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Equilibrium Bank Runs

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Introduction

- Diamond and Dybvig's classic model laid the theoretical groundwork for bank runs
  - However their model does not seem to explain equilibrium bank runs under optimal contracts
- Green and Lin also show that mechanisms that support constrained efficient allocation preclude bank runs equilibria
- However bank runs do occur…
  - Peck and Shell show that bank runs can still occur in equilibrium with "optimal contracts"
  - Also formalize the conditions under which pre and post deposit run equilibria can still occur
  - Possible to eliminate bank runs but not without sacrificing welfare

The Model

- Three periods $t = 0, 1, 2$
- Finite number of consumers N, each endowed with y units of wealth at $t = 0$
- Let α denote the number of impatient consumers with probability $f(\alpha)$
- Given a consumer is patient, update conditional probability of impatient consumers to

$$f_p(\alpha) = \frac{[1 - (\alpha/\mathrm{N})]\mathrm{f}(\alpha)}{\sum_{\alpha'=0}^{N-1}[1 - (\alpha'/\mathrm{N})]\mathrm{f}(\alpha')}$$

The Model Continued

- Impatient only get utility from consumption in $t = 1$
- Patient and Impatient consumers can have different utility functions
  - Patient: $\mathrm{u}(c^1)$, $\frac{xu''(x)}{\mathrm{u}'(\mathrm{x})} < -1$
  - Impatient: $\mathrm{v}(c^1 + c^2)$, $\frac{\mathrm{xv}''(\mathrm{x})}{\mathrm{v}'(\mathrm{x})} < -1$
- Investment Technology $\begin{cases} 1 & t = 1 \\ R > 1 & t = 2 \end{cases}$

Timeline

- In period 0, bank designs the contract by maximizing ex ante expected utility of consumers
- In period 1, each consumer learns her type and decides when to arrive at the bank
- Mechanism satisfies the following sequential service constraint
  - Consumers arrive in a random order at period 1
  - Consumption is allocated as a function of the history of transactions up until that point
- Then the resource condition can be written as

$$c^2(\alpha_1) = \frac{[\mathrm{Ny} - \sum_{z=1}^{\alpha_1} c^1(z)]R}{N - \alpha_1}, \qquad c^1(N) = Ny - \sum_{z=1}^{N-1} c^1(z)$$

- Banking mechanism $\boldsymbol{m} = (c^1(1), \dots, c^1(z), \dots, c^1(N), c^2(0), \dots, c^2(N-1))$

Green and Lin - Key Differences

- Patient consumers do not show up to declare their type
  - Showing up is inferred to be a report of “impatient”
  - Consumption cannot depend on the number of people who have declared themselves to be patient
  - Appendix B has an example where this assumption is dropped but equilibrium bank run still exists
- No clock. Individuals do not know their number in queue
  - Iterated elimination of dominated strategies by backward elimination not possible
- Patient and Impatient consumers can have different utility functions

Welfare

- Ex ante welfare

$$\widehat{W}(\boldsymbol{m}) = \sum_{\alpha=0}^{N-1} f(\alpha)\left[\sum_{z=1}^{\alpha} u\big(c^1(z)\big) + (N-\alpha)v\left(\frac{[Ny - \sum_{z=1}^{\alpha} c^1(z)]R}{N-\alpha}\right)\right]$$
$$+ f(N)\left[\sum_{z=1}^{N-1} u\big(c^1(z)\big) + u\left(Ny - \sum_{z=1}^{N-1} c^1(z)\right)\right]$$

- Welfare when all patient consumers choose t = 1

$$W^{run}(\boldsymbol{m}) = \sum_{\alpha=0}^{N} f(\alpha)\left[\frac{\alpha}{N}\sum_{z=1}^{N} u\big(c^1(z)\big) + \frac{N-\alpha}{N}\sum_{z=1}^{N} v\big(c^1(z)\big)\right]$$

Optimal Contract

- Incentive Compatibility Constraint

$$\sum_{\alpha=0}^{N-1} f_p(\alpha)\left[\frac{1}{\alpha+1}\sum_{z=1}^{\alpha+1} v\left(c^1(z)\right)\right] \leq \sum_{\alpha=0}^{N-1} f_p(\alpha) v\left(\frac{[Ny - \sum_{z=1}^{\alpha} c^1(z)]R}{N-\alpha}\right) \ldots (1)$$

- The "optimal contract" solves

$$\max_{(c^1(1),\ldots,c^1(N-1))} \widehat{W}(\boldsymbol{m})$$
$$subject\ to\ (1)$$

Run Equilibrium

- Definition: Given a mechanism $\boldsymbol{m} \in \boldsymbol{M}$, the post deposit game is, said to have a run equilibrium if there is a Bayes-Nash equilibrium in which all consumers choose to withdraw in period 1, independent of the realization of their type
- Incentive compatibility is different when other patient consumers choose period 1

$$\frac{1}{N}\sum_{z=1}^{N} v\left(c^1(z)\right) \geq v\left(\left[Ny - \sum_{z=1}^{N-1} c^1(z)\right]R\right) \ldots (2)$$

Run Equilibrium

- Proposition: For some economies, a run equilibrium exists at “optimal contract” **m***
- Proof by example with $N = 2$
- Impatience is i.i.d (patient with probability p and impatient with probability $1 - p$)
- Let the utility functions be given by

$$u(x) = \frac{Ax^{1-a}}{1-a}, \qquad v(x) = \frac{x^{1-b}}{1-b}$$

- Let $A = 10, a = 1.01, b = 1.01, p = \frac{1}{2}, R = 1.05, y = 3$
- Optimal mechanism with $c^1(1) = 3.1481$, satisfies (2) so a run equilibrium exists
- A more general example with 300 consumers and correlated impatience in Appendix A
- Same example but where all consumers have to explicitly declare impatience is also done in Appendix B

Pre-Deposit Game Timeline

- The bank announces its mechanism
- In period 0, consumers decide whether or not to deposit
- In period 1, each consumer:
  - Observes a sunspot variable $\sigma \sim U[0,1]$
  - Learns her type
  - Decides the period of arrival on both of these factors
- Assumptions
  - The space of mechanisms **M** is the same for pre-deposit and post-deposit game
  - Sunspots do not affect preferences, the likelihood of being impatient, endowments or technology
  - Banks cannot choose a withdrawal schedule as a function of σ

Pre-Deposit Game Run Equilibrium

- Definition: Given a mechanism $\boldsymbol{m} \in \boldsymbol{M}$, the pre-deposit game is said to have a run equilibrium if there is a subgame-perfect Nash equilibrium in which (i) consumers are willing to deposit, and (ii) for some set of realizations of σ occurring with positive probability, all consumers choose to withdraw in period 1, independent of the realization of their type.

Pre-Deposit Game Run Equilibrium Continued

- Proposition: Consider a mechanism $\boldsymbol{m} \in \boldsymbol{M}$, for which the post-deposit game has an equilibrium in which all patient consumers choose period 2, yielding welfare strictly higher than welfare under autarky. Then the pre-deposit game has a run equilibrium if and only if the post-deposit game has a run equilibrium
- $(necessity)$ Easy
- $(sufficiency)$ Sketch
  - $\sigma < s$, all consumers choose period 1
  - $\sigma \geq s$, impatient consumers choose period 1 and patient consumers choose period 2
  - Each sub-game after deposits are made and σ observed is in equilibrium
  - Run equilibrium with $\sigma < s$ and non-run equilibrium with $\sigma \geq s$
  - Consumers are willing to deposit because for "Sufficiently Small s"

$$sW^{run}(\boldsymbol{m}) + (1-s)\widehat{W}(\boldsymbol{m}) \gg W^{autarky}$$

Propensity to Run

- Suppose the economy has propensity to run s if whenever:
  - $\sigma < s$, all consumers choose period 1 if post-deposit game admits run equilibrium
  - $\sigma \geq s$, equilibrium is selected in which patient consumers choose period 2
- Definition: Given a mechanism $\boldsymbol{m} \in \boldsymbol{M}$, and a propensity to run s, ex ante welfare for the pre-deposit game denoted as W(**m**, s), is given by

$$W(\boldsymbol{m}, s) = \begin{cases} sW^{run}(\boldsymbol{m}) + (1-s)\widehat{W}(\boldsymbol{m}) & if\ \boldsymbol{m}\ has\ a\ run\ equilibrium \\ \widehat{W}(\boldsymbol{m}) & if\ \boldsymbol{m}\ does\ not\ have\ a\ run\ equilibrium \end{cases}$$

- The s-optimal mechanism solves

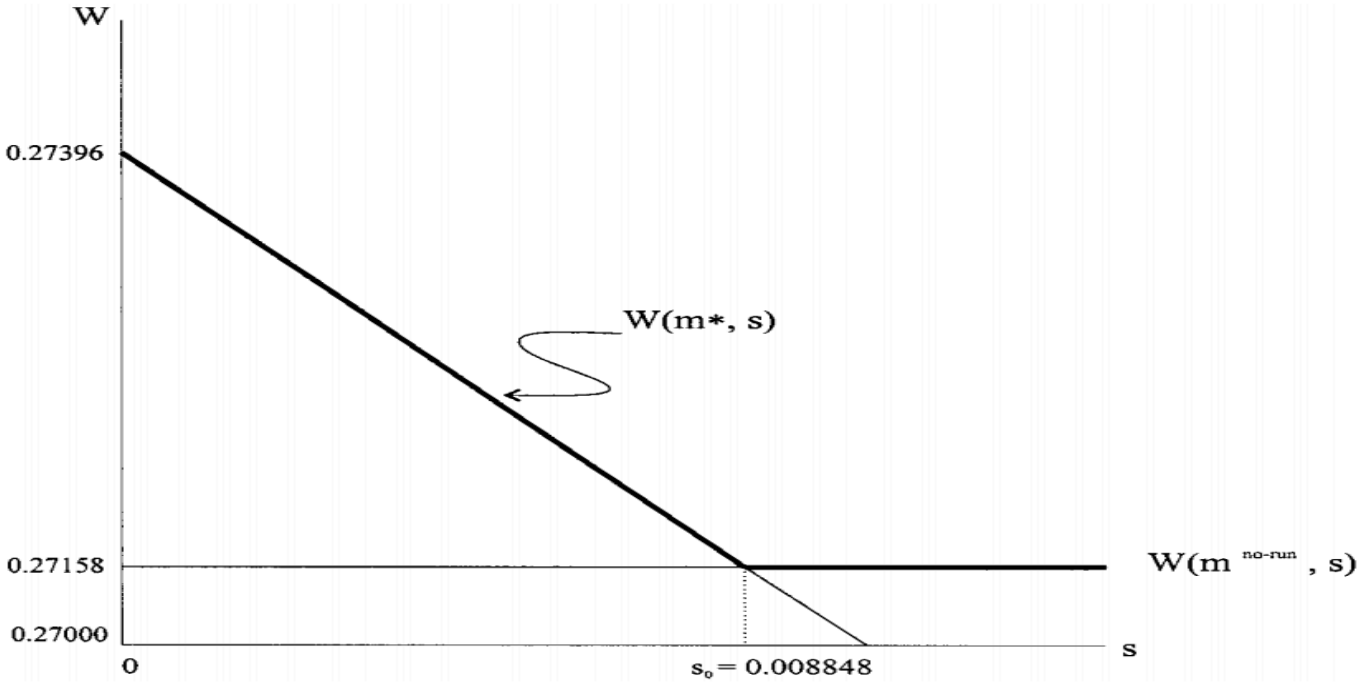
$$\max_{(c^1(1),\ldots,c^1(N-1))} W(\boldsymbol{m}, s)$$
$$subject\ to\ (1)$$

Propensity to Run

- Proposition: For some economies with a sufficiently small propensity to run, s, the optimal mechanism for the pre-deposit game has a run equilibrium.
- Proof sketch
  - Consider the 2 consumer example from before
  - Patient consumers choose period 2 when $\sigma \geq s$ so (1) holds and is binding as shown before
  - So (1) must hold as an equality and be binding for sufficiently small s i.e. $\boldsymbol{m}^*$ is still optimal
  - By continuity, $W(\boldsymbol{m}, s)$ can be made sufficiently close to the previous optimal solution
  - Which is better than the welfare under autarky so consumers deposit
  - Since $\boldsymbol{m}^*$ has a post-deposit run equilibrium, it has a pre-deposit equilibrium
- Not so easy for to calculate s-optimal mechanism for general economies

Propensity to Run

- As s increases, welfare under $\boldsymbol{m}^*$ falls



- Largest propensity to run s_0 solves

$$(1 - s_0)\widehat{W}(\boldsymbol{m}^*) + s_0 W^{run}(\boldsymbol{m}^*) = \widehat{W}(\boldsymbol{m}^{no-run})$$

Conclusion

- Possibility of equilibrium bank run does not depend on a simple and suboptimal specification of the deposit contract
- Types of economies that allow bank runs
  - Significant uncertainty about number of patient and impatient consumers
  - Utility functions reflect a high degree of “impulse demand”
  - Incentive to choose period 1 for patient consumers
- Eliminating these bank runs may require a sacrifice of welfare