Economics 4905
Financial Fragility and the Macroeconomy
Cornell University, Fall 2017
Prelim 1 Solutions
Wednesday, September 27, 2017, 2:55PM to 4:10PM
104 Rockefeller Hall

1 Question 1 (30 minutes)

Assume $\omega = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5) = (150, 80, 75, 25, 10)$. For each of the following, calculate the set of equilibrium money prices, $\mathcal{P}^m$, and the set of equilibrium allocations $x = (x_1, x_2, x_3, x_4, x_5)$.

a) $\tau = (50, 25, 15, -15, -30)$

b) $\tau = (50, 10, 0, -20, -40)$

c) $\tau = (30, 20, -5, -10, -35)$

d) $\tau = (3, 0, 0, -1, -2)$

e) $\tau = (2, 2, 1, 1, -1)$

f) $\tau = (0, 0, 0, 0, 0)$

g) $\tau = (5, 4, 0, -4, -5)$

Solution:

a) The tax system is not balanced ($\sum_h \tau_h = 45$), hence the set of equilibrium prices of money must be $\mathcal{P}^m = \{0\}$. The equilibrium allocation is

$$x = (150, 80, 75, 25, 10)$$

b) First we establish that the tax system is balanced ($\sum_h \tau_h = 0$). The equilibrium price has to satisfy the following inequalities:

$$150 - 50P^m > 0$$
$$80 - 10P^m > 0$$

This implies that P^m has to satisfy:

$$P^m < 3$$
$$P^m < 8$$

The set of equilibrium prices of money is then

$$\mathcal{P}^m = [0, 3)$$

The set of equilibrium allocations is

$$x = (150, 80, 75, 25, 10) - P^m(50, 10, 0, -20, -40)$$
$$x(P^m) \in \{(150 - 50P^m, 80 - 10P^m, 75, 25 + 20P^m, 10 + 40P^m)|P^m \in [0, 3)\}$$

c) Using similar method as in (b), we can obtain

$$\mathcal{P}^m = [0, 4)$$

$$x(P^m) \in \{(150 - 30P^m, 80 - 20P^m, 75 + 5P^m, 25 + 10P^m, 10 + 35P^m)|P^m \in [0, 4)\}$$

d)

$$\mathcal{P}^m = [0, 50)$$
$$x(P^m) \in \{(150 - 3P^m, 80, 75, 25 + 1P^m, 10 + 2P^m)|P^m \in [0, 50)\}$$

e) As in (a), the tax system is not balanced, so $\mathcal{P}^m = \{0\}$

$$x = (150, 80, 75, 25, 10)$$

f) Since there is no tax, it does not matter what price of money is.

$$\mathcal{P}^m = [0, \infty)$$

$$x = (150, 80, 75, 25, 10)$$

g)

$$\mathcal{P}^m = [0, 20)$$
$$x(P^m) \in \{(150 - 5P^m, 80 - 4P^m, 75, 25 + 4P^m, 10 + 5P^m)|P^m \in [0, 20)\}$$

2 Question 2: (30 minutes)

There are two monies, Red (R), and Blue (B). The units are $R\$$ and $B\$$. Calculate the exchange rate between $R\$$ and $B\$$ for each of the following tax and transfer systems (giving units in your answers). Calculate in each case the **set** of equilibrium allocations $x = (x_1, x_2, x_3)$ when the endowment vector is $\omega = (\omega_1, \omega_2, \omega_3) = (150, 80, 10)$.

a) $\tau^R = (2, 1, 0), \tau^B = (5, 3, -12)$

b) $\tau^R = (5, 4, -2), \tau^B = (1, 0, 0)$

c) $\tau^R = (8, -2, -6), \tau^B = (4, 1, -5)$

d) $\tau^R = (7, 2, -12), \tau^B = (6, 5, -2)$

e) $\tau^R = (1, 1, -2), \tau^B = (2, 2, -4)$

f) $\tau^R = (0, 0, 0), \tau^B = (1, 0, -1)$

g) In (a) - (f), are the exchange rates independent of ω? Why? Are the allocations independent of ω? Why?

Solution:

Recall from homework 1 that

$$\frac{P^R}{P^B} = -\frac{\sum_h \tau_h^B}{\sum_h \tau_h^R}$$

P^R is the price of $R\$$ in terms of chocolate, which is chocolate/$R\$$. P^B is the price of $B\$$ in terms of chocolate, which is chocolate/$B\$$. The unit for the exchange rate is then

$$\frac{chocolate/R\$}{chocolate/B\$} = \frac{B\$}{R\$}$$

a)

$$\frac{P^R}{P^B} = -\frac{\sum_h \tau_h^B}{\sum_h \tau_h^R} = -\frac{5+3-12}{2+1+0} = \frac{4}{3}\frac{B\$}{R\$}$$

$$\begin{aligned} x &= \omega - P^R\tau^R - P^B\tau^B \\ &= (150, 80, 10) - P^R(2, 1, 0) - \frac{3}{4}P^R(5, 3, -12) \\ x(P^m) &\in \{(150 - \frac{23}{4}P^R, 80 - \frac{13}{4}P^R, 10 + 9P^R) | P^R \in [0, \frac{320}{13})\} \end{aligned}$$

b) Neither tax system is balanced ($\sum_h \tau_h^R > 0$ and $\sum_h \tau_h^B > 0$), so we know that $P^R = P^B = 0$. hence the equilibrium exchange rate is indeterminate ($\frac{P^R}{P^B} = \frac{0}{0}\frac{B\$}{R\$}$).

$$x = (150, 80, 10)$$

c) Now both tax system are balanced, and the equilibrium exchange rate is again indeterminate ($\frac{P^R}{P^B} = \frac{0}{0}\frac{B\$}{R\$}$).

$$\begin{aligned} x &= (150, 80, 10) - P^R(8, -2, -6) - P^B(4, 1, -5) \\ x(P^R, P^B) &\in \{(150 - 8P^R - 4P^B, 80 + 2P^R - P^B, 10 + 6P^R + 5P^B) | \\ &\quad 8P^R + 4P^B < 150, P^R \geq 0, P^B \geq 0\} \end{aligned}$$

d) $\frac{P^R}{P^B} = -\frac{\sum_h \tau_h^B}{\sum_h \tau_h^R} = -\frac{6+5-2}{7+2-12} = 3\frac{B\$}{R\$}$

$$\begin{aligned} x &= (150, 80, 10) - P^R(7, 2, -12) - \frac{1}{3}P^R(6, 5, -2) \\ x(P^R) &\in \{(150 - 9P^R, 80 - \frac{11}{3}P^R, 10 + \frac{38}{3}P^R) | P^R \in [0, \frac{50}{3})\} \end{aligned}$$

e) Exchange rate case is similar to (c).

$$\begin{aligned} x &= (150, 80, 10) - P^R(1, 1, -2) - P^B(2, 2, -4) \\ x(P^R, P^B) &\in \{(150 - P^R - 2P^B, 80 - P^R - 2P^B, 10 + 2P^R + 4P^B) | \\ &\quad P^R + 2P^B < 80, P^R \geq 0, P^B \geq 0\} \end{aligned}$$

f) Exchange rate case is similar to (c).

$$x = (150, 80, 10) - P^R(0, 0, 0) - P^B(1, 0, -1)$$
$$x(P^B) \in \{(150 - P^B, 80, 10 + P^B) | P^B \in [0, 150)\}$$

g) In part (a) and (d), we can determine the equilibrium exchange rate independent of endowments. These are purely financial markets.

In part (b), we cannot determine the equilibrium exchange rate, irrespective of endowments.

In part (c),(e) and (f), with endowments given, we can pin down say $P^R \in [0, \bar{P}^R)$ and $P^B \in [0, \bar{P}^B)$ such that consumption of all individuals remains positive. Then holding fixed a possible $P^B > 0$ and taking P^R to zero, the exchange rate goes to zero. Likewise, if $P^R > 0$ is fixed, and we take P^B to zero, we find the exchange rate going to infinity. Thus we have an indeterminate exchange rate with balanced taxes, irrespective of endowments.

In all (a)-(f), the equilibrium allocations are dependent on ω.