Equilibrium Bank Runs

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Introduction

- Diamond and Dybvig’s classic model laid the theoretical groundwork for bank runs
  - However their model does not seem to explain equilibrium bank runs under optimal contracts
- Green and Lin also show that mechanisms that support constrained efficient allocation preclude bank runs equilibria
- However bank runs do occur…
  - Peck and Shell show that bank runs can still occur in equilibrium with “optimal contracts”
  - Also formalize the conditions under which pre and post deposit run equilibria can still occur
  - Possible to eliminate bank runs but not without sacrificing welfare
The Model

- Three periods $t = 0, 1, 2$
- Finite number of consumers $N$, each endowed with $y$ units of wealth at $t = 0$
- Let $\alpha$ denote the number of impatient consumers with probability $f(\alpha)$
- Given a consumer is patient, update conditional probability of impatient consumers to

$$f_p(\alpha) = \frac{[1 - (\alpha/N)]f(\alpha)}{\sum_{\alpha' = 0}^{N-1}[1 - (\alpha'/N)]f(\alpha')}$$
The Model Continued

- Impatient only get utility from consumption in \( t = 1 \)
- Patient and Impatient consumers can have different utility functions
  - Patient: \( u(c^1), \frac{xu''(x)}{u'(x)} < -1 \)
  - Impatient: \( v(c^1 + c^2), \frac{xv''(x)}{v'(x)} < -1 \)
- Investment Technology
  \[
  \begin{cases} 
  1 & t = 1 \\
  R > 1 & t = 2 
  \end{cases}
  \]
Timeline

- In period 0, bank designs the contract by maximizing ex ante expected utility of consumers.
- In period 1, each consumer learns her type and decides when to arrive at the bank.
- Mechanism satisfies the following sequential service constraint:
  - Consumers arrive in a random order at period 1.
  - Consumption is allocated as a function of the history of transactions up until that point.
- Then the resource condition can be written as:
  \[
  c^2(\alpha_1) = \frac{[Ny - \sum_{z=1}^{\alpha_1} c^1(z)]R}{N - \alpha_1}, \quad c^1(N) = Ny - \sum_{z=1}^{N-1} c^1(z)
  \]
- Banking mechanism \( m = (c^1(1), \ldots, c^1(z), \ldots, c^1(N), c^2(0), \ldots, c^2(N - 1)) \)
Green and Lin - Key Differences

- Patient consumers do not show up to declare their type
  - Showing up is inferred to be a report of “impatient”
  - Consumption cannot depend on the number of people who have declared themselves to be patient
  - Appendix B has an example where this assumption is dropped but equilibrium bank run still exists

- No clock. Individuals do not know their number in queue
  - Iterated elimination of dominated strategies by backward elimination not possible

- Patient and Impatient consumers can have different utility functions
Welfare

- Ex ante welfare

\[ \bar{W}(m) = \sum_{\alpha=0}^{N-1} f(\alpha) \left[ \sum_{z=1}^{\alpha} u(c_1(z)) + (N - \alpha)\nu \left( \frac{Ny - \sum_{z=1}^{\alpha} c_1(z)}{N - \alpha} \right) \right] \]
\[ + f(N) \left[ \sum_{z=1}^{N-1} u(c_1(z)) + u \left( Ny - \sum_{z=1}^{N-1} c_1(z) \right) \right] \]

- Welfare when all patient consumers choose \( t = 1 \)

\[ W^{run}(m) = \sum_{\alpha=0}^{N} f(\alpha) \left[ \frac{\alpha}{N} \sum_{z=1}^{N} u(c_1(z)) + \frac{N - \alpha}{N} \sum_{z=1}^{N} v(c_1(z)) \right] \]
Optimal Contract

- Incentive Compatibility Constraint

\[ \sum_{\alpha=0}^{N-1} f_p(\alpha) \left[ \frac{1}{\alpha + 1} \sum_{z=1}^{\alpha+1} v(c^1(z)) \right] \leq \sum_{\alpha=0}^{N-1} f_p(\alpha) v \left( \frac{[Ny - \sum_{z=1}^\alpha c^1(z)]R}{N - \alpha} \right) \quad \ldots (1) \]

- The “optimal contract” solves

\[ \max_{(c^1(1), \ldots, c^1(N-1))} \bar{W}(m) \]

subject to \((1)\)
Run Equilibrium

- Definition: Given a mechanism \( m \in M \), the post deposit game is, said to have a run equilibrium if there is a Bayes-Nash equilibrium in which all consumers choose to withdraw in period 1, independent of the realization of their type.

- Incentive compatibility is different when other patient consumers choose period 1

\[
\frac{1}{N} \sum_{z=1}^{N} v(c^1(z)) \geq v \left( N y - \sum_{z=1}^{N-1} c^1(z) R \right) \quad \text{(2)}
\]
Run Equilibrium

- Proposition: For some economies, a run equilibrium exists at “optimal contract” $m^*$
- Proof by example with $N = 2$
- Impatience is i.i.d (patient with probability $p$ and impatient with probability $1 - p$)
- Let the utility functions be given by
  \[ u(x) = \frac{Ax^{1-a}}{1-a}, \quad v(x) = \frac{x^{1-b}}{1-b} \]
- Let $A = 10, a = 1.01, b = 1.01, p = \frac{1}{2}, R = 1.05, y = 3$
- Optimal mechanism with $c^1(1) = 3.1481$, satisfies (2) so a run equilibrium exists
- A more general example with 300 consumers and correlated impatience in Appendix A
- Same example but where all consumers have to explicitly declare impatience is also done in Appendix B
Pre-Deposit Game Timeline

- The bank announces its mechanism
- In period 0, consumers decide whether or not to deposit
- In period 1, each consumer:
  - Observes a sunspot variable $\sigma \sim U[0,1]$
  - Learns her type
  - Decides the period of arrival on both of these factors
- Assumptions
  - The space of mechanisms $\mathcal{M}$ is the same for pre-deposit and post-deposit game
  - Sunspots do not affect preferences, the likelihood of being impatient, endowments or technology
  - Banks cannot choose a withdrawal schedule as a function of $\sigma$
Pre-Deposit Game Run Equilibrium

- Definition: Given a mechanism $m \in M$, the pre-deposit game is said to have a run equilibrium if there is a subgame-perfect Nash equilibrium in which (i) consumers are willing to deposit, and (ii) for some set of realizations of $\sigma$ occurring with positive probability, all consumers choose to withdraw in period 1, independent of the realization of their type.
Pre-Deposit Game Run Equilibrium Continued

- Proposition: Consider a mechanism \( m \in M \), for which the post-deposit game has an equilibrium in which all patient consumers choose period 2, yielding welfare strictly higher than welfare under autarky. Then the pre-deposit game has a run equilibrium if and only if the post-deposit game has a run equilibrium.

- \((necessity)\) Easy

- \((sufficiency)\) Sketch
  - \( \sigma < s \), all consumers choose period 1
  - \( \sigma \geq s \), impatient consumers choose period 1 and patient consumers choose period 2
  - Each sub-game after deposits are made and \( \sigma \) observed is in equilibrium
  - Run equilibrium with \( \sigma < s \) and non-run equilibrium with \( \sigma \geq s \)
  - Consumers are willing to deposit because for “Sufficiently Small \( s \)"

\[
sw^{\text{run}}(m) + (1-s)\bar{W}(m) \gg W^{\text{autarky}}
\]
Propensity to Run

- Suppose the economy has propensity to run $s$ if whenever:
  - $\sigma < s$, all consumers choose period 1 if post-deposit game admits run equilibrium
  - $\sigma \geq s$, equilibrium is selected in which patient consumers choose period 2

- Definition: Given a mechanism $m \in M$, and a propensity to run $s$, ex ante welfare for the pre-deposit game denoted as $W(m, s)$, is given by
  \[
  W(m, s) = \begin{cases} 
  sW^{\text{run}}(m) + (1 - s)\hat{W}(m) & \text{if } m \text{ has a run equilibrium} \\
  \hat{W}(m) & \text{if } m \text{ does not have a run equilibrium}
  \end{cases}
  \]

- The $s$-optimal mechanism solves
  \[
  \max_{(c_1^1(1), \ldots, c^1(N-1))} W(m, s)
  \]
  subject to (1)
Propensity to Run

- Proposition: For some economies with a sufficiently small propensity to run, $s$, the optimal mechanism for the pre-deposit game has a run equilibrium.

- Proof sketch
  - Consider the 2 consumer example from before
  - Patient consumers choose period 2 when $\sigma \geq s$ so (1) holds and is binding as shown before
  - So (1) must hold as an equality and be binding for sufficiently small $s$ i.e. $m^*$ is still optimal
  - By continuity, $W(m, s)$ can be made sufficiently close to the previous optimal solution
  - Which is better than the welfare under autarky so consumers deposit
  - Since $m^*$ has a post-deposit run equilibrium, it has a pre-deposit equilibrium

- Not so easy for to calculate $s$-optimal mechanism for general economies
Propensity to Run

- As $s$ increases, welfare under $m^*$ falls

\[
W(m^*, s) = (1 - s_0)\hat{W}(m^*) + s_0 W^{run}(m^*) = \hat{W}(m^{no-run})
\]

- Largest propensity to run $s_0$ solves

\[
W(m^{so-run}, s) = 0.27396
\]

\[
W(m^{so-run}, s) = 0.27158
\]

\[
W(m^{so-run}, s) = 0.27000
\]
Conclusion

- Possibility of equilibrium bank run does not depend on a simple and suboptimal specification of the deposit contract

- Types of economies that allow bank runs
  - Significant uncertainty about number of patient and impatient consumers
  - Utility functions reflect a high degree of “impulse demand”
  - Incentive to choose period 1 for patient consumers

- Eliminating these bank runs may require a sacrifice of welfare