Economics 4905
Financial Fragility and the Macroeconomy
Fall 2018
Problem Set 1
Due before the beginning of class on Monday, September 17, 2018

1 Money Taxation

Consider an economy with a single commodity, $\ell = 1$, chocolate. There are 5 consumers, so $n = 5$. The endowments are defined by

$$\omega = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5)$$

$$= (900, 800, 700, 600, 500)$$

measured in ounces of chocolate.

1.1 A Single Currency

There is one money, dollars. The chocolate price of money is $P^m \geq 0$. In each of the following cases, solve for the set $\mathcal{P}^m$ of equilibrium prices P^m, given the following tax policies τ, and the set of equilibrium commodity allocations, $x = (x_1, x_2, \ldots, x_5)$. Provide the units in which the variables are measured.

a) $\tau = (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5) = (2, 1, 0, 0, -3)$

a) Solution:
First check if the tax system is balanced

$$\sum_h \tau_h = 2 + 1 - 3 = 0$$

In general,

$$x_h = \omega_h - \tau_h P^m > 0$$

Taxes τ are in money, but Mr. h's endowment ω_h is in chocolate. The price of money, P^m, is the rate of exchanging one unit of currency (like dollars) with a unit of real good, here chocolate; P^m is therefore in *chocolate/dollars*.

For Mr. 1, we have $900 - 2P^m > 0$. Therefore $P^m < 450$.

For Mr. 2, $800 - P^m > 0$. Therefore $P^m < 800$.

Thus, we have $\mathcal{P}^m = [0, 450)$. Note that a worthless currency, $P^m = 0$, is an equilibrium outcome.

b) $\tau = (10, 5, 0, -8, -7)$

b) Solution:
Check if tax system is balanced

$$\sum_h \tau_h = 10 + 5 - 8 - 7 = 0$$

For Mr. 1, $900 - 10P^m > 0$, so $P^m < 90$.
For Mr. 2, $800 - 5P^m > 0$. Thus, $P^m < 160$.
We have $\mathcal{P}^m = [0, 90)$.

c) $\tau = (20, 2, 1, -2, -20)$
We may immediately note that $\sum_h \tau_h = 20 + 2 + 1 - 2 - 20 = 1 \neq 0$. Thus, taxes are not balanced in this finite economy. The equilibrium price of money must therefore be $\mathcal{P}^m = \{0\}$, such taxes fail to be bonafide. The result will be an economy in autarky, as money will be worthless.

1.2 Two Monies

Consider a scenario where there are 2 monies, red dollars R and blue dollars B, with respective chocolate prices of money, $P^R \geq 0$ and $P^B \geq 0$.

In each of the following cases, solve for the equilibrium exchange rate between B and R. Do these depend on the endowments ω? Give the economic explanation for your answer.

For each of the 3 cases, solve for the set of equilibrium allocations.

a) $\tau^R = (1, 1, 1, 1, -3)$ and $\tau^B = (1, 0, 0, 0, -2)$

a) Solution:
Recalling that $x_h = \omega_h - P^R \tau_h^R - P^B \tau_h^B$, we may rearrange the equation to get

$$x_h - \omega_h = -P^R \tau_h^R - P^B \tau_h^B$$

If we sum over h consumers, we get

$$\sum_h (x_h - \omega_h) = -P^m \sum_h \tau_h^R - P^m \sum_h \tau_h^B$$

And since when markets clear, $\sum_h (x_h - \omega_h) = 0$,

$$P^R \sum_h \tau_h^R + P^B \sum_h \tau_h^B = 0 \quad \Rightarrow \quad P^R \sum_h \tau_h^R = -P^B \sum_h \tau_h^B$$

Rearranging further, we get the exchange rate as

$$\frac{P^R}{P^B} = -\frac{\sum_h \tau_h^B}{\sum_h \tau_h^R}$$

In this case, $\sum_h \tau_h^R = 1 + 1 + 1 + 1 - 3 = 1$, while $\sum_h \tau_h^B = 1 - 2 = -1$, so

$$\frac{P^R}{P^B} = -\left(\frac{-1}{1}\right) = 1$$

Of course, this is also equivalent to $\frac{P^B}{P^R} = 1$ as well.
The equilibrium allocations are:

$$\begin{aligned} x &= (900, 800, 700, 600, 500) - P^R(1, 1, 1, 1, -3) - P^R(1, 0, 0, 0, -2) \\ &= (900 - 2P^R, 800 - P^R, 700 - P^R, 600 - P^R, 500 + 5P^R) \end{aligned}$$

where $P^R \in [0, 450)$.

b) $\tau^R = (1, 1, 0, -1, -2)$ and $\tau^B = (1, 1, 1, 0, -2)$

b) Solutions:
Here, $\sum_h \tau_h^R = 1 + 1 - 1 - 2 = -1$, while $\sum_h \tau_h^B = 1 + 1 + 1 - 2 = 1$. Thus, it again holds that $\frac{P^R}{P^B} = -\left(\frac{-1}{1}\right) = 1$ (and exchanging in the other direction, $\frac{P^B}{P^R} = 1$).
The equilibrium allocations are:

$$\begin{aligned} x &= (900, 800, 700, 600, 500) - P^R(1, 1, 0, -1, -2) - P^R(1, 1, 1, 0, -2) \\ &= (900 - 2P^R, 800 - 2P^R, 700 - P^R, 600 + P^R, 500 + 4P^R) \end{aligned}$$

where $P^R \in [0, 400)$

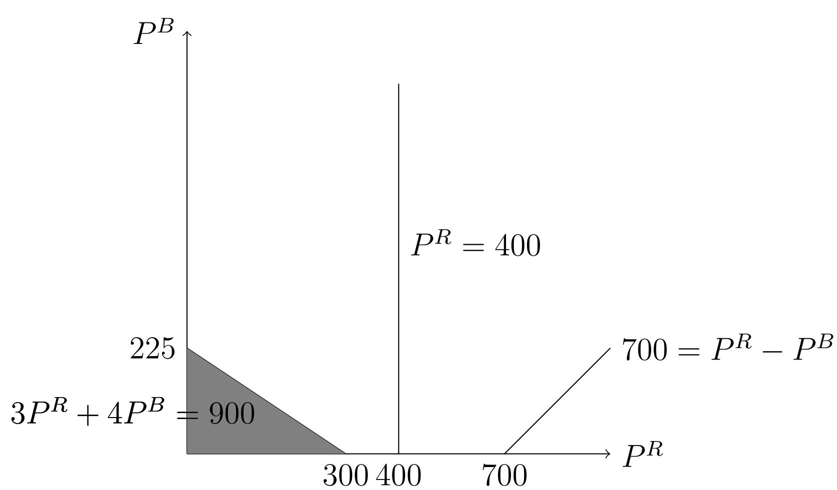
c) $\tau^R = (3, 2, 1, 0, -6)$ and $\tau^B = (4, 0, -1, -1, -2)$ Finally, we have $\sum_h \tau_h^R = 3+2+1-6 = 0$, while $\sum_h \tau_h^B = 4-1-1-2 = 0$. The exchange rate is therefore indeterminate, as $\frac{P^R}{P^B} = \frac{0}{0}$ is not well-defined. Let e be the exchange rate. The exchange rate could be anything $e \in [0, \infty)$.

These exchange rates are independent of the endowments ω; the supply and demand for the currencies completely determines the exchange rate between them unless one or both currencies are worthless. If both tax policies are balanced, then the exchange rate is indeterminate since there are no currency trades.
The equilibrium allocations are:

$$\begin{aligned} x &= (900, 800, 700, 600, 500) - P^R(3, 2, 1, 0, -6) - P^B(4, 0, -1, -1, -2) \\ &= (900 - 3P^R - 4P^B, 800 - 2P^R, 700 - P^R + P^B, 600 + P^B, 500 + 6P^R + 2P^B) \end{aligned}$$

The prices have to satisfy the following constraints:

$$\begin{aligned} 900 - 3P^R - 4P^B &> 0 \\ 800 - 2P^R &> 0 \\ 700 - P^R + P^B &> 0 \end{aligned}$$



By graphing the three inequalities on the positive domain $(P^R, P^B) \in \mathbb{R}^2_+$, we can see that out of the three constraints, only one matters.

$$900 > 3P^R + 4P^B$$

Any non-negative prices (P^R, P^B) that satisfy the above inequality could be an equilibrium price combination.