We recommend that you read or re-read Lectures 3 and 4 from www.karlshell.com > COURSES > CURRENT > ECON 4905 FALL 2018.
Review (Outside) Money Taxation:

- Bonafide Taxes $\tau$
- Balanced Taxes $\tau$
- Bonafide $\tau \equiv$ Balanced $\tau$
- $l = 1$, $\mathcal{P}^m$ is interval $[0, \bar{P}^m)$
- $l > 1$, $\mathcal{P}^m$ is the union of intervals
- Multiple Currencies
- Problem Set 1
- Simple, *finite* static model
Enriching the model to include dynamics and uncertainty: Debreu’s isomorphism

- "Debreu" Isomorphism
  - Expand definition of commodity, $x_h^{i,s,t}$
  - Commodity type $i$, state of nature $s$, time $t$
  - Contingent claims
  - Futures Market

- Profit Maximization
  - Theorem
  - Not assumption
  - Diagrams
Futures Market

- $t = 1, 2$
- $I = 1$
- present prices, $(p^1, p^2) = (1, p^2)$

**CP:**

$$\max u_h(x^1_h, x^2_h)$$

* e.g. $u_h(x^1_h, x^2_h) = \phi_h(x^1_h) + \beta_h \phi_h(x^2_h)$

* s.t. 

$$p^1(x^1_h - \omega^1_h) + p^2(x^2_h - \omega^2_h) = 0$$

**CE:** $(p^1, p^2)$ such that

$$\sum_h x^t_h = \sum_h \omega^t_h \text{ for } t = 1, 2$$

where $x^t_h$ solves CP for $t = 1, 2$. 
Criticism of Futures Market Interpretation:

- Do we really choose today all our future consumptions?
- Do ordinary people use futures market for personal choices over time?
- Everyone on a “meal plan” for everything?
- FM model is real, i.e., non-financial. More stable, but less realistic?
Inside Money Market for Dynamic Economy

- Spot market at each date, $t = 1, 2$
- Saving and dis-saving through "money-market"
- Rational expectations about future spot prices
- Expectations play no role in FM model
Inside Money, continued

**CP:**

\[
\max u_h(x_1^h, x_2^h)
\]

s.t.
\[
\begin{align*}
p^1 x_1^h + p^{m1} x_{m1}^h &= p^1 \omega_1^h \\
p^2 x_2^h + p^{m2} x_{m2}^h &= p^2 \omega_2^h \\
x_{m1}^h + x_{m2}^h &= 0 \text{ or } x_{m2}^h = -x_{m1}^h
\end{align*}
\]

**CE:**

\[
(p^1, p^2; p^{m1}, p^{m2})
\]

s.t.
\[
\begin{align*}
\sum_h x_t^h &= \sum_h \omega_t^h \text{ for } t = 1, 2 \\
\sum_h x_{m,t}^h &= 0 \text{ for } t = 1, 2
\end{align*}
\]

where \(x_t^h\) and \(x_{m,t}^h\) satisfy CP for \(t = 1, 2\).
Simplifying and substituting

- $p^1(x_h^1 - \omega_h^1) = -p^{m1}x_{h}^{m1}$
- $p^2(x_h^2 - \omega_h^2) = p^{m2}x_{h}^{m1}$
- $x_{h}^{m1}$ is a slack variable permitting us to combine terms (discuss):

$$p^1(x_h^1 - \omega_h^1) + p^2(x_h^2 - \omega_h^2) = (p^{m2} - p^{m1})x_{h}^{m1}$$

- In MM, buy low sell high, try to arbitrage the $(p^{m2} - p^{m1})$ gap
- allowing unbounded consumptions denying CE
- therefore in CE: $p^{m1} = p^{m2} = p^m \geq 0$
Assume $p^m > 0$

- We have
  \[ p^1(x_h^1 - \omega_h^1) + p^2(x_h^1 - \omega_h^1) = 0 \]

- MM equilibrium allocation is identical to FM equilibrium allocation

- Irving Fisher (isomorphic to the Arrow article)

- Very important caveat
  - If $p^m = 0$, the money market is closed. No inter-temporal trades
  - This important outcome does not occur in the FM model
Uncertainty

- Two states, \( s = \alpha, \beta \)
- One commodity, \( l = 1 \)
- Finite model, as before
- See Arrow RES article: History of article
- Contingent claims
  - "AD"
  - buy and sell contracts to deliver commodity contingent on the realization of \( s \)
- CP:
  \[
  \begin{align*}
  \max \pi(\alpha)u_h(x_h(\alpha)) + \pi(\beta)u_h(x_h(\beta)) \\
  \pi(\alpha) + \pi(\beta) &= 1 \\
  \text{s.t.} \quad p(\alpha)x_h(\alpha) + p(\beta)x_h(\beta) &= p(\alpha)\omega_h(\alpha) + p(\beta)\omega_h(\beta)
  \end{align*}
  \]
CE for "AD" Economy

\[ p(\alpha), p(\beta) \]

such that

\[ \sum_h x_h(s) = \sum_h \omega_h(s) \text{ for } s = \alpha, \beta \]

where \( x_h(s) \) solves CP for \( s = \alpha, \beta \).
Arrow Securities

- Arrow money
- Buy and sell commodities on spot markets
- Buy and sell Arrow monies $b_h(s)$ for $s = \alpha, \beta$ before $s$ is realized

**CP:**

$$\max E_s u_h(s) = \pi(\alpha)u_h(x_h(\alpha)) + \pi(\beta)u_h(x_h(\beta))$$

s.t.  
$$x_h(s) = \omega_h(s) + b_h(s) \text{ for } s = \alpha, \beta$$

and  
$$p^b(\alpha)b_h(\alpha) + p^b(\beta)b_h(\beta) = 0$$

Hidden assumption: 1 unit of $b_h(s)$ pays 1 unit of commodity in state $s$, 0 otherwise.
CE:

\[(p(\alpha), p(\beta); p^b(\alpha), p^b(\beta))\]

such that

\[\sum_h x_h(s) = \sum_h \omega_h(s) \text{ for } s = \alpha, \beta\]

where \(x_h(s)\) solves CP.
Results

- Every CE allocation in the AD economy can be decentralized as a CE allocation in the Arrow Securities economy.

- Every CE allocation in the AS economy in which we have \((p^b(\alpha), p^b(\beta)) > 0\) is also an equilibrium in the AD contingent claims economy.