On Sunspots, Bank Runs, and Glass-Steagall*

Karl Shell[†]
Cornell University

Yu Zhang
Xiamen University

March 21, 2018

*We dedicate this paper to Jess Benhabib and Roger Farmer, the fathers of empirical sunspot modelling of the macro-economy.

[†]Corresponding Author: Karl Shell, karl.shell@cornell.edu

Abstract

We analyze the *pre-deposit* game in a banking model based on Peck and Shell (2010). The Glass-Steagall bank (the GSB) is assumed to be restricted to holding only liquid assets. Consumers tolerate a sunspot-driven, panic-based run if its probability of occurrence, s, is small. How does s affect the consumers' allocation of assets between the GSB and the illiquid asset? We would expect that as s is increased consumers would choose a more conservative allocation by increasing their deposits in the GSB. In some cases, consumers follow this intuition by *strictly* increasing GSB deposits when faced with small increases in s. In these cases, the consumer incentive compatibility constraint (ICC) is not binding. In other cases, GSB deposits are unchanged by small changes in s. In these cases, the ICC is binding. We offer this paper as a contribution to the literature on banking-and-financial fragility. We also offer this paper as a contribution to the broader literature on sunspot equilibrium. When the consumer incentive compatibility constraint is not binding, the sunspot allocation is not a mere randomization over the run and non-run outcomes under the so-called "optimal contract" for the post-deposit game. This reminds us of other situations in which the sunspot-equilibrium allocation is not a mere randomization over the equilibrium allocations in the corresponding certainty economy.

Keywords: bank runs, deposit contract, Glass-Steagall banking, illiquid assets, liquid assets, merchant banking, pre-deposit game, post-deposit game, run probability, sunspots

JEL classification numbers: G21; E44

1 Introduction

We analyze in some detail the *pre-deposit* game in a banking model based on Peck and Shell (2010), hereafter PS. As in Wallace (1996), there are two investment assets: one liquid and the other illiquid. Two financial systems are compared: (1) In *the separated financial system*, there are two separate institutions. One holds only the liquid asset. This might be thought of as a Glass-Steagall bank (GSB), or a narrow bank. The other financial institution holds only the illiquid asset. It is like a stock brokerage or mutual fund. (2) In the *unified financial system*, or consolidated system, one institution holds the two assets. This might be thought of as a merchant bank, or merely a post-Glass-Steagall, pre-Dodd-Frank, modern bank.

As in PS, we introduce intrinsic aggregate uncertainty by assuming that the realized fraction of impatient consumers is itself stochastic. PS assume that there is a continuum of consumers. We assume that there is only a finite number of consumers.

In the post-deposit game, the unified financial system is immune from sunspot-driven runs, but for some parameters the unified system runs out of cash when the realized fraction of impatient consumers is high. The GSB is always susceptible to a sunspot-driven run. How does the run probability s affect the optimal contract? For small s, runs are tolerated. In some cases, the contract becomes — as one would expect — strictly more conservative as s increases. In other cases, the contract is locally constant in s. In the first cases — the intuitive ones — the incentive constraint is not binding. In the second cases, the incentive constraint is binding.

Our paper is a contribution to the banking and related finance literatures. Our paper is also a contribution to the general sunspots literature — making it appropriate for inclusion in an IJET issue in honor of Roger Farmer. In the present model, the sunspot equilibrium allocation is sometimes a mere randomization over the allocations from the post-deposit game. Otherwise it is an entirely different animal. This second is like the sunspot equilibria that are not mere randomizations over the competitive equilibria from the corresponding certainty economy.

2 The Model

2.1 Preferences and Technologies

As in Shell and Zhang (2017), there are two consumers and three periods: 0, 1 and 2. In period 0, each consumer is endowed with y units of the consumption good. There are no endowments in periods 1 and 2. In period 1, either (1) one consumer becomes patient and the other becomes impatient, or (2) both consumer are patient. The probability of the two cases are q and $(1-q)$ respectively. Thus, the aggregate number of patient depositors is stochastic.[1]

Following PS, an impatient consumer has an indivisible "consumption opportunity"[2] in period 1, yielding incremental utility of $\overline{u}$ for 1 unit of consumption in period 1.[3] If the consumption opportunity goes unfulfilled in period 1, these consumers face a discounted consumption opportunity in period 2, yielding incremental utility of $\beta\overline{u}$ for 1 unit of consumption in period 2, where the scalar β is less than unity. For a patient consumer, the indivisible "consumption opportunity" arrives in period 2. Beyond these urgent "consumption opportunities," both types of consumers derive utility from "left-over" consumption in period 2 with the strictly concave utility $u()$ for "left over" balances. Thus, impatient and patient consumers, respectively, have the reduced-form utility functions:

$$U_I(C_I^1, C_I^2) = \begin{cases} \overline{u} + u(C_I^1 + C_I^2 - 1) & \text{if } C_I^1 \geq 1 \\ \beta\overline{u} + u(C_I^1 + C_I^2 - 1) & \text{if } C_I^1 < 1 \end{cases} \tag{1}$$

and

$$U_P(C_P^1, C_P^2) = \overline{u} + u(C_P^1 + C_P^2 - 1), \tag{2}$$

[1] As in PS, nature first determines how many consumers are patient in the economy. Conditional on there being a patient consumer, the probability of each consumer being patient is $\frac{1}{2}$. The ex ante probability that a given consumer becomes impatient is $\frac{q}{2}$.

[2] This "indivisibility" is meant to capture the idea that if payment is not made at par the purchase is lost. See PS.

[3] We adopt the notation of the Peck and Shell (2008) working paper, which agrees in most – but not all – cases with that of the published paper PS.

where C_j^t is the total withdrawal of a type-j depositor from the bank in period t. I stands for impatient depositor and P stands for patient depositor.

There are two constant-returns-to-scale technologies, an illiquid, higher-yield technology A and a liquid, lower-yield technology B. For the illiquid technology, investing one unit of period 0 consumption yields R_A units of consumption if harvested in period 2 and nothing if harvested in period 1. For the liquid technology, investing one unit of period 0 consumption yields R_B units of consumption if harvested in period 2 and 1 unit of consumption if harvested in period 1.We assume that $1 < R_B < R_A$ holds. Consumers can also costlessly store consumption across periods.

2.2 The Space of Deposit Contracts

In period 0, the bank designs the demand-deposit contract. Following the bank runs literature, we assume that the competitive bank maximizes the ex-ante expected utilities of depositors. At the beginning of period 1, each depositor learns his type and observes a sunspot variable δ distributed uniformly on $[0, 1]$. Then, the depositors decide whether to arrive at the bank in period 1 or in period 2. Depositors who choose period 1 arrive in random order.

Let γ denote the fraction of a depositor's endowment invested in technology B. The resources available to the bank in period 1 is thus $2y\gamma$. Sequential service constraints are part of the physical environment.[4] Let $c^1(z)$ be the withdrawal in period 1 for a depositor with position z in the queue, where $z \in \{1, 2\}$. Given the indivisible "consumption opportunity", we know that in the optimal contract, $c^1(z)$ will take either the value of 1 or 0. Furthermore, as in PS, we restrict attention to environments in which it is beneficial to provide for consumption opportunities whenever the resources are available.[5]

$$c^1(z) = \begin{cases} 1, & \text{if } 2y\gamma \geq z. \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

[4] See Wallace (1988).

[5] That is, we assume $\overline{u}$ is sufficiently large.

Let α denote the number of depositors who have made a withdrawal in period 1. We have

$$\alpha \in \{0, 1, 2\} \text{ and } \alpha \leq 2y\gamma.$$

In period 2, the bank chooses how to divide its remaining resources between those who have withdrawn in period 1 and those who have not as functions of α, denoted respectively by $c_I^2(\alpha)$ and $c_P^2(\alpha)$. Therefore, for the deposit contract to be feasible, all remaining resources must be distributed in period 2. The resource constraint (RC) is given by

$$\alpha c_I^2(\alpha) + (2-\alpha)c_P^2(\alpha) = [2y\gamma - \sum_{z=1}^{\alpha} c^1(z)]R_B. \tag{4}$$

The space of deposit contracts, or mechanisms, for the unified financial system, M^U, is given by

$$M^U = \{\gamma, c_I^2(\alpha), c_P^2(\alpha) \,|\, \textit{Equation (4) holds}\}.$$

The unified bank can access the illiquid asset in period 2, and therefore, $c_I^2(\alpha)$ and $c_P^2(\alpha)$ can be negative. For the separated financial system, neither $c_I^2(\alpha)$ nor $c_P^2(\alpha)$ can be negative since the GSB does not have access to the illiquid asset. The space of deposit contracts or mechanisms for the separated financial system, M^S, is given by

$$M^S = \{\gamma, c_I^2(\alpha) \geq 0, c_P^2(\alpha) \geq 0 \,|\, \textit{Equation (4) holds}\}.$$

It follows that $c_I^2(0)$ is not relevant. When $\alpha = 0$, no one has withdrawn in period 1. Similarly, $c_P^2(2)$ is not relevant. Therefore, both $c_P^2(0)$ and $c_I^2(2)$ can be solved directly from the resource constraint as a function of γ. What we are left to determine in the space of the deposit contract is $\{\gamma,\, c_I^2(1),\, c_P^2(1)\}$. We will show later that $c_I^2(1)$ and $c_P^2(1)$ will be determined by smoothing of left-over consumption and the ICC. The contract is characterized by the fraction γ.

3 The Post-Deposit Game

A non-run equilibrium in the *post-deposit* game is a Bayes-Nash equilibrium (BNE) in which only impatient depositors arrive and withdraw in period 1. The ex-ante welfare at the non-run equilibrium, $W^{non-run}$, depends on the total liquidity available to the bank in period 1, namely $2y\gamma$. For $\gamma \in [0, \frac{1}{2y})$, no depositor can withdraw in period 1. Therefore, we have

$$\begin{aligned} W^{non-run} &= q\{\overline{u} + \beta\overline{u} + 2u[(1-\gamma)yR_A + \gamma yR_B - 1]\} \\ &+(1-q)\{2\overline{u} + 2u[(1-\gamma)yR_A + \gamma yR_B - 1]\}. \end{aligned} \tag{5}$$

In this case, even though a depositor becomes impatient, he has to defer his indivisible consumption opportunity. For $\gamma \in [\frac{1}{2y}, 1]$, the impatient depositor (if he exists), does not have to wait for the next period. Therefore, we have

$$\begin{aligned} W^{non-run} &= q\{2\overline{u} + u[(1-\gamma)yR_A + c_I^2(1)] + u[(1-\gamma)yR_A + c_P^2(1) - 1]\} \\ &+(1-q)\{2\overline{u} + 2u[(1-\gamma)yR_A + \gamma yR_B - 1]\}. \end{aligned} \tag{6}$$

Since depositors' types are private information, non-run equilibrium requires that the incentive compatibility constraint (ICC) holds. That is, a patient depositor weakly prefers period-2 arrival to period-1 arrival, when he assumes that the other patient depositor chooses period-2 arrival. Since a patient depositor's "consumption opportunity" is in period 2 and consumption can be costlessly stored, his comparison between arrivals in the two periods only depends on the left-over consumption. For $\gamma \in [0, \frac{1}{2y})$, no one can withdraw in period 1, and therefore the ICC is irrelevant. For $\gamma \in [\frac{1}{2y}, 1]$, at most one depositor can withdraw in period 1, the ICC is given by

$$
\begin{aligned}
&qu[(1-\gamma)yR_A + c_P^2(1) - 1] + (1-q)u[(1-\gamma)yR_A + c_P^2(0) - 1] \qquad (7)\\
&\geq q\{\frac{1}{2}u[(1-\gamma)yR_A + c_I^2(1)] + \frac{1}{2}u[(1-\gamma)yR_A + c_P^2(1) - 1]\}\\
&+ (1-q)u[(1-\gamma)yR_A + c_I^2(1)].
\end{aligned}
$$

The LHS and RHS of inequality *(7)* are a patient depositor's expected utilities from the left-over consumption if he arrives in period 2 and 1 respectively, when he assumes that the other patient depositor chooses period-2 arrival. If a patient depositor is the second in the queue, he knows that there will be no more withdrawals in period 1 after him and therefore, he would always refuse to withdraw.[6] Other parts of the ICC are verified in like manner.

3.1 The Unified Financial System

The non-run optimal contract (NROC) for the unified bank is a contract belonging to M^U which maximizes $W^{non-run}$ subject to the ICC. Several observations follow: First, the ICC never binds in the unified system. For $\gamma \in [0, \frac{1}{2y})$, the ICC is not relevant. For $\gamma \in [\frac{1}{2y}, 1]$, the incentive to smooth the left-over consumption across the depositors and RC implies that

$$
c_I^2(1) = c_P^2(1) - 1 = \gamma y R_B - \frac{R_B + 1}{2}.
$$

From the RC, we also have

$$
c_P^2(0) - 1 = \gamma y R_B - 1.
$$

Hence the ICC does not bind for $\gamma \in [\frac{1}{2y}, 1]$. Furthermore, since the ICC does not bind and complete consumption smoothing is available, the incentive to economize on the liquid asset is the only concern when choosing γ in both $[0, \frac{1}{2y})$ and $[\frac{1}{2y}, 1]$. Therefore, to find the NROC for the unified system, we

[6] As in PS, we assume that a depositor in the queue who refuses to withdraw can return in period 2 without prejudice.

merely need to compare the values of $W^{non-run}$ when $\gamma = 0$ versus when $\gamma = \frac{1}{2y}$.

Proposition 1 *If $\overline{u}$ is larger than the threshold u_0, the unified bank never runs out of liquidity in period 1 (i.e., $\gamma = \frac{1}{2y}$). Otherwise, the unified bank holds only the illiquid asset (i.e., $\gamma = 0$). The threshold u_0 is equal to*

$$[u(yR_A - 1) - qu(yR_A - \frac{R_A + 1}{2}) - (1-q)u(yR_A - \frac{R_A - R_B}{2} - 1)]/[q(1-\beta)].$$

All Proofs are in the Appendix.

In PS, the unified system *always* runs out of cash when the realized fraction of impatient consumers is high. This difference is due to the fact that in PS there is a continuum of depositors and the fraction of impatient depositors, α, is continuously distributed with support $[0, \overline{\alpha}]$. Therefore, the probability of running out of liquidity increases continuously as the liquidity is lowered below $\overline{\alpha}$. Hence if liquidity is smaller than but sufficiently close to $\overline{\alpha}$, the *expected* cost of running out of liquidity diminishes while the benefit due to higher return from the illiquid asset does not. However, for a model like ours with a finite number of depositors, that probability increases abruptly, from 0 to q as γ is lowered below $\frac{1}{2y}$. Therefore, the expected cost of running out of liquidity cannot be made arbitrarily small and is fixed at $q(1-\beta)\overline{u}$, which is the expected utility loss due to the postponement of the consumption opportunity by the impatient depositor. Hence, depending on the parameters, the NROC might entail that the unified bank never runs out of liquidity. In fact, this is the case when $\overline{u} > u_0$. If the consumption opportunity is more important (i.e., larger $\overline{u}$), or the discounting factor is smaller (i.e., smaller β), or it is more likely to have an impatient depositor (i.e., q is higher), the expected cost of running out of liquidity is larger. That is why the threshold level u_0 is negatively related to $(1-\beta)$ and q.

If $\overline{u} \leq u_0$, the bank is redundant since in autarky the depositors can do equally well by holding only the illiquid asset. Hence for the remainder of the analysis, we focus on the case in which $\overline{u}$ is sufficiently large such that we only need to consider $\gamma \in [\frac{1}{2y}, 1]$.

A run equilibrium in the post-deposit game is defined as a BNE in which both depositors, regardless of types, arrive in period 1. A run equilibrium exists if and only if a patient depositor strictly prefers arriving in period 1. That is,

$$\frac{1}{2}u[(1-\gamma)yR_A + c_I^2(1)] + \frac{1}{2}u[(1-\gamma)yR_A + c_P^2(1) - 1] \\ > u[(1-\gamma)yR_A + c_P^2(1) - 1]. \tag{8}$$

As with the ICC, inequality (*8*) only involves utilities from left-over consumption. The LHS and (the RHS) of inequality (*8*) is a patient depositor's expected utility from left-over consumption if he arrives in period 1 (and period 2), while assuming the other patient depositor (if he exists) chooses period-1 arrival. Obviously, inequality (*8*) can be reduced to

$$c_I^2(1) > c_P^2(1) - 1. \tag{9}$$

Since the NROC entails complete left-over consumption smoothing, $c_I^2(1) = c_P^2(1) - 1$, the unified banking system is immune to panic-based runs.

3.2 The Separated Financial System

The non-run optimal contract (NROC) for the separated banking system is a contract belonging to M^S which maximizes $W^{non-run}$ subject to the ICC. Unlike the unified system, we must have non-negative $c_I^2(1)$ in the separated system since the GSB does not have access to the illiquid asset. Therefore, the incentive to smooth the left-over consumption across the depositors is restricted by the non-negativity constraint if the bank does not have sufficient liquidity. To be more specific, for $\gamma \in [\frac{1}{2y}, \frac{1}{2y} + \frac{1}{2yR_B}]$, the non-negativity constraint binds and

$$c_P^2(1) - 1 < c_I^2(1) = 0.$$

For $\gamma \in (\frac{1}{2y} + \frac{1}{2yR_B}, 1]$, the non-negativity constraint does not bind so there is complete consumption smoothing. Combining this with the resource constraint, we have

$$c_I^2(1) = \max\{0, \gamma y R_B - \frac{R_B + 1}{2}\}$$

and

$$c_P^2(1) - 1 = \min\{(2\gamma y - 1)R_B - 1, \gamma y R_B - \frac{R_B + 1}{2}\}.$$

If the bank has sufficient liquidity so that left-over consumption is completely smoothed across the depositors, the ICC does not bind (as in the unified system). Furthermore, from condition (*8*), we know that the non-run equilibrium is the unique equilibrium. However, if the GSB does not have sufficient liquidity, the ICC might bind depending on the parameters. Furthermore, from the condition (*8*), we know that a run equilibrium also exists. To summarize, we have the following lemma.

Lemma 1 *If the GSB has sufficient liquidity (i.e., $\gamma \geq \frac{1}{2y} + \frac{1}{2yR_B}$), the ICC does not bind and non-run is the unique equilibrium in the post-deposit game. Otherwise, the ICC might bind depending on the parameters and we also have a run equilibrium in the post-deposit game.*

Let $\gamma^{run-proof}$ denote the value of $\frac{1}{2y} + \frac{1}{2yR_B}$. From the lemma above we know that a run equilibrium exists if and only if $\gamma < \gamma^{run-proof}$. The next proposition shows that the NROC in the separated system also has a run equilibrium. This result is the same as the result in PS, where there is a continuum of depositors.

Proposition 2 *Let $\widehat{\gamma}$ denote the fraction of the liquid asset investment specified by the NROC. We have $\widehat{\gamma} < \gamma^{run-proof}$.*

The NROC only considers the non-run equilibrium, so the benefit from eliminating runs is not included in this calculation. Therefore, the only benefit of increasing γ to $\gamma^{run-proof}$ is smoothing of left-over consumption when there is an impatient depositor. The cost of doing this is the lower return compared to the illiquid asset. But when γ sufficiently close to $\gamma^{run-proof}$, left-over consumption is sufficiently close to complete smoothing. So the benefit of raising γ diminishes while the cost does not.

4 The Pre-Deposit Game

Since the NROC in the unified system is immune to runs, the optimal contract in the pre-deposit game is the same as the NROC from the post-deposit game. So in this section, we only analyze the pre-deposit game for the separated system. In Section 3, we showed that in the post-deposit game of the separated system, we have both a non-run equilibrium and a run equilibrium for any $\gamma \in [\frac{1}{2y}, \gamma^{run-proof})$. For any $\gamma \in [\gamma^{run-proof}, 1]$, we have a unique non-run equilibrium. We assume – as in PS – that bank runs are sunspot-driven and the run probability is s. Hence whether a run equilibrium exists in the *pre-deposit* game depends on whether the optimal contract for the pre-deposit game, denoted by $\gamma^*(s)$, belongs to the set $[\frac{1}{2y}, \gamma^{run-proof})$.

Denote the ex-ante welfare by $W(\gamma; s)$. We have

$$W(\gamma; s) = \begin{cases} sW^{run}(\gamma) + (1-s)W^{non-run}(\gamma) & \text{if } \gamma \in [\frac{1}{2y}, \gamma^{run-proof}], \\ W^{non-run}(\gamma) & \text{if } \gamma \in [\gamma^{run-proof}, 1], \end{cases} \tag{10}$$

where

$$\begin{aligned} W^{run}(\gamma) = {} & q(\frac{\overline{u} + \beta\overline{u}}{2} + \frac{2\overline{u}}{2}) + (1-q)(2\overline{u}) \\ & + u[(1-\gamma)yR_A + c_I^2(1)] + u[(1-\gamma)yR_A + c_P^2(1) - 1]. \end{aligned} \tag{11}$$

The ex-ante welfare in the run equilibrium is denoted by W^{run}. In the run equilibrium, both depositors, regardless of their types, arrive in period 1 and a patient depositor chooses to withdraw if and only if he is the first in the queue. If there is one impatient depositor, the probability of the patient depositor being the first in the queue is $\frac{1}{2}$ and, in this case, there is misallocation of liquidity since the impatient depositor is unable to make his withdrawal. However, if the patient depositor is the second in the queue or both depositors are patient, there is no misallocation of liquidity even in the run equilibrium.

As we show in Section 3, depending on the parameters, the ICC might bind for $\gamma \in [\frac{1}{2y}, \gamma^{run-proof}]$. To analyze the ICC, let $D(\gamma)$ denote the difference between the LHS and the RHS of the inequality (*7*). Let Δ denote the difference between asset returns, that is

$$\Delta = R_A - R_B > 0.$$

The next lemma shows that, if the difference in asset returns is small, a patient depositor's incentive to arrive in period 2 strictly increases with the liquid asset investment.

Lemma 2 *If $\Delta < R_B$, $D(\gamma)$ is strictly increasing in γ.*

For any $\gamma \in [\frac{1}{2y}, \gamma^{run-proof}]$, complete smoothing of left-over liquidity is restricted by the non-negativity constraint. Hence, if a depositor withdraws in period 1, the remaining depositor receives in period 2 all the liquid asset left in the GSB including its return, namely $(2\gamma y - 1)R_B$. Therefore, with more liquidity, a patient depositor can get more from the GSB in period 2. However, it is costly to have more liquidity since the remaining depositor's total left-over consumption also includes the illiquid asset and its return, $(1-\gamma)yR_A$. Hence, if the difference in the asset returns is small (i.e., $\Delta < R_B$), a patient depositor's incentive to arrive in period 2 strictly increases with the bank liquidity.

The condition $\Delta < R_B$ is equivalent to $R_A < 2R_B$, which seems to be a reasonable assumption. We focus on the case of $R_B < R_A < 2R_B$ in the analysis of the optimal contract. We characterize the ICC as a constraint on γ in the following lemma.

Lemma 3 *If $\Delta < R_B$, the ICC is equivalent to $\gamma \geq \gamma^{IC}$. If $D(\frac{1}{2y}) \geq 0$, we have $\gamma^{IC} = 1$. If $D(\frac{1}{2y}) < 0$, we have γ^{IC} equal to the level of γ such that $D(\gamma) = 0$.*

From the above analysis, we know that $\gamma^*(s)$ is the level of $\gamma \in [\gamma^{IC}, 1]$ which maximizes $W(\gamma; s)$. For any $\gamma \in [\gamma^{run-proof}, 1]$, run equilibria can be eliminated and complete consumption smoothing is provided. The best γ is $\gamma^{run-proof}$. Therefore, we only have to compare $\gamma^{run-proof}$ with the best γ in the range of $[\gamma^{IC}, \gamma^{run-proof})$ to find $\gamma^*(s)$. Due to the postponement of the consumption opportunity by the impatient depositor, the expected utility in the run equilibrium is always smaller than the expected utility with $\gamma = \gamma^{run-proof}$. Therefore, the optimal contract tolerates runs if and only if s is sufficiently small.

Proposition 3 *The optimal contract tolerates runs if and only if the run probability is smaller than a threshold s_0. If the run probability is larger than that threshold, the optimal contract is run proof (i.e., $\gamma = \gamma^{run-proof}$).*

Now let us analyze how the optimal contract responds as the run probability increases from 0 to a sufficiently small value such that it is still optimal to tolerate the runs.

Proposition 4 *If the ICC constraint does not bind at the NROC, then $\gamma^*(s)$ strictly increases with s when s is sufficiently small. If the ICC binds at the NROC, we have $\gamma^*(s) = \gamma^{IC}$ when s is sufficiently small.*

Three numerical examples follow. In the first example, banks, are redundant.

Example 1: $\overline{u} = 3$, $\beta = 0.5$, $q = 0.9$, $y = 1.1$, $R_A = 1.5$, $R_B = 1.3$, $u(c) = \frac{c^{1-\theta}-1}{1-\theta}$, where $\theta = 2$. For the unified system, we have $W^{non-run} = 4.173077$ when $\gamma = 0$ and $W^{non-run} = 3.681818$ when $\gamma = \frac{1}{2y} = 0.454545$. Therefore $\gamma = 0$ for the NROC.

In the next example, the ICC does not bind for the GSB; for small s, $\gamma^*(s)$ is strictly increasing.

Example 2: The parameters are the same as Example 1 except that $\overline{u} = 10$. For the unified system, we have $W^{non-run} = 16.423077$ when $\gamma = 0$ and $W^{non-run} = 17.681818$ when $\gamma = \frac{1}{2y} = 0.454545$. Therefore $\gamma = \frac{1}{2y} = 0.454545$ for the NROC. For the GSB, we have $\gamma^{IC} = 0.765910$ and $\gamma^{run-proof} = 0.804196$. If $s > s_0 = 0.017708$, the optimal contract is run-proof ($i.e.$ $\gamma^*(s) = \gamma^{run-proof}$). If $s < s_0$, the optimal contract tolerates runs and $\gamma^*(s)$ is strictly increasing in s. If $s = s_0$, run-tolerating contract and the run-proof contract would deliver the same ex-ante welfare. In Figure 1, we plot $\gamma^*(s)$. The jump of γ to the $\gamma^{run-proof}$ at $s = s_0$ dwarfs the increase in $\gamma^*(s)$ below s_0. To show more clearly that $\gamma^*(s)$ does strictly increase in s when $s < s_0$, we also plot the constant $\widehat{\gamma}$ as the dotted red horizontal line for comparison to $\gamma^*(s)$. We have $\gamma^*(0) = \widehat{\gamma} = 0.778153$.

− − − − − − − − −

Figure 1 about here

− − − − − − − − −

In the next example, $\gamma^*(s)$ is a step function.

Example 3: All the parameters are the same as in Example 2 except that $\theta = 1.2$. For the unified system, we have $W^{non-run} = 16.600230$ when $\gamma = 0$ and $W^{non-run} = 18.359332$ when $\gamma = \frac{1}{2y} = 0.454545$. Therefore $\gamma = \frac{1}{2y} = 0.454545$ for the NROC. For the GSB, the ICC binds and we have $\gamma^{IC} = \widehat{\gamma} = 0.760669$. Since $\gamma^{run-proof}$ only depends on y and R_B, we have the same $\gamma^{run-proof}$ as in Example 2. If $s > s_0 = 0.019148$, the optimal contract is run-proof (*i.e.* $\gamma^*(s) = \gamma^{run-proof}$). If $s < s_0$, the optimal contract tolerates runs and is locally constant $\gamma^*(s) = \gamma^{IC}$. In Figure 2, we plot the step function $\gamma^*(s)$.

− − − − − − − − −

Figure 2 about here

− − − − − − − − −

5 Summary and Concluding Remarks

In the separated financial system, the GSB has access only to the liquid asset. Therefore, to eliminate runs the GSB would have to hold too much liquid asset, which is costly given the difference in the asset returns of the illiquid and liquid assets. The cost cannot be justified unless the run probability is sufficiently high. On the contrary, there is no portfolio restriction for the consolidated bank. It is able to use the illiquid asset to provide enough incentive for the patient depositor to wait. It is immune to panic-based runs.

We have focused on the GSB where sunspot-driven runs can occur. The optimal banking contract is completely determined by $\gamma^*(s)$, the fraction of resources deposited in the GSB as a function of the exogenous run probability s. For small s, consumers tolerate runs. How does $\gamma^*(s)$ vary with small changes in s? In some cases, $\gamma^*(s)$ is invariant to small changes in s. In these cases, the ICC is binding. Other cases are more intuitive: The fraction $\gamma^*(s)$ increases with small increases in s. That is, depositors respond to increased risk by choosing to be more conservative (i.e., to be more liquid). In these intuitive cases, the ICC is not binding.

If the ICC does not bind, then for small, positive s the sunspot allocation in the pre-deposit game is not a mere randomization over outcomes from the post-deposit game. This places this result in the general sunspot equilibrium literature, from which we know: Sunspot equilibria are sometimes mere randomizations over equilibria from the corresponding certainty economy; the GSB with binding ICC is analogous. Other times, sunspot equilibria are not randomizations over the corresponding certainty equilibria. Indeed they are entirely different animals; the GSB with non-binding ICC is like one of these different sunspot animals.

Appendix: Proofs

Proof of Proposition 1

For $\gamma = 0$,

$$W^{non-run} = q(\overline{u} + \beta\overline{u}) + (1-q)(2\overline{u}) + u[yR_A - 1].$$

For $\gamma = \frac{1}{2y}$,

$$W^{non-run} = 2\overline{u} + qu(yR_A - \frac{R_A + 1}{2}) + (1-q)u(yR_A - \frac{R_A - R_B}{2} - 1).$$

Therefore, if $\overline{u} > u_0$, the NROC implies that $\gamma = \frac{1}{2y}$. If $\overline{u} \leq u_0$, the NROC implies that $\gamma = 0$.

Proof of Proposition 2

It can be shown that

$$
\begin{aligned}
(\frac{\partial W^{non-run}}{\partial \gamma})_{\gamma=\gamma^{run-proof}} &= -2yqu'[yR_A - \frac{R_A}{2} - \frac{R_A}{2R_B}](R_A - R_B) \\
&- 2y(1-q)u'[yR_A - \frac{R_A}{2} - \frac{R_A}{2R_B} + \frac{R_B - 1}{2}](R_A - R_B) \\
&< 0.
\end{aligned}
$$

Hence we have $\widehat{\gamma} < \gamma^{run-proof}$.

Proof of Lemma 2

It can be shown that

$$
\begin{aligned}
D'(\gamma) = y\{&(1 - \frac{q}{2})u'[(1-\gamma)yR_A]R_A \\
&+ \frac{q}{2}u'[(1-\gamma)yR_A + (2\gamma y - 1)R_B - 1](2R_B - R_A) \\
&- (1-q)u'[(1-\gamma)yR_A + (\gamma y - 1)R_B - 1](R_A - R_B)\}.
\end{aligned}
$$

Therefore, if $\Delta < R_B$, we have $(2R_B - R_A) > 0$. Since

$$
\begin{aligned}
& u'[(1-\gamma)yR_A] \\
> \;& u'[(1-\gamma)yR_A + (\gamma y - 1)R_B - 1]
\end{aligned}
$$

for any $\gamma \in [\frac{1}{2y}, \gamma^{run-proof}]$, we have $D'(\gamma) > 0$.

Proof of Lemma 3

If $D(\frac{1}{2y}) \geq 0$, we have $D(\gamma) > 0$ for any $\gamma \in [\frac{1}{2y}, \gamma^{run-proof}]$ since $D(\gamma)$ is strictly increasing. Since $D(\gamma^{run-proof})$ is always positive, if $D(\frac{1}{2y}) < 0$ there must be a unique level of $\gamma \in [\frac{1}{2y}, \gamma^{run-proof}]$ such that $D(\gamma) = 0$. Therefore, the ICC is equivalent to $\gamma \geq \gamma^{IC}$.

Proof of Proposition 4

If the ICC does not bind at NROC, we have $(\frac{\partial W^{non-run}}{\partial\gamma})_{\gamma=\gamma^{IC}} \geq 0$. Then, we have $(\frac{\partial W^{run}}{\partial\gamma})_{\gamma=\gamma^{IC}} > 0$. This is because

$$\begin{aligned}\frac{\partial W^{non-run}}{\partial\gamma} &= -yqu'[(1-\gamma)yR_A]R_A \\ &+ yqu'[(1-\gamma)yR_A + (2\gamma y - 1)R_B - 1](2R_B - R_A) \\ &- 2y(1-q)u'[(1-\gamma)yR_A + (\gamma y - 1)R_B - 1](R_A - R_B)\end{aligned}$$

and

$$\begin{aligned}&\frac{\partial W^{run}}{\partial\gamma} \\ &= -yu'[(1-\gamma)yR_A]R_A \\ &+ yu'[(1-\gamma)yR_A + (2\gamma y - 1)R_B - 1](2R_B - R_A).\end{aligned}$$

We see that if $\frac{\partial W^{non-run}}{\partial\gamma} \geq 0$, we have $\frac{\partial W^{run}}{\partial\gamma} > \frac{\partial W^{non-run}}{\partial\gamma} > 0$ and therefore,

$$\begin{aligned}&(\frac{\partial W(\gamma;s)}{\partial\gamma})_{\gamma=\gamma^*(s)} \\ = \; & s(\frac{\partial W^{run}}{\partial\gamma})_{\gamma=\gamma^*(s)} + (1-s)(\frac{\partial W^{non-run}}{\partial\gamma})_{\gamma=\gamma^*(s)} \\ = \; & 0\end{aligned}$$

for s sufficiently small. According to the implicit function theorem, we have

$$\begin{aligned}\frac{\partial\gamma^*(s)}{\partial s} &= [\frac{\frac{\partial W^{non-run}}{\partial\gamma} - \frac{\partial W^{run}}{\partial\gamma}}{s\frac{\partial^2 W^{run}}{\partial\gamma^2} + (1-s)\frac{\partial^2 W^{non-run}}{\partial\gamma^2}}]_{\gamma=\gamma^*(s)} \\ &> 0.\end{aligned}$$

If the ICC does not bind at the NROC, we have $(\frac{\partial W^{non-run}}{\partial\gamma})_{\gamma=\gamma^{IC}} < 0$. Then for s sufficiently small,

$$\begin{aligned} & (\frac{\partial W(\gamma; s)}{\partial \gamma})_{\gamma=\gamma^{IC}} \\ = \;& s(\frac{\partial W^{run}}{\partial \gamma})_{\gamma=\gamma^{IC}} + (1-s)(\frac{\partial W^{non-run}}{\partial \gamma})_{\gamma=\gamma^{IC}} \\ < \;& 0. \end{aligned}$$

Therefore, $\gamma^*(s) = \gamma^{IC}$ for s is small.

References

[1] Peck, J., and K. Shell (2008), "Bank portfolio restrictions and equilibrium bank runs", working paper.

[2] Peck, J., and K. Shell (2010), "Could making banks hold only liquid assets induce bank runs?", Journal of Monetary Economics 57,420–427.

[3] Shell, K. (2008), "Sunspot equilibrium", L. Blume and S. Durlauf, eds, The New Palgrave: A Dictionary of Economics, 2nd Edition, New York: Palgrave Macmillan.

[4] Shell, K., and Y. Zhang (2017), "Bank runs: the pre-deposit game", working paper.

[5] Wallace, N. (1988), "Another attempt to explain an illiquid banking system: The Diamond and Dybvig model with sequential service taken seriously", Quarterly Review of the Federal Reserve Bank of Minneapolis 12 (4),3–16.

[6] Wallace, N. (1996), "Narrow banking meets the Diamond-Dybvig model", *Quarterly Review of the Federal Reserve Bank of Minneapolis* **20 (1)**,3-13.

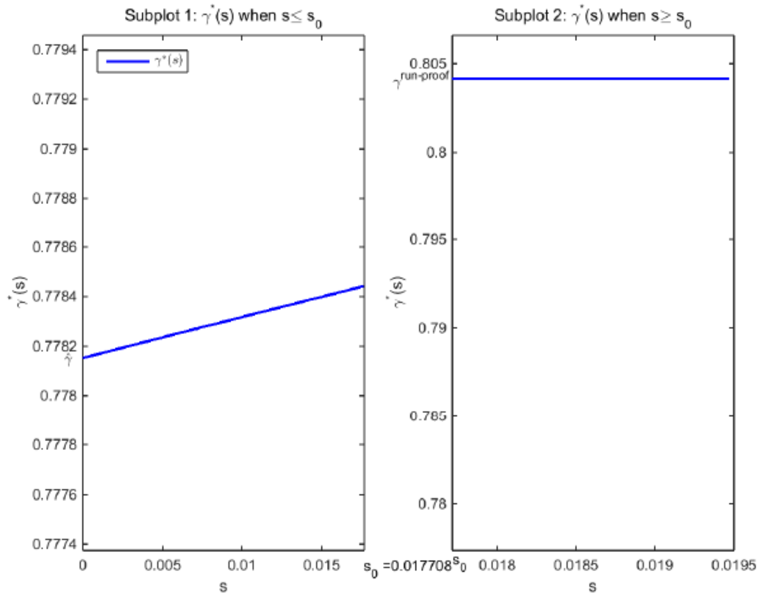

Figure 1. $\gamma^*(s)$ for Example 2

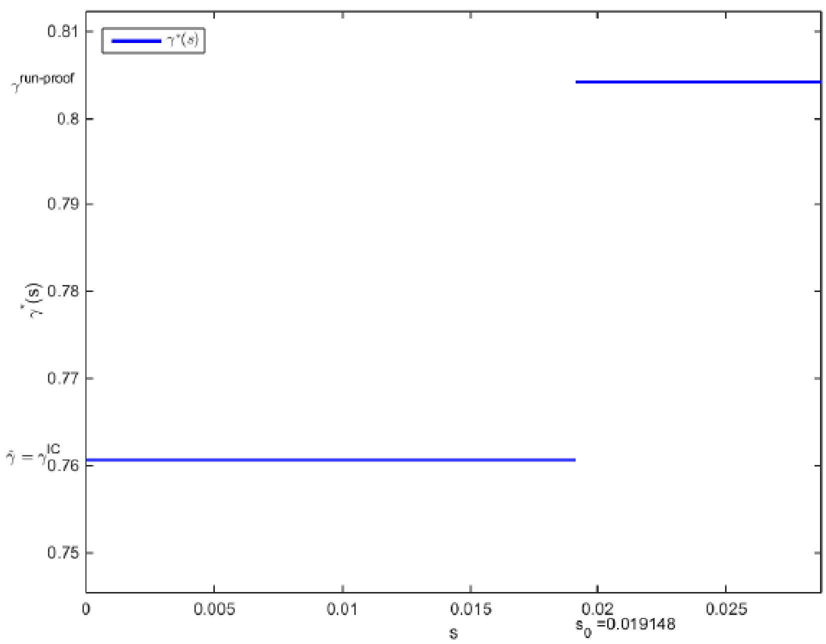

Figure 2. $\gamma^*(s)$ for Example 3