1 Connections between Futures Markets Economy and Money Market Economy

One good per period, $\ell = 1$, two periods, $t = 1, 2$.

**Futures Market:**

$$\max_{x_1, x_2} u_h(x_1^h, x_2^h)$$
$$\text{s.t. } p^1 x_1^h + p^2 x_2^h = p^1 \omega_1^h + p^2 \omega_2^h$$

Equilibrium is a price vector $(p^1, p^2)$ such that:

$$\sum_h x_t^h = \sum_h \omega_t^h \text{ for } t = 1, 2$$

Define the interest factor $R$ and the interest rate $r$ in terms of the equilibrium commodity prices $(p^1, p^2)$.

**Money Market:**

$$\max_{x_1, x_2} u_h(x_1^h, x_2^h)$$
$$\text{s.t. } p^1 x_1^h + p^m_1 m_1^h = p^1 \omega_1^h$$
$$p^2 x_2^h + p^m_2 m_2^h = p^2 \omega_2^h$$

Equilibrium $(p^1, p^2, p^m_1, p^m_2)$ such that:

$$\sum_h x_t^h = \sum_h \omega_t^h \text{ and } \sum_h m_t^h = 0 \text{ for } t = 1, 2$$

1) Prove that in equilibrium $p^m_1 = p^m_2 = p^m \geq 0$. This is a no-arbitrage-property result.

2) Show that if $(x_1^h, x_2^h), h = 1, \ldots, n$ solves the futures market problem, it also solves the money market problem.

3) Show that if $(x_1^h, x_2^h), h = 1, \ldots, n$ solves the money market problem with $p^m > 0$, then it also solves the futures market problem.

4) **Example A:** 1 good, 2 individuals $h = 1, 2$, 2 periods $t = 1, 2$. Futures markets.

$$u_h = \log(x_1^h) + \beta \log(x_2^h) \text{ for } h = 1, 2$$
Set up the CP and CE for when there is (only) perfect futures markets.

Solve for the CE allocations, the CE prices, the interest factors $R$, and the CE interest rate $r$ for the following cases:

a) $\beta = 1$
b) $\beta = 5$
c) $\beta = 1/5$

Discuss the economics of your answers to parts (a), (b) and (c).

**Example B:** 1 good, 2 individuals, 1 inside money. Money markets.

Replace futures markets in Example A with (inside) money markets.

Set up the CP and the CE in this problem. Show that the CE allocations in Example A are also a CE allocations in Example B. Identify in (a), (b) and (c) which individual is a borrower and which one is a lender. Discuss the economics of your answers.

Show that there is a CE allocation in Example B that is not a CE allocation in Example A. Discuss the economics.