Economics 4905
Financial Fragility and the Macroeconomy
Cornell University, Fall 2017
Prelim 2 Solutions
Monday, November 6, 2017, 2:55PM to 4:10PM
104 Rockefeller Hall

Instructions: This is designed to take 60 minutes but you have 75 minutes to write your answers. Do not take, seek or give advice from any source, animate or inanimate. Do not use calculators. There is no need to simplify numerical answers. Place all personal items including books, papers, and computers in a place determined by the proctor. Sit away from neighbors if possible.

1 Overlapping Generations (45 minutes)

- 1 good per period, $\ell = 1$
- 2 -period lives
- 1 consumer per generation

$$u_0(x_0^1) = B \log x_0^1 \text{ for } t = 0$$
$$u_t(x_t^t, x_t^{t+1}) = A \log x_t^t + B \log x_t^{t+1} \text{ for } t = 1, 2, \ldots$$

- Endowments are as follow:

$$\omega_0^1 = C \text{ for } t = 0$$
$$(\omega_t^t, \omega_t^{t+1}) = (C, D) \text{ for } t = 1, 2, \ldots$$

- The parameters A, B, C, and D are positive scalars

(a) Derive the equation for Mr t's indifference curve (IC) through his endowment.

Solution:

The utility evaluated at the endowment is

$$u_t(C, D) = A \log C + B \log D$$

The equation of the indifference curve that contains the endowment point is

$$A \log x_t^t + B \log x_t^{t+1} = A \log C + B \log D$$

(b) What is the slope of the IC at the endowment point in terms of A, B, C, and D? What is R? What is r?

Solution:

Solving the IC equation for x_t^{t+1} gives

$$x_t^{t+1} = D\left(\frac{C}{x_t^t}\right)^{\frac{A}{B}}$$

Taking the derivative with respect to x_t^t gives

$$\frac{dx_t^{t+1}}{dx_t^t} = \frac{AD}{B}\left(\frac{C}{x_t^t}\right)^{\frac{A}{B}-1}\left(-\frac{C}{(x_t^t)^2}\right)$$

Evaluating the derivative at the endowment (C, D) gives

$$\frac{dx_t^{t+1}}{dx_t^t} = \frac{AD}{B}\left(\frac{C}{C}\right)^{\frac{A}{B}-1}\left(-\frac{C}{C^2}\right) = -\frac{AD}{BC}$$

To find R and r

$$R = 1 + r = \frac{p^t}{p^{t+1}} = -\frac{dx_t^{t+1}}{dx_t^t} = \frac{AD}{BC}$$

$$r = \frac{AD}{BC} - 1$$

Alternative Solution:

Taking implicit derivative of the equation from part (a) with respect to x_t^t:

$$\frac{A}{x_t^t} + \frac{B}{x_t^{t+1}}\frac{dx_t^{t+1}}{dx_t^t} = 0$$

Rearranging gives

$$\frac{dx_t^{t+1}}{dx_t^t} = -\frac{Ax_t^{t+1}}{Bx_t^t}$$

Plugging in the endowment (C, D) gives

$$\frac{dx_t^{t+1}}{dx_t^t} = -\frac{AD}{BC}$$

R and r can then be calculated accordingly.

(c) Derive a condition on A, B, C, and D for this economy to be Samuelson.

Solution:

If the interest rate $r < 0$, then the economy is Samuelson, which implies that

$$\frac{AD}{BC} - 1 < 0$$

or

$$AD < BC$$

(d) Derive a condition on A, B, C, and D for this economy to be Ricardo.

Solution

If the interest rate $r > 0$, then the economy is Ricardo, which implies that

$$AD > BC$$

($r = 0$ is the borderline case. For log utilities, it is a Ricardo case.)

Let $z^t = x_t^t - \omega_t^t$ and $z^{t+1} = \omega_t^{t+1} - x_t^{t+1}$.

(e) Derive z^{t+1} as a function of z^t on Mr t's reflected offer curve (ROC).

Solution: The utility maximization problem is

$$\begin{aligned} \max_{x_t^t, x_t^{t+1}} \quad & A \log x_t^t + B \log x_t^{t+1} \\ \text{subject to} \quad & p^t x_t^t + p^{t+1} x_t^{t+1} = p^t \omega_t^t + p^{t+1} \omega_t^{t+1} \end{aligned}$$

Taking the derivative of the Lagrangian gives

$$\frac{p^{t+1}}{p^t} = \frac{B}{A} \frac{x_t^t}{x_t^{t+1}}$$

Plugging this and the given definitions into the budget constraint gives

$$(z^t + \omega_t^t) + \frac{B}{A} \frac{(z^t + \omega_t^t}{\omega_t^{t+1} - z^{t+1}} (\omega_t^{t+1} - z^{t+1}) = \omega_t^{t+1} + \frac{B}{A} \frac{(z^t + \omega_t^t)}{(\omega_t^{t+1} - z^{t+1}} \omega_t^{t+1}$$

Solving for z^{t+1} gives

$$z^{t+1} = \frac{A \omega_t^{t+1} z^t}{B \omega_t^t - (A+B) z^t}$$

Plugging in the endowments

$$z^{t+1} = \frac{AD z^t}{BC - (A+B) z^t}$$

(f) Assume $A = C = D = 1$, $B = 2$, $m_0^1 = 1$ and $m_t^s = 0$ otherwise.

(i) Calculate the stationary equilibria.

Solution:

Setting $\bar{z} = z^t = z^{t+1}$ in the ROC

$$\bar{z} = \frac{\bar{z}}{2 - 3\bar{z}}$$

$$\bar{z} = 0 \quad \text{or} \quad \bar{z} = \frac{1}{3}$$

(ii) What is the set of equilibrium money prices, $\mathcal{P}^m$?

Solution:

Since the monetary base is $m_0^1 = 1$, the set of equilibrium money prices is

$$\mathcal{P}^m = [0, 1/3]$$

(iii) Do the full dynamic and welfare analyses for this particular OG economy.

Solution:

There are two stationary equilibria. The first one is $\bar{z} = 0$ which is stable but not Pareto-optimal. There is another stationary equilibrium in which $P^m = \bar{P}^m = 1/3$. This equilibrium is Pareto-optimal but unstable. For P^m in $(0, 1/3)$, the economy is inflationary: the money bubble gradually fades away. For $P^m > 1/3$, the economy is deflationary: the money bubble bursts in finite time.

2 Diamond-Dybvig Bank (15 minutes)

(a) What does the DD bank offer its depositors? In particular, what does the DD bank do to increase the welfare of consumers who choose to deposit in the DD bank?

Solution:
The DD bank offers its depositors consumption smoothing across liquidity states (patient or impatient). For risk-averse depositors, consumption smoothing increases welfare. Consumers cannot buy traditional insurance as protection against liquidity risk, because the liquidity impulse is private knowledge. Insurance companies would not offer this type of policy, because they cannot verify "impatience".

(b) What is a panic-based bank run?

Solution:
A panic-based bank run is when patient depositors withdraw only because they believe that the other patient depositors will withdraw.

(c) What is the incentive compatibility constraint (ICC)?

Solution:
The ICC says that the banking contract should not encourage patient depositors to withdraw when they expect the other patient depositors will wait. In the DD model, this translates into the constraint that the banking contract not allow for the early withdrawal d_1 to be greater than the late withdrawal d_2.

(d) What is the difference between the post-deposit banking game and the pre-deposit banking game?

Solution:
The DD post-deposit game assumes that customers exogenously deposit in the bank before they know the banking contract. In the pre-deposit game, the bank designs the optimal deposit contract, and then consumers decide on whether or not to deposit in the bank based on the contract offered by the bank, but before their types are known.

The pre-deposit game is the realistic game. The corresponding optimal banking contract can allow for runs if the probability of the run is small.

(e) Should bank runs be avoided at all costs?

Solution:
No. Sometimes when the probability of a bank run is low enough, run-permitting contracts give higher welfare. This is like crossing the street. We do it because we believe the gain from crossing outweighs the risks.