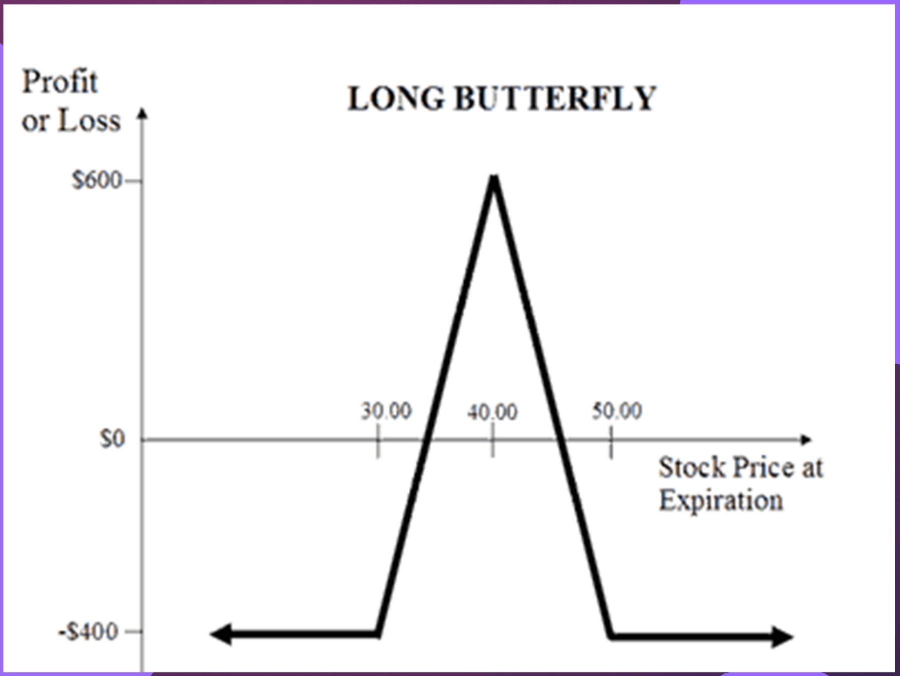
Options and the Black-Scholes Model

BY CHASE JAEGER



Defining Options

- A put option (usually just called a "put") is a financial contract between two parties, the writer (seller) and the buyer of the option. The buyer acquires a short position with the right, but not the obligation, to sell the underlying instrument at an agreed-upon price (the strike price). If the buyer exercises his right to sell the option, the seller is obliged to buy it at the strike price. In exchange for having this option, the buyer pays the writer a fee (the option premium).
- A Call option gives the buyer of the option the right, but not the obligation to buy an agreed quantity of a particular commodity or financial instrument (the underlying instrument) from the seller of the option at a certain time (the expiration date) for a certain price (the strike price). The seller (or "writer") is obligated to sell the commodity or financial instrument should the buyer so decide. The buyer pays a fee (called a premium) for this right.

Options and Insurance

- It is OK to ‘think’ of options as insurance but it is incorrect to call them insurance.
- A key differences is simply that for insurance an indemnity must be tied to a specific and measured loss
- Financial options have no requirement meaning that they have speculative and tradable characteristics

Uses of Options

- A money manager whose portfolio has reaped huge gains can safeguard them by buying index put options (*portfolio insurance*).
- A meat processor can hedge input price from rising by buying a call option on pork belly futures.
- An American manufacturer buying machines from Germany for which payment is due in three months can remove price risk from dollar/euro exchange rates by buying an option on a euro futures contract.
- A speculator can make leveraged bets by trading options.
- A sophisticated investor can alter portfolio's risk-return tradeoff by trading options.
- An investor can avoid short selling restrictions on the New York Stock Exchange by taking a "sell" position in the options market.

History of Options

- The astronomer Thales (624-547 BC), took options on olive oil presses by paying in advance for the right to hire them. He based his moves on astrology and when harvest was strong sold his rights by renting the presses.
- Most famous options markets was the great tulip bubble craze of the 1630’s in Holland
- Unregulated options started to trade on the London exchange in the late 1800s

Background to Options Pricing

THE BLACK-SCHOLES FORMULA

In the Beginning....

- 1840 Reverend Brown a Scottish botanist observes pollen in a fluid and outlines the dynamic motion of particles in motion

- Hence… Brownian motion

Followed by…

- 1880 T.N. Thiele (Copenhagen)
- 1900 L. Bachelier (Paris)
  - Looked at randomness in Paris stock market but was ignored because the application of pure mathematics to economics was frowned upon
- 1905 A. Einstein (Berlin)
  - Laid out the basic form for stochastic differential equation but mathematics required for general proof not yet invented
- 1923 N. Wiener (Berlin) was able to establish a proof of Einstein's model
  - This is where the term Wiener Process comes from

In the mean time…

- Kolmogorov in Russia, Markov in Russia and Levy in Paris were applying new discoveries in pure mathematics to the study of probability.

  - The mathematicians seek out universal proofs that hold under all conditions.

While in Tokyo…

- Kiyosi Ito, a 24 year old mathematician studied Markov, Kolmogorov, Wiener, and Levy
- Looking for an approach to unify their various theories.

Kiyosi Ito

- In 1941 published a mimeo in Japanese defining the first stochastic integral for a Brownian motion.
- In 1951 this was published in English
- And became known as Ito's Lemma
- But the Lemma saw little use and lingered through the mid 1960's

$$I(\tau,\omega,f) \equiv \int_0^t f(\tau,\omega)\, d_\tau g(\tau,\omega)$$

$$\frac{dP_i}{P_i} = \mu_i(P,t)\,dt + \sigma_i(P,t)\,dz_i$$

$$dP = f(P,t)\,dt + g(P,t)\,dz$$

$$P_i(t) = P_0(0) + \int_0^t f_i(P,s)\,ds + \int_0^t g_i(P,s)\,dz_i, \qquad i = 1,\ldots,n,$$

$$dY = \sum_{i=1}^{n} \frac{\partial F}{\partial P_i} dP_i + \frac{\partial F}{\partial t} dt + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial^2 F}{\partial P_i \partial P_j} dP_i dP_j$$

Ito Goes to Princeton

- When published in English Ito’s papers become notable
- 1954 took leave to go to Princeton’s Institute for Advanced Studies where he met up with the young mathematician Henry McKean.
- McKean was working with the mathematicians Bochner and Feller who were also looking at diffusion processes.
- McKean traveled to work with Ito in Tokyo for two years in 1957/58 where they started training Japanese mathematicians in diffusion processes
- It was this group that fine tuned the stochastic calculus and coined the term ‘Ito’s Lemma’ in the late 1960’s

And….

- Robert Merton, Samuelson's student at MIT starts examining options pricing from the rational point of view developing certain boundary conditions and is the first to apply Ito's lemma in its current form to financial economics
- AND
- Myron Scholes also at MIT was taking another look at the pricing of options and teams up with practitioner Fischer Black.
- It was McKean, Samuelson, Merton and Scholes all at MIT at the same time that came together to solve the options pricing formula.

Black and Scholes

- Applying Ito's Lemma they come up with a stochastic differential equation for the dynamics of the options price based on the Brownian motion of the underlying

$$dw = \frac{\partial w(p,t)}{\partial p} dp + \frac{\partial w(p,t)}{\partial t} dt + \frac{1}{2} \frac{\partial^2 w(p,t)}{\partial p^2} dp^2$$

$$dw = \frac{\partial w(p,t)}{\partial p} (\mu pdt + \sigma pdz) + \frac{\partial w(p,t)}{\partial t} dt + \frac{1}{2} \frac{\partial^2 w(p,t)}{\partial p^2} (\mu pdt + \sigma pdz)^2$$

$$dw = \frac{\partial w(p,t)}{\partial p} (\mu pdt + \sigma pdz) + \frac{1}{2} \frac{\partial^2 w(p,t)}{\partial p^2} \sigma^2 p^2 dt + \frac{\partial w(p,t)}{\partial t} dt$$

Black and Scholes

- They employ CAPM and risk neutral valuations arguing that because the hedge position is riskless the value of the portfolio must equal the risk free rate

$$dE = \frac{-1}{\frac{\partial w(p,t)}{\partial p}}\left[\frac{1}{2}\frac{\partial^2 w(p,t)}{\partial p^2}\sigma^2 p^2 + \frac{\partial w(p,t)}{\partial t}\right]dt \quad \longleftrightarrow \quad dE = \left(p - \frac{w(p,t)}{\frac{\partial w(p,t)}{\partial p}}\right) r\,dt$$

$$\frac{-1}{\frac{\partial w(p,t)}{\partial p}}\left[\frac{1}{2}\frac{\partial^2 w(p,t)}{\partial p^2}\sigma^2 p^2 + \frac{\partial w(p,t)}{\partial t}\right]dt = \left(\frac{p\frac{\partial w(p,t)}{\partial p} - w(p,t)}{\frac{\partial w(p,t)}{\partial p}}\right) r\,dt$$

$$\frac{\partial w(p,t)}{\partial t} = r w(p,t) - r p \frac{\partial w(p,t)}{\partial p} - \frac{1}{2}\frac{\partial^2 w(p,t)}{\partial p^2}\sigma^2 p^2$$

The Black Scholes Formula for a Call & Put Option on non-dividend paying stocks

B-S Call:

$$w(p,t) = pN(d_1) - xe^{-rt}N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{p}{x}\right) + \left(r + \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}$$

$$d_2 = \frac{\ln\left(\frac{p}{x}\right) + \left(r - \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}$$

B-S Put:

$$w(p,t) = xe^{-rt}N(-d_2) - pN(-d_1)$$

$$d_1 = \frac{\ln\left(\frac{p}{x}\right) + \left(r + \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}$$

$$d_2 = \frac{\ln\left(\frac{p}{x}\right) + \left(r - \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}$$

B-S Assumptions

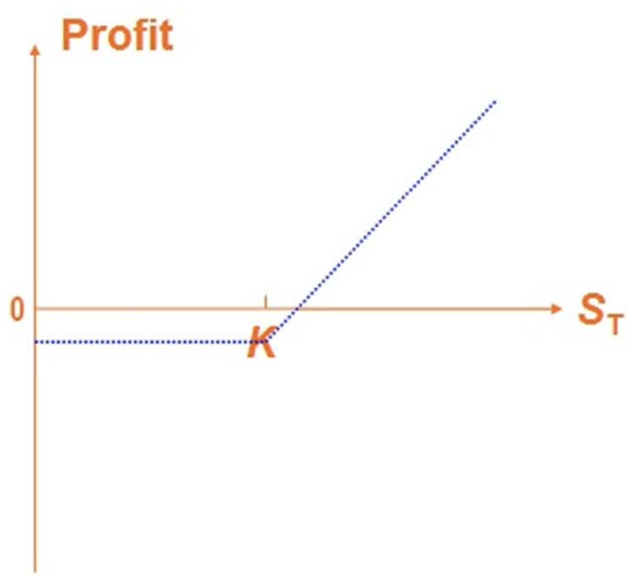
- 1: Price of Underlying
- 2: Strike Price
- 3 Time to Maturity
- 4: Interest Rates
- 5: Volatility
- 6: Dividends

Trading Strategies

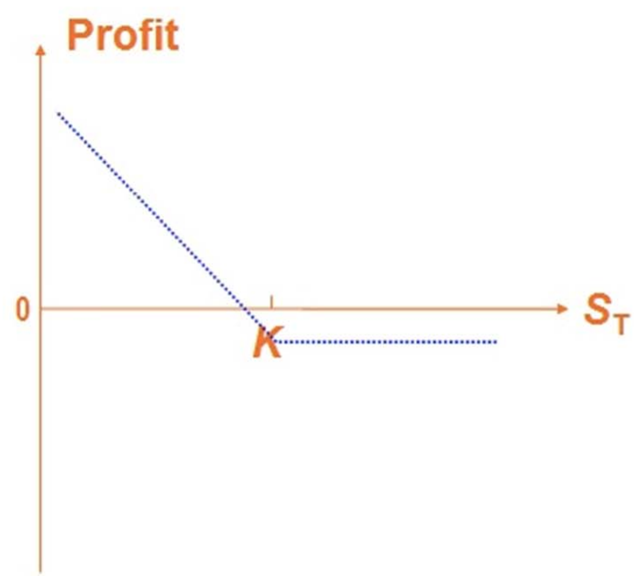
THE BLACK-SCHOLES FORMULA

Vanilla Call/Put Option:

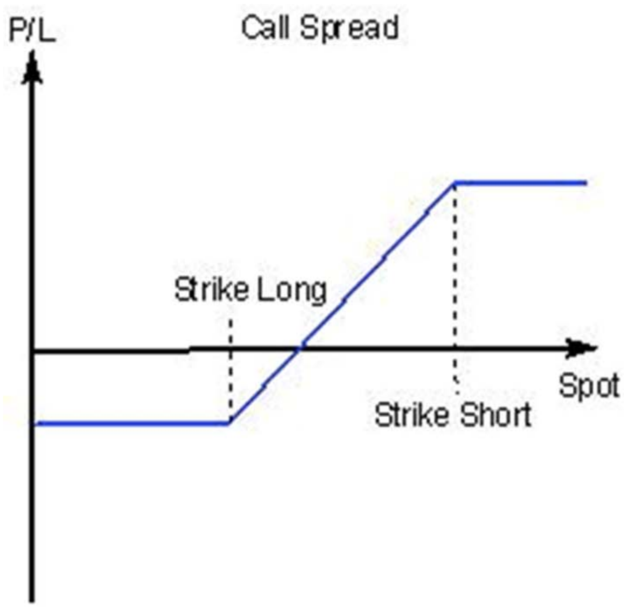
Call Option

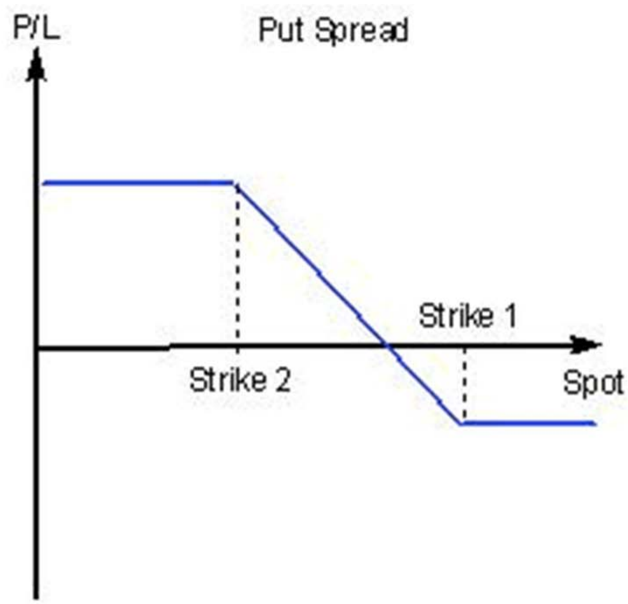


Put Option



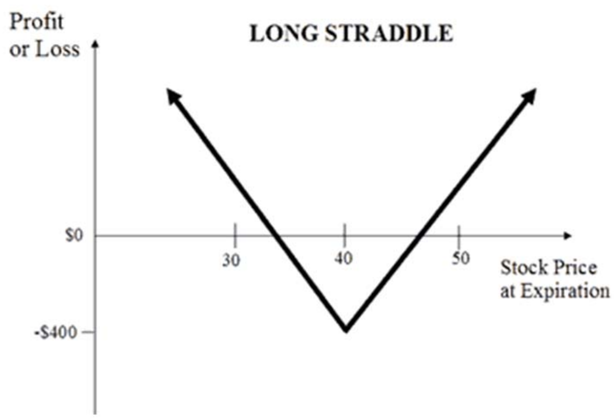
Call/Put Spread



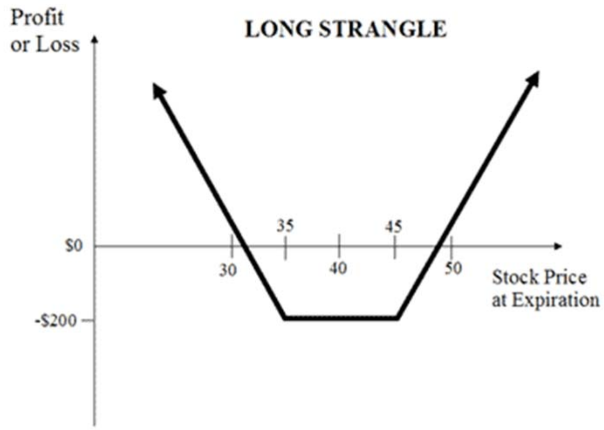


Volatility Trades

- Straddle:



- Strangle:

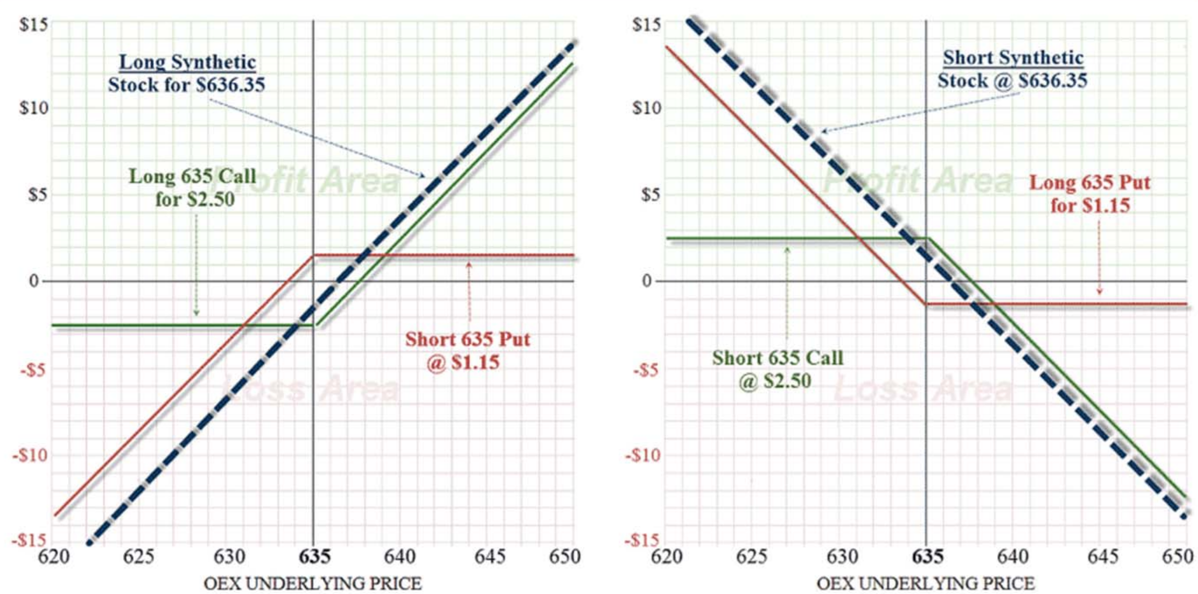


Synthetic Long/Short

- Used Heavily During Financial Crisis

SYNTHETIC POSITIONS

The long synthetic (left) and short synthetic stock positions are mirror images of each other, created by combining offsetting positions in a call and put option. By establishing these positions, we can take advantage of early exercise.



Source: RandomWalkTrading

The Greeks

Delta:

- Measures change in the price of an option relative to a change in the price of the underlying security

- For a call option: this will be between 0 and 1

- For a put option: this will be between 0 and -1

Gamma:

- Gamma is the derivative of Delta

- Gamma measures the change in delta relative to a change in the price of the underlying security

Theta

- Theta is the time decay of the option
- Theta is always negative
- Theta decreases the closer an option gets to maturity

Vega

- Measures change in the price of an option relative to a 1% change in the volatility of the underlying security

- This will be positive for both call and put options

- Thus, when you own an option... you always want volatility of the underlying security to increase

Rho

- Measures the change in the price of the price of an option relative to a change in the risk free rate
- This will be positive for Call Options and negative for Put Options
- Easier to think of why if you think of the alternative to buying a call and put respectively

Benefits of Exchange Traded Options

- a) ***Orderly, efficient and liquid markets with low transactions costs***- Standardized options trade in orderly, efficient, and liquid markets.

Benefits of Exchange Traded Options

- b) ***Flexibility***- Options are an extremely versatile investment tool. Because of their unique risk/reward structure, options can be used in many combinations with other option contracts and/or other financial instruments to create either a hedged or speculative position.

Benefits of Exchange Traded Options

- c) ***Leverage- Options make it cheaper to speculate***. Allows you to forward buy or sell in multiples of what could otherwise be purchased in full.

Benefits of Exchange Traded Options

- d) ***Limited risk for buyer-*** Unlike other investments where the risks may have no limit, options offer a known risk to buyers. The maximum you can lose is the amount you paid for the option. On the other hand, the nature of the game makes call writer face unlimited risk.

Benefits of Exchange Traded Options

- f) ***Guaranteed contract performance***- Exchange-traded options have no credit risk (counterparty risk) as their performances are guaranteed by the OCC (The Options Clearing Corporation).

Benefits of Exchange Traded Options

- g) ***Overcome stock market restrictions***- such as restrictions on shortselling when the market is going down. Options don’t have such restrictions- in fact they are designed to give flexibility of investment in a variety of market conditions.

Options versus Equities

Common with Equities

- Both options and stocks are listed securities.
- Like stocks, options trade with buyers making bids and sellers making offers.
- Option investors, like stock investors, have the ability to follow price movements, trading volume and other pertinent information

Not Common with Equities

- option has a limited life
- There is not a fixed number of options
- Stocks have certificates evidencing their ownership, options are certificateless
- stock ownership provides the holder with a share of the company, certain voting rights and rights to dividends (if any), option owners participate only in the potential benefit of the stock's price movement

How does this Relate to Our Course

- In our Fragile Macro-Economic Economy, the Stock Market is particularly Fragile
- Options can act as insurance against tail-risk events
- Through proper use of options, your portfolio can be robust against Bank Runs, inflation, periods of high volatility, and Black-Swan events

Works Cited

- https://www.google.com/search?biw=1399&bih=700&tbm=isch&sa=1&q=call+and+put+options&oq=call+and+put+options&gs_l=psy-ab.3..0l3j0i8i30k1j0i24k1l6.7643.8801.0.8921.8.7.0.0.0.0.160.635.3j3.6.0....0...1.1.64.psy-ab..6.2.186...0i13k1j0i8i7i30k1.0.eRsmxXD3Tn4#imgrc=SXHRXxUOdg36LM:
- https://www.google.com/search?q=butterfly+options&source=lnms&tbm=isch&sa=X&ved=0ahUKEwiepJHOrozXAhXMKyYKHVBKD8gQ_AUICygC&biw=1399&bih=700#imgrc=XCeTL3CTY4DdGM:
- https://www.google.com/search?biw=1399&bih=700&tbm=isch&sa=1&q=straddle&oq=straddle&gs_l=psy-ab.3..0i67k1j0l4j0i67k1l2j0l3.16909.17997.0.18290.8.8.0.0.0.0.108.572.6j1.7.0....0...1.1.64.psy-ab..1.7.569....0.OQlxh_R5fMg#imgrc=1NcExiMdT8C9zM:
- https://www.google.com/search?q=strangle+options&source=lnms&tbm=isch&sa=X&ved=0ahUKEwjVzfzgrozXAhXJLSYKHVDlBpAQ_AUICigB&biw=1399&bih=700#imgrc=iVfJh9TqShPfKM:

- *some slides taken from lecture by Prof. Calum Turvey (with permission)