1 Economics of Uncertainty

Consider an individual who owns houses at the coast of Florida. There is a probability of 0.1 that a hurricane will hit the coast. Suppose the total wealth of the individual is $100 without hurricane but falls down to $20 after a hurricane. The utility of the individual is $u(x)$ where $x$ is her total wealth. Consider the following utility functions:

a) $u(x) = -\frac{1}{x}$
b) $u(x) = \sqrt{x}$
c) $u(x) = x + 50$
d) $u(x) = x^2$

For each of the utility function, answer the following questions:

i) Is the individual risk averse, risk neutral, or risk loving?

ii) Would the individual prefer a “fair” bet to being unhedged?

iii) How large a premium permits him to buy insurance?

2 Solutions

2.1 Part (a)

i) Taking the second derivative:

\[ u'(x) = \frac{1}{x^2} \]

\[ u''(x) = \frac{-2}{x^3} < 0 \]

The individual is risk averse.

ii) The expected utility from being unhedged:

\[ 0.9u(100) + 0.1u(20) = 0.9\left(-\frac{1}{100}\right) + 0.1\left(-\frac{1}{20}\right) = -0.014 \]

The expected utility from a “fair” bet:

\[ u(0.9(100) + 0.1(20)) = u(92) = -\frac{1}{92} = -0.01087 \]

Since $-0.01087 > -0.014$, the individual prefers a “fair” bet.
iii) Let $P$ be the largest premium that still permits him to buy insurance. $P$ solves the following equation

$$u(0.9(100) + 0.1(20) - P) = 0.9u(100) + 0.1u(20)$$

where the individual is indifferent between the “fair” bet after paying the premium $P$ and being unhedged. Solving the equation gives:

$$\frac{1}{92 - P} = -0.014$$

$$P = 20.57$$

2.2 Part (b)

i) Taking the second derivative:

$$u'(x) = \frac{1}{2\sqrt{x}}$$

$$u''(x) = -\frac{1}{4x^{3/2}} < 0$$

The individual is risk averse.

ii) The expected utility from being unhedged:

$$0.9u(100) + 0.1u(20) = 0.9\sqrt{100} + 0.1\sqrt{20} = 9.4472$$

The expected utility from a “fair” bet:

$$u(0.9(100) + 0.1(20)) = u(92) = \sqrt{92} = 9.5917$$

Since $9.5917 > 9.4472$, the individual prefers a “fair” bet.

iii) Let $P$ be the largest premium that still permits him to buy insurance. $P$ solves the following equation

$$u(0.9(100) + 0.1(20) - P) = 0.9u(100) + 0.1u(20)$$

where the individual is indifferent between the “fair” bet after paying the premium $P$ and being unhedged. Solving the equation gives:

$$\sqrt{92 - P} = 0.9\sqrt{100} + 0.1\sqrt{20}$$

$$P = 2.7502$$

2.3 Part (c)

i) Taking the second derivative:

$$u'(x) = 1$$

$$u''(x) = 0$$

The individual is risk neutral.
ii) The expected utility from being unhedged:
\[ 0.9u(100) + 0.1u(20) = 0.9(100 + 50) + 0.1(20 + 50) = 142 \]
The expected utility from a “fair” bet:
\[ u(0.9(100) + 0.1(20)) = u(92) = 92 + 50 = 142 \]
The individual is indifferent between a “fair” bet and being unhedged.

iii) Let \( P \) be the largest premium that still permits him to buy insurance. \( P \) solves the following equation
\[ u(0.9(100) + 0.1(20) - P) = 0.9u(100) + 0.1u(20) \]
where the individual is indifferent between the “fair” bet after paying the premium \( P \) and being unhedged. Solving the equation gives:
\[ (92 - P) + 50 = 142 \]
\[ P = 0 \]

2.4 Part (d)

i) Taking the second derivative:
\[ u'(x) = 2x \]
\[ u''(x) = 2 > 0 \]
The individual is risk loving.

ii) The expected utility from being unhedged:
\[ 0.9u(100) + 0.1u(20) = 0.9(100)^2 + 0.1(20)^2 = 9040 \]
The expected utility from a “fair” bet:
\[ u(0.9(100) + 0.1(20)) = u(92) = 92^2 = 8464 \]
Since 9040 > 8464, the individual prefers being unhedged.

iii) Let \( P \) be the largest premium that still permits him to buy insurance. \( P \) solves the following equation
\[ u(0.9(100) + 0.1(20) - P) = 0.9u(100) + 0.1u(20) \]
where the individual is indifferent between the “fair” bet after paying the premium \( P \) and being unhedged. Solving the equation gives:
\[ (92 - P)^2 = 9040 \]
\[ P = 92 \pm 4\sqrt{565} \]
\[ = -3.0789 \text{ or } 187.0789 \]
\( P = 187.0789 \) cannot be a solution here because this amount is larger than the wealth of the individual in any state ($100 and $20). It is not possible for the individual to be willing to pay this much premium. Therefore, \( P = -3.0789 \). In words, the individual is willing to pay up to $3.0789 to go from having a “fair” bet to being unhedged.