t+1)}{1+r(t)}\right] \end{aligned}$$