Economics 4905

Financial Fragility and the Macroeconomy

Fall 2018

Problem Set 2

Due Monday, September 24, 2018

1 Economics of Uncertainty

Consider an individual who owns houses at the coast of Florida. There is a probability of 0.1 that a hurricane will hit the coast. Suppose the total wealth of the individual is \$100 without hurricane but falls down to \$20 after a hurricane. The utility of the individual is $u(x)$ where x is her total wealth. Consider the following utility functions:

a) $u(x) = -\frac{1}{x}$

b) $u(x) = \sqrt{x}$

c) $u(x) = x + 50$

d) $u(x) = x^2$

For each of the utility function, answer the following questions:

i) Is the individual risk averse, risk neutral, or risk loving?

ii) Would the individual prefer a "fair" bet to being unhedged?

iii) How large a premium permits him to buy insurance?

2 Solutions

2.1 Part (a)

i) Taking the second derivative:

$$u'(x) = \frac{1}{x^2}$$
$$u''(x) = -\frac{2}{x^3} < 0$$

The individual is risk averse.

ii) The expected utility from being unhedged:

$$0.9u(100) + 0.1u(20) = 0.9\left(-\frac{1}{100}\right) + 0.1\left(-\frac{1}{20}\right) = -0.014$$

The expected utility from a "fair" bet:

$$u(0.9(100) + 0.1(20)) = u(92) = -\frac{1}{92} = -0.01087$$

Since $-0.01087 > -0.014$, the individual prefers a "fair" bet.

iii) Let P be the largest premium that still permits him to buy insurance. P solves the following equation

$$u(0.9(100) + 0.1(20) - P) = 0.9u(100) + 0.1u(20)$$

where the individual is indifferent between the "fair" bet after paying the premium P and being unhedged. Solving the equation gives:

$$\begin{aligned} -\frac{1}{92 - P} &= -0.014 \\ P &= 20.57 \end{aligned}$$

2.2 Part (b)

i) Taking the second derivative:

$$\begin{aligned} u'(x) &= \frac{1}{2\sqrt{x}} \\ u''(x) &= -\frac{1}{4x^{3/2}} < 0 \end{aligned}$$

The individual is risk averse.

ii) The expected utility from being unhedged:

$$0.9u(100) + 0.1u(20) = 0.9\sqrt{100} + 0.1\sqrt{20} = 9.4472$$

The expected utility from a "fair" bet:

$$u(0.9(100) + 0.1(20)) = u(92) = \sqrt{92} = 9.5917$$

Since $9.5917 > 9.4472$, the individual prefers a "fair" bet.

iii) Let P be the largest premium that still permits him to buy insurance. P solves the following equation

$$u(0.9(100) + 0.1(20) - P) = 0.9u(100) + 0.1u(20)$$

where the individual is indifferent between the "fair" bet after paying the premium P and being unhedged. Solving the equation gives:

$$\begin{aligned} \sqrt{92 - P} &= 0.9\sqrt{100} + 0.1\sqrt{20} \\ P &= 2.7502 \end{aligned}$$

2.3 Part (c)

i) Taking the second derivative:

$$\begin{aligned} u'(x) &= 1 \\ u''(x) &= 0 \end{aligned}$$

The individual is risk neutral.

ii) The expected utility from being unhedged:

$$0.9u(100) + 0.1u(20) = 0.9(100 + 50) + 0.1(20 + 50) = 142$$

The expected utility from a "fair" bet:

$$u(0.9(100) + 0.1(20)) = u(92) = 92 + 50 = 142$$

The individual is indifferent between a "fair" bet and being unhedged.

iii) Let P be the largest premium that still permits him to buy insurance. P solves the following equation

$$u(0.9(100) + 0.1(20) - P) = 0.9u(100) + 0.1u(20)$$

where the individual is indifferent between the "fair" bet after paying the premium P and being unhedged. Solving the equation gives:

$$\begin{aligned}(92 - P) + 50 &= 142\\ P &= 0\end{aligned}$$

2.4 Part (d)

i) Taking the second derivative:

$$\begin{aligned}u'(x) &= 2x\\ u''(x) &= 2 > 0\end{aligned}$$

The individual is risk loving.

ii) The expected utility from being unhedged:

$$0.9u(100) + 0.1u(20) = 0.9(100)^2 + 0.1(20)^2 = 9040$$

The expected utility from a "fair" bet:

$$u(0.9(100) + 0.1(20)) = u(92) = 92^2 = 8464$$

Since $9040 > 8464$, the individual prefers being unhedged.

iii) Let P be the largest premium that still permits him to buy insurance. P solves the following equation

$$u(0.9(100) + 0.1(20) - P) = 0.9u(100) + 0.1u(20)$$

where the individual is indifferent between the "fair" bet after paying the premium P and being unhedged. Solving the equation gives:

$$\begin{aligned}(92 - P)^2 &= 9040\\ P &= 92 \pm 4\sqrt{565}\\ &= -3.0789 \text{ or } 187.0789\end{aligned}$$

$P = 187.0789$ cannot be a solution here because this amount is larger than the wealth of the individual in any state (\$100 and \$20). It is not possible for the individual to be willing to pay this much premium. Therefore, $P = -3.0789$. In words, the individual is willing to pay up to \$3.0789 to go from having a "fair" bet to being unhedged.