Economics 4905: Lecture 8

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Problem Set #2

Problem Set 2 has been posted. It is due on Monday, September 24, at the beginning of the lecture.

Next Lecture

Next lecture will include a review of deriving demand functions from utility functions. It will also introduce the basics of optimization theory. Preparation is not necessary. (For the first consult — if you like — your micro text. For the second — if you like — consult in a math text "the rule of Lagrange" or "mathematical programming" or "Kuhn-Tucker Theorem".

Futures Market

Consider a market with one good per period $\ell = 1$, and two periods $t = 1, 2$.

$$\begin{aligned} \max \quad & u_h(x_h^1, x_h^2) \\ \text{s.t.} \quad & p^1 x_h^1 + p^2 x_h^2 = p^1 \omega_h^1 + p^2 \omega_h^2 \end{aligned}$$

The equilibrium is a price vector (p^1, p^2) such that

$$\sum_h x_h^t = \sum_h \omega_h^t \text{ for } t = 1, 2$$

Futures Market

The interest factor R and interest rate r are defined as follow:

$$p^1 = Rp^2$$
$$r = R - 1$$

Normalizing $p^1 = 1$:

$$(p^1, p^2) = (1, p^2)$$

Then

$$p^2 = \frac{1}{R} = \frac{1}{1 + r}$$

Money Market

Again, consider a market with one good per period $\ell = 1$, and two periods $t = 1, 2$.

$$\begin{aligned} \max \quad & u_h(x_h^1, x_h^2) \\ \text{s.t.} \quad & p^1 x_h^1 + p^{m^1} m_h^1 = p^1 \omega_h^1 \\ & p^2 x_h^2 + p^{m^2} m_h^2 = p^2 \omega_h^2 \end{aligned}$$

An equilibrium in the money market is $(p^1, p^2, p^{m^1}, p^{m^2})$ such that

$$\sum_h x_h^t = \sum_h \omega_h^t \text{ and } \sum_h m_h^t = 0 \text{ for } t = 1, 2$$

Money Market

I will show that in equilibrium $p^{m^1} = p^{m^2} = p^m$. This is the no arbitrage property result.

Consumer h maximizes his financial wealth in order to maximize his inter-temporal utility:

$$\begin{aligned} \max \quad & W_h = p^{m^1} m_h^1 + p^{m^2} m_h^2 \\ \text{s.t.} \quad & m_h^1 + m_h^2 = 0 \end{aligned}$$

Rearranging the constraint and plugging into the wealth equation give:

$$\begin{aligned} m_h^2 &= -m_h^1 \\ W_h &= (p^{m^1} - p^{m^2}) m_h^1 \end{aligned}$$

Money Market

- If $p^{m^1} > p^{m^2}$, the consumer can borrow (high) in period 1 and re-pay (low) in period 2. This means $m_h^1 > 0$.
- If $p^{m^1} < p^{m^2}$, the consumer can lend (high) in period 1 and receive repayment (low) in period 2. This means $m_h^1 < 0$.
- If $p^{m^1} \neq p^{m^2}$, W_h can be arbitrarily large by choosing $|m_h^1|$ arbitrarily large. This allows x_h^1 and x_h^2 to be arbitrarily large.
- So,
$$\sum_h x_h^t > \sum_h \omega_h^t \text{ for } t = 1, 2$$
- Therefore, $p^{m^1} = p^{m^2}$ is required for competitive equilibrium.
- Let $p^m = p^{m^1} = p^{m^2}$.

Equivalence

I will next show that if (x_h^1, x_h^2) solves the futures market problem, it also solved the money market problem.

The excess demand is

$$z_h^t = x_h^t - \omega_h^t$$

Since $p^1 = 1$,

$$z_h^1 + p^2 z_h^2 = 0$$

$$z_h^1 = -p^2 z_h^2$$

Define scalar k by

$$k = z_h^1 - p^2 z_h^2$$

Equivalence

Set $k = p^m m_h^1 = -p^m m_h^2$. Then

$$
\begin{aligned}
z_h^1 + p^m m_h^2 &= 0 \\
p^2 z_h^2 + p^m m_h^2 &= 0 \\
m_h^1 + m_h^2 &= 0
\end{aligned}
$$

Hence, if (x_h^1, x_h^2) is an equilibrium allocation in the futures market, it is also an equilibrium in the money market.

Equivalence

Next, I will show that if (x_h^1, x_h^2) for $h = 1, \ldots, n$ solves the money market problem with $p^m > 0$, then it also solves the futures market problem.

Equivalence

$$m_h^1 + m_h^2 = 0$$
$$m_h^1 = -m_h^2$$

Since $p^m > 0$,

$$z_h^1 = -p^m m_h^1 \qquad\qquad p^2 z_h^2 = p^m m_h^1$$
$$\frac{z_h^1}{p^m} = -m_h^1 \qquad\qquad \frac{p^2 z_h^2}{p^m} = m_h^1$$

m_h^1 is a slack variable. Hence, we have

$$z_h^1 + p^2 z_h^2 = 0$$

Hence, if (x_h^1, x_h^2) is an equilibrium allocation in the money market economy with $p^m > 0$, then (x_h^1, x_h^2) is also an equilibrium allocation in the corresponding futures market economy.