Economics 4905

Financial Fragility and the Macroeconomy
Fall 2017
Problem Set 2
Due Wednesdy, October 5, 2017

1 Connections between Futures Markets Economy and Money Market Economy

One good per period, $\ell = 1$, two periods, $t = 1, 2$.

Futures Market:

$$\begin{aligned} \max \quad & u_h(x_h^1, x_h^2) \\ \text{s.t.} \quad & p^1 x_h^1 + p^2 x_h^2 = p^1 \omega_h^1 + p^2 \omega_h^2 \end{aligned}$$

Equilibrium is a price vector (p^1, p^2) such that:

$$\sum_h x_h^t = \sum_h \omega_h^t \text{ for } t = 1, 2$$

Define the interest factor R and the interest rate r in terms of the equilibrium commodity prices (p^1, p^2).

Money Market:

$$\begin{aligned} \max \quad & u_h(x_h^1, x_h^2) \\ \text{s.t.} \quad & p^1 x_h^1 + p^{m^1} m_h^1 = p^1 \omega_h^1 \\ & p^2 x_h^2 + p^{m^2} m_h^2 = p^2 \omega_h^2 \end{aligned}$$

Equilibrium $(p^1, p^2, p^{m^1}, p^{m^2})$ such that:

$$\sum_h x_h^t = \sum_h \omega_h^t \text{ and } \sum_h m_h^t = 0 \text{ for } t = 1, 2$$

1) Prove that in equilibrium $p^{m^1} = p^{m^2} = p^m \geq 0$. This is a no-arbitrage-property result.

2) Show that if (x_h^1, x_h^2), $h = 1, \ldots, n$ solves the futures market problem, it also solves the money market problem.

3) Show that if $(x_h^1, x_h^2), h = 1, \ldots, n$ solves the money market problem with $p^m > 0$, then it also solves the futures market problem.

4) **Example A:** 1 good, 2 individuals $h = 1, 2$, 2 periods $t = 1, 2$. Futures markets.

$$u_h = \log(x_h^1) + \beta \log(x_h^2) \text{ for } h = 1, 2$$

$$\text{Mr 1:} \quad \omega_1 = (100, 50) = (\omega_{11}, \omega_{12})$$
$$\text{Mr 2:} \quad \omega_2 = (50, 100) = (\omega_{21}, \omega_{22})$$

Set up the CP and CE for when there is (only) perfect futures markets.

Solve for the CE allocations, the CE prices, the interest factors R, and the CE interest rate r for the following cases:

a) $\beta = 1$

b) $\beta = 5$

c) $\beta = 1/5$

Discuss the economics of your answers to parts (a), (b) and (c).

Example B: 1 good, 2 individuals, 1 inside money. Money markets.

Replace futures markets in Example A with (inside) money markets.

Set up the CP and the CE in this problem. Show that the CE allocations in Example A are also a CE allocations in Example B. Identify in (a), (b) and (c) which individual is a borrower and which one is a lender. Discuss the economics of your answers.

Show that there is a CE allocation in Example B that is not a CE allocation in Example A. Discuss the economics.