Economics 4905: Lecture 3

Karl Shell

Cornell University

Fall 2018
Suppose the endowment is given as $(20, 15, 10, 5)$
Let $\tau = (5, 2, -2, -5)$ and $\tau' = 2\tau$

- $x_1 = 20 - 5P^m > 0 \Rightarrow P^m < 4$  
- $x_2 = 15 - 2P^m > 0 \Rightarrow P^m < \frac{15}{2}$
- $0 < P^m < 4 < \frac{15}{2}$
- $\bar{P}^m = 4$
- $\tau' = 2(5, 2, -2, -5) = (10, 4, -4, -10)$

- $x_1' = 20 - 10P^{m'} > 0 \Rightarrow P^{m'} < 2$
- $x_2' = 15 - 4P^{m'} > 0 \Rightarrow P^{m'} < \frac{15}{4}$
- $0 < P^{m'} < 2 < \frac{15}{4}$
- $\bar{P}^{m'} = 2$
$P^m \in [0, 4), P^m = [0, 4), P^{m'} \in [0, 2), P^{m'} = [0, 2)$

- This is a statement about sets, not price levels
- If everyone believes QTM, then QTM is REE
- If not, not
Take-Away

- Indeterminacy of the price level
- Beliefs about $P^m$ and fundamentals $\omega$ jointly determine outcomes
- Beliefs matter
- The quantity theory of money is (too) subtle. Doubling $\tau$ will affect $P^m$ but not necessarily according to QTM.
Two Currencies, R and B:

- Bi-metalism in the US
- "Cross of Gold" speech
- Borrowers hurt by deflation
Two Currencies, R and B:

- \( l = 1, n = 5, \omega = (25, 20, 15, 10, 5) \)
- \( \tau^B = (1, 1, 1, -1, -1), \tau^R = (1, 1, -1, -1, -1) \)
- \( \sum \tau^B_h = 1, \sum \tau^R_h = -1 \)
- \( P^B \sum \tau^B_h + P^R \sum \tau^R_h = 0 \)
- \( P^B - P^R = 0 \Rightarrow P^B = P^R \)
Two Currencies, R and B:

\[ x_1 = 25 - P^B - P^R = 25 - 2P^B > 0 \Rightarrow P^B < \frac{25}{2} \]
\[ x_2 = 20 - P^B - P^R \Rightarrow P^B < 10 \]
\[ x_3 = 15 - P^B + P^R = 15 \]
\[ 0 \leq P^B < 10 < \frac{25}{2} \]

Solving for exchange rate:

\[ \frac{P^R}{P^B} = -\frac{\sum \tau_h^B}{\sum \tau_h^R} = -\frac{1+1+1+(-1)+(-1)}{1+1+(-1)+(-1)+(-1)} = 1 \]
\[ \mathcal{P}^m = \{ P^B, P^R \mid P^B = P^R, P^B \in [0, 10) \} \]

\[ \{ (x_1, x_2, x_3, x_4, x_5) \mid x_1 = 25 - 2P^B, x_2 = 20 - 2P^B, \]
\[ x_3 = 15, x_4 = 10 + 2P^B, \]
\[ x_5 = 5 + 2P^B, P^B \in [0, 10) \} \]

- The elements of \( x \) are not independent. They are constrained by \( \mathcal{P}^m \).
In General

- If $\sum \tau^B_h$ and $\sum \tau^R_h$ agree in sign, then $P^B = P^R = 0$.
- If $\sum \tau^B_h$ and $\sum \tau^R_h$ disagree in sign, then either the exchange rate is

  \[
  \frac{P^B}{P^R} = -\frac{\sum \tau^R_h}{\sum \tau^B_h}
  \]

  or

  \[P^B = P^R = 0\]
Why?

If \( \sum \tau_h^B = \sum \tau_h^R = 0 \), then \( \frac{P^B}{P^R} \) is indeterminate.

Why?
Some Take-aways:

- Surpluses in both "countries" lead to de-monetization.
- Deficits in both "countries" lead to de-monetization.
- In this simple economy, (real) fundamentals such as endowments do not affect exchange rates. They are purely financial.