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Diamond and Dybvig's Classic Theory of Financial Intermediation: What's Missing?

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Summary

- Is the existence of a bank run equilibrium an inevitable result of the assumptions of the Diamond and Dybvig model, or are the feasible bank contracts that lead to a unique run-proof equilibrium with optimal risk-sharing?
- Lin and Green examine a finite-trader version of Diamond of Dybvig with sequential service. They allow the bank to adjust payouts to depositors as they accumulate information from agents who show up at the bank.
- Findings: There is a contract structure that:
  - Allows optimal insurance of liquidity preference risk.
  - Provides an incentive for all agents to truthfully report their type (patient or impatient) when they arrive at the bank. The contract is thus run-proof.

Background: Diamond and Dybvig

- Four key features of the Diamond and Dybvig modeling environment:
    1. Uncertainty among agents regarding desired timing of consumption
    2. Privacy of information regarding these preferences (investor type is unverifiable)
    3. Sequential service constraint: Depositors arrive at the bank at different times and the bank fulfills deposit requests in the order in which depositors arrive.
    4. A long-term investment project which is costly to interrupt before completion.
- Assumption 2 means agents cannot write insurance contracts that condition on time 1 preferences. Without a bank, all agents will invest in the project, but will choose to interrupt if they are impatient at time 1. This arrangement does not maximize ex-ante expected utility.

Risk-Sharing and Bank Runs

- Bank contracts improve upon autarky consumption allocation. The bank allows investors to choose at time 1 between ex-ante optimal time 1 consumption for impatient depositors and a contract that yields $R(1-rf)/(1-f)$ where f is the fraction of investors who withdraw early.
- If all impatient investors choose to consume at time 1 and patient investors consume at time 2, this is an optimal equilibrium. If patient consumers choose to consume early, the bank will not be able to pay out deposits to all consumers.
- The question of whether patient investors withdraw early depends on the value of f above, and thus on the assumed actions of other depositors.
- A bank run equilibrium is the pure strategy Nash Equilibrium where all depositors, patient or impatient, attempt to withdraw at time 1.

Responses to Bank Run Equilibrium

- Diamond and Dybvig considers two cases:
    1. No aggregate uncertainty regarding fraction of impatient investors
    2. Fraction of impatient investors is a random variable
- For case 1, a suspension of payments scheme is sufficient to eliminate bank-run equilibrium.
- For case 2, we need some other scheme (such as deposit insurance). If the bank doesn't know the proportion of impatient investors, it risks suspending deposits "too late" to avoid a run.

Green and Lin Respond

- Claim: The existence of a bank-run equilibrium relies on restrictions to the sent of admissible bank contracts, not on the key assumptions of the modeling environment (assumptions 1-4 from earlier).
- Consider expanding the set of admissible banking contracts (beyond forcing the bank to pay every depositor a fixed amount at time 1). If we allow the bank to vary payouts as subsequent depositors approach the bank at time 1, we can devise a scheme that excludes the bank run equilibrium.
- Such contracts are still optimal: They maximize ex-ante expected utility of investors given uncertainty regarding investor type and the order in which each investor arrives at the bank.

Model Set-Up

- We will consider a finite-trader version of Diamond and Dybvig. Each trader will be impatient with probability p and patient with probability (1-p) and be given an initial 1 unit endowment.
- Assume there are 3 traders (necessary only for ease of exposition) with CRRA utility, with parameter $\gamma>1$. The bank invests in a single project with non-random time 1 payout of $R>1$.
- All traders pool their resources and set up a bank before types are revealed. The bank is a mechanism that describes what each trader gets as a payout depending on the time at which they arrive at the bank (which is random), and whether they claim to be patient or impatient.
- At date 0, traders send a signal to the bank where the signal takes the form $m_i \in \{0,1\}$ where *i* is the order at which they arrive at the bank. A signal of 0 implies that the trader is impatient and wants to withdraw, and a signal of 1 implies that the trader is patient and will wait until date 1 to consume.
- The sequential service constraint implies that the payout to any depositor can be a function only of information provided by investors who have already withdrawn. That is m_i can depend on the withdrawal information of the first *i-1* investors.

The Model Without a Sequential Service Constraint

- First consider the less realistic case where the bank learns all investors' types prior to filling deposits. We will use this to show our modeling strategy.
- Here we can choose payouts based on any revealed distribution of patient and impatient investors. Thus, maximizing ex-post realized utility is the same as maximizing ex-ante expected utility.
- We let $\theta(\omega)$ be the number of patient traders in any realized state ω. Let I be the amount of money deposited into the bank (equal to the number of traders). We seek to maximize:

$$\big(I-\theta(\omega)\big)v(c_0)+\theta(\omega)v(c_1)\ subject\ to\ (I-\theta(\omega))c_0+R^{-1}\theta(\omega)c_1)=I$$

- We thus have the first order condition $v'(c_0(\theta(\omega)))=Rv'(c_1(\theta(\omega)))$ the marginal utility of a trader impatient and patient traders balance.
- This gives us equation (3): $c_0(\theta)=\frac{I}{I+\theta\left(R^{(1/\gamma)-1}-1\right)}$ and (4): $c_1(\theta)=\frac{IR^{1/\gamma}}{I+\theta(R^{(1/\gamma)-1}-1)}$
- Since R>1 we see that period 1 consumption is greater than period zero consumption so a patient trader would always choose to wait until period 1 to consumer.

Adding the Sequential Service Constraint: Model Set-up

- Now look at how our results change with the sequential service constraint. We will maximize the sum of expected utilities of the three investors. We will find these allocations by assuming investors truthfully report their type and then assess whether truth-telling is optimal.
- We will now let $x_i(m)$ be the amount paid depositor i where m is now a vector of the first i realizations of the investors' reported type. That is, if i=2, m=(0,1) implies that the first investor announced himself to be impatient and the second investor to arrive announces himself to be patient.
- We have the following maximization:

$$
\begin{aligned}
(6) \qquad \max\ & (1-p)v(x_1(0)) + pE_{m_2,m_3}v(x_1(1,m_2,m_3)) \\
& + (1-p)E_{m_1}v(x_2(m_1,0)) \\
& + pE_{m_1,m_3}v(x_2(m_2,1,m_3)) \\
& + (1-p)E_{m_1,m_2}v(x_3(m_1,m_2,0)) \\
& + pE_{m_1,m_2}v(x_3(m_1,m_2,1))
\end{aligned}
$$

Subject to:

$$\sum_{i\in I,m_i=1} x_i(m) = R\left[I - \sum_{i\in I,m_i=0} x_i(m)\right]$$

Solving for Optimal Payouts: Trader 3

- We have an additively separable objective function, which suggests we can solve our optimization problem using backward induction, starting with the last trader to arrive at the bank and moving backward toward earlier arriving customers.
- We first let $y(m_1, m_2)$ be the amount of the initial deposits left at the bank after the first two traders have had the opportunity to withdraw. Y is clearly dependent on how many of the traders claimed to be impatient.
- A trader who claims to be patient receives the payout $\frac{Ry}{\theta}$. If the trader is impatient, his consumption will be allocated to maximize the sum of his utility and the traders who have not yet received payouts (the previous traders claiming to be patient).
- So we have $x_3(m_1, m_2, 0) = \arg\max x_3 \;\; v(x_3) + (m_1 + m_2)v(\frac{R(y-x_3)}{m_1+m_2})$ with first order condition:

$$v'(x_3) = Rv'\left(\frac{R(y - x_3)}{m_1 + m_2}\right).$$

Optimality of Truth-telling for Trader 3

- The fact that R>1 implies that $v'(x_3) > v'\left(\frac{R(y-x_3)}{m_1+m_2}\right)$ and since $v'' < 0$ this implies that the amount consumed by an impatient third trader is less than the amount consumed by the patient traders who arrived at the bank earlier.
- We can also state the following lemma (8):

$$\frac{y}{m_1 + m_2 + 1} < x_3(m_1, m_2, 0) < \frac{Ry}{m_1 + m_2 + 1}.$$

- The impatient third trader is thus allocated consumption that is less than what he would receive if he claimed to be patient. So a patient third investor will never find it optimal to lie about his type. Since an impatient investor doesn't value time 1 consumption at all, it is clear trader 3 finds truth-telling to be optimal.
- The explicit solution for impatient third trader consumption is (9): $\frac{y}{\left[1+(m_1+m_2)R^{\frac{1}{\gamma}-1}\right]}$.

The Second Trader with a Patient First Trader

- We next move backward to the second trader to approach the bank. Let us suppose this trader is impatient and that trader 1 was patient.
- When we determine the amount to allocate to trader 2, we balance this trader's expected utility with the expected utility of the third and first trader.
- With probability p trader 3 is patient in which case traders 1 and 3 consume $\frac{R(I-x_2(1,0))}{2}$. With probability 1-p trader 3 is impatient, and consumes the amount given by equation (9). Thus, we choose trader 2 consumption to solve:

$$max_{x2}\ v(x_2) + 2pv\left(\frac{R[I-x_2(1,0)]}{2}\right) + (1-p)[v(x_3(1,0,0)) + v(x_1(1,0,0))].$$

- The explicit solution is $x_2 = \frac{I}{I+A^{\frac{1}{\gamma}}}$ where $A = (1-p)\left(1 + R^{\frac{1}{\gamma}-1}\right)^{\gamma} + \ 2^{\gamma}pR^{1-\gamma}$.

The Second Trader with an Impatient First Trader

- If instead, the first trader was impatient, then this trader is assigned consumption immediately and when the second trader approaches the bank, his consumption is assigned only to maximize the sum of the expected utility of traders 2 and 3.
- Again, with probability p trader 3 is patient and gets consumption $R(I - x_1(0) - x_2(0,0))$ and with probability 1-p is assigned $I - x_2 - x_3$. Thus trader 2 consumption here generates the first order condition:

$$v'(x_2(0,0)) = (1-p)v'(I - x_1(0) - x_2) + pRv'(R[I - x_1(0) - x_2]).$$

- This yields the solution $x_2(0,0) = \frac{[I - x_1(0)]}{1+B}$ where $B = [1 - p + pR^{1-\gamma}]^{1/\gamma}$.
- Finally, we can state the impatient first trader's consumption as

$$x_1(0) = \frac{I}{1 + [pA + (1-p)(1+B)^{\gamma}]^{\frac{1}{\gamma}}}.$$

Some Key Results

- We will discuss three results that follow from our analysis. The key point to remember, is that in every maximization problem we solve, we are, in some way balancing marginal utilities of each of the three traders. When we balance marginal utilities of patient vs. impatient traders, the marginal utility of the patient trader will be scaled by R, the added return on the initial investment that is gained by waiting to consume.
- We now state 3 results:
  1. $x_1(0)\ is\ an\ increasing\ function\ of\ p$
  2. $For\ any\ probability\ p, x_1(0) > x_2(0,0) > x_3(0,0,0)$
  3. The payouts to each trader are satisfy incentive compatibility. That is, each trader has an incentive to truthfully report his type to the bank.

Intuition Behind the Results

- Start with the first proposition. The amount allocated to an impatient first trader increases in p because as p rises, it becomes more likely that subsequent investors will be patient. Each of these investors benefits from the positive return R, so we need to leave less money in the bank to guard against the risk that subsequent investors will be impatient. Similarly, we have that the consumption of the impatient second trader also increases with p.
- Next, look at the second proposition. The logic is similar. The impatient first trader receives more than the other two traders when both are impatient. This is because, once the first trader reveals himself to be impatient, we know for certain that there will be one impatient trader, but the subsequent two traders may still be patient, and have the benefit of the positive return technology. In turn, when the second trader arrives, we now know with certainty that two traders are impatient, but the third might still be patient. We then weight that trader's possible utilities probabilistically, placing a positive weight on the higher utility associated with being patient.

Truthful Reporting of Type

- It is already clear that an impatient trader would never claim to be patient, since this trader only values time 0 consumption.
- Can a patient trader benefit from claiming to be impatient? We know that trader 3 can never gain from lying about his type. Thus when trader 2 makes his decision, he will evaluate his expected payoffs, using the objective probability measure, p and (1-p).
- We will have that trader 2 will choose to tell the truth if and only if utility from the payout where he claims to be impatient is less than his expected utility of the payout where he claims to be patient.
- Recall, each time we calculate payoffs we take a linear weighting of investors' marginal utilities. Suppose for example we are calculating this trader's payoff when trader 1 was impatient. We then look only at trader's 1 and 3 when we maximize utilities. If trader 2 claims to be impatient, then he must be paid out of the assets currently available in the bank, and must balance his utility against the potential future utility of investor 3 who may be patient or impatient.
- If trader 2 is patient, we do the same balancing of utilities, but this time trader 2 benefits from the higher value of bank resources that result from leaving deposits in the bank for an extra period.