

Economics 4905
 Financial Fragility and the Macroeconomy
 Fall 2017
 Problem Set 3
 Due Monday, October 23, 2017

1. Diamond-Dybvig Bank #1

The probability λ of being impatient is 60%. The utility function is:

$$u(c) = 5 - \frac{1}{\sqrt{c}}.$$

The rate of return to the asset harvested late is 300%, i.e.,

$$R = 4$$

- (a) What is the depositor's *ex-ante* expected utility W as a function of c_1 , consumption in period 1, and c_2 , consumption in period 2?
- (b) Show that the depositor prefers consumption smoothing.
- (c) Why can't she insure on the market or self-insure against liquidity shocks?

Assume that her endowment is 100 and that she deposits her entire endowment in the bank.

- (d) What is her utility W in autarky?
- (e) What is her utility W under perfect smoothing, i.e. when $c_1 = c_2$?
- (f) What is the bank's resource constraint RC? Write this down precisely. Explain this in words.
- (g) What is the incentive problem? Write this down precisely and explain in words the incentive constraint IC.
- (h) Find the optimal deposit contract for this bank. What is W if there is no run?
- (i) Why is there a run equilibrium for this bank?
- (j) Calculate the following numerical values of *ex-ante* utility W and rank them in numerical ascending order: $W_{autarky}$, $W_{perfect\ smoothing}$, $W_{no\ run}$, W_{run} .
- (k) Assume that the run probability s is 5%. Will individuals deposit in this bank? That is, will they accept this banking contract? Explain.

2. Diamond-Dybvig Bank #2

The probability of being impatient is $\lambda = 0.4$. The type (patient or impatient) is realized in period 1 and it is private information. The utility function is:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

where $\gamma = \frac{3}{2} > 0$. Each individual has one unit of endowment in period 0. There is costless storage. If the endowment $\omega = 1$ is invested in period 0 and is harvested in period 1, the return is 0. If harvested late, the return is 4. Assume that the banking industry is free-entry.

- (a) What is the depositor's *ex-ante* expected utility W as a function of c_1 consumption in period 1, and c_2 , consumption in period 2?
- (b) Show that the depositor prefers consumption smoothing.
- (c) What is the bank's resource constraint RC? Write this down precisely and explain in words.
- (d) What is the incentive problem? Write down the incentive constraint IC precisely, and explain it in words.
- (e) Solve for the optimal deposit contract for the post-deposit bank assuming that there is no run. (That is: Write down the optimal first-period payment d_1^* as a function of λ , R and γ)
- (f) Is there a run equilibrium to this "optimal contract"?

3. The 2-Consumer, Pre-Deposit Bank

The utility function of the impatient agent is

$$u(x) = \frac{Ax^{1-\gamma}}{1-\gamma}$$

and the utility function of the patient agent is

$$v(x) = \frac{x^{1-\gamma}}{1-\gamma},$$

where $\gamma > 1$. The probability λ of being impatient is 50%. The parameter γ is 1.01. The endowment is $y = 3$. The rate of return on the asset if harvested late is 50%, i.e., $R = 1.5$. The probability of being first in line if 2 agents withdraw early is 50%.

- (a) Solve for the numerical values of c^{early} and c^{wait} . Show that they are independent of the impulse parameter A .
- (b) Write down the expression for the *ex-ante* expected utility of the depositor, W .
- (c) Solve for \hat{c} , the value of c that maximizes W in the post-deposit game, as a function of A .
- (d) Calculate the critical values A^{early} and A^{wait} .
- (e) Let $A = 7$. Describe the optimal contract for the pre-deposit game, $c^*(s)$, as a function of s , the exogenous run probability.