Verification and Commitment

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In a cheap-talk game commitment by sender to a signal structure was assumed by Kamenica & Gentzkow 11

- Q: But how do we get it?
- A: Verification by the receiver as in Townsend 79

Two models:

1. Sec. 1-5 two state model Borrower-Lender interpretation

2. Sec. 6 Continuum of states – CS 82 model not a clear economic interpretation, unfinished

Project yields revenue

$$y = zk. \tag{1}$$

Borrower-lender relation. Borrower will see z first
Townsend 79, Bernanke & Gertler 89, Gale & Hellwig 85

Their timing is:

1. Contract
2. Lender advances k
3. production
4. Borrower sees z and reports it
5. Verification and transfers

This paper: different timing

1. Contract
2. Borrower sees z and reports it
3. Verification,
4. Lender may advance k
5. Production, Transfers, penalties

Who uses verification?—

1. VCs use informal audits
2. Acquirers in takeovers
3. Regulators, IRS, auditors...

Our mechanism is a cheap talk game + verification

Other mechanisms

1. sale of decision rights Grossman & Hart 86
2. Repetition Clementi & Hopenhayn 06
3. Collateral as a signal Besanko & Thakor 87

Model: 2 risk-neutral agents B and L

$$y = zk, \tag{2}$$

where $k \in \{0, 1\}$

$$z = \begin{cases} Z & \text{with prob. } 1 - \pi \\ 0 & \text{with prob. } \pi \end{cases}$$

common prior.

Endowments.—
B has endowment $\omega < 1$, illiquid until transfer time
L has an endowment exceeding unity.

Conflict of interest.—If $k = 1$ but $z = 0$, B still gets a salvage value δ from the project, and L gets nothing.

Outside option. rk Assume

$$\omega < \delta < 1 \leq r < Z. \tag{3}$$

First best.—

$$k_{\text{FB}} = \begin{cases} 1 \text{ if } z = Z \\ 0 \text{ if } z = 0 \end{cases}.$$

Info transfer necessary for trade:

$$(1-\pi) Z + \pi\delta < r. \tag{4}$$

$\iff$

$$\pi > \frac{Z-r}{Z-\delta}. \tag{5}$$

Collateral contract $= \omega, b$ is unacceptable to L

$$(1-\pi) b + \pi\omega < r. \tag{6}$$

$\iff$

$$\pi > \frac{b-r}{b-\omega}. \tag{7}$$

Commitment solution
Kamenica & Gentzkow 11 Goex and Wagenhofer 09
B's strategy:

1. If $z = Z$, reveal truthfully
2. If $z = 0$, lie with probability

$$\rho = \Pr(s = Z \mid z = 0).$$

and drive L to indifference so that

$$EU_{\mathrm{L}} = r. \tag{8}$$

Then

$$\rho_{\mathrm{KG}} = \frac{(1-\pi)(b-r)}{\pi(r-\omega)}. \tag{9}$$

Probability that the loan is made in KG $> 1 - \pi$.

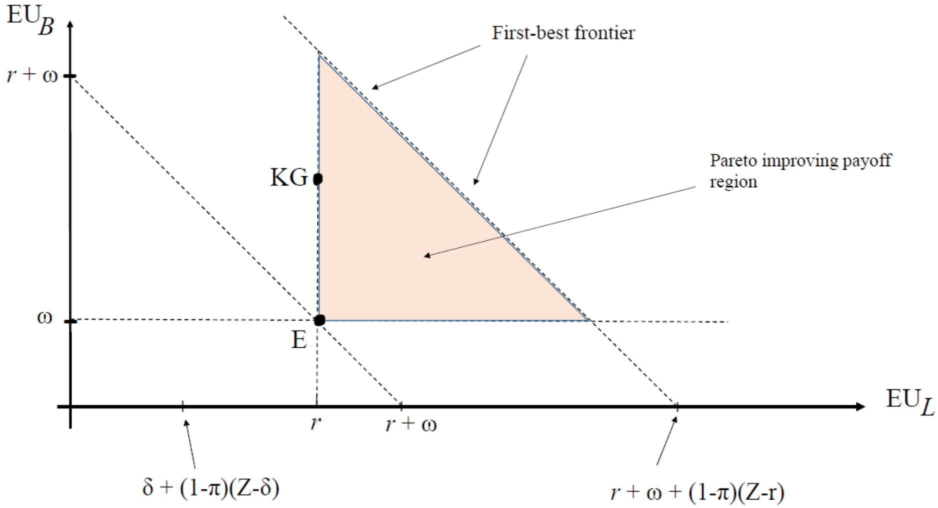

Figure: THE KG EQUILIBRIUM AND. THE PARETO IMPROVEMENT REGION, THE NO-TRADE OUTCOME E AND

Parameter	Z	b	r	δ	ω	π
Value	2	1.5	1.1	0.8	0.6	0.8

Table 1: PARAMETER VALUES USED IN FIG. 2

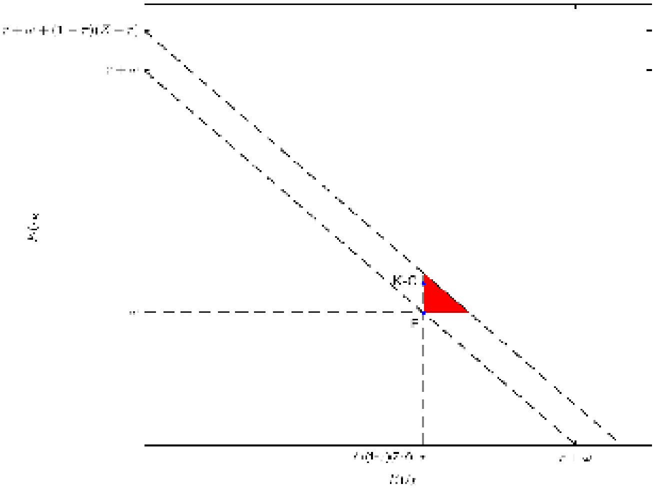

Figure: Reproduction of Fig. 1 for the parameter values in Table 1

Verification

b = repayment by B to L ($b \leq Z$).

P = penalty B pays if he is caught lying about z.

1. B sees z privately,
2. B sends message s to L, (this is B's only action),
3. L may verify at a the cost c. L's action is $v \in \{0, 1\}$, with $v = 1$ denoting "verify,"
4. B pays P to L if $s = Z$, but $z = 0$,
5. L chooses $k \in \{0, 1\}$ irreversibly,
6. z is observed publicly,
7. B pays b to L if $k = 1$ and $z = Z$. If $z = 0$ and $k = 1$, then B pays ω to L

Multiple equilibria

- Babbling equilibria in cheap talk games always exist
- Emir suggested refinement: Any $\varepsilon > 0$ chance of random verification eliminates the babbling equilibrium because babbling does nothing for B

Payoffs

$$U_{\mathrm{B}}(s = Z \mid z = 0) = -vP + (1 - v)(\delta - \omega) k$$

and

$$\begin{aligned} U_{\mathrm{L}}(v, k \quad \mid \quad s = Z) &= v[q(P + r) + (1 - q)b - c] \\ &+ (1 - v)[k((1 - q) b + q\omega) + (1 - k)r]. \end{aligned}$$

L's choices of (v, k) summarized as follows

1. *Take the outside option.*—Set $v = k = 0$ and get r.
2. *Invest without verifying.*—Set $v = 0$ and $k = 1$.
3. *Verify.*—Set $v = 1$ and then set $k = 1$ iff $z = Z$.

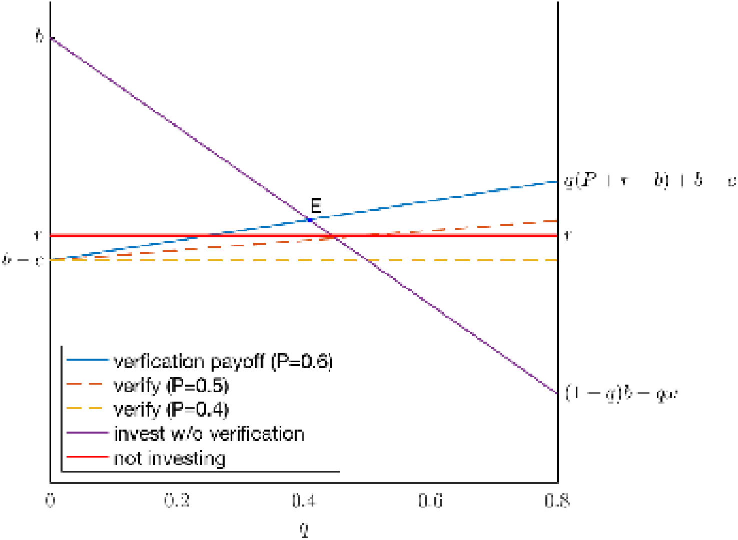

Figure: Lender's payoff conditional on $s = Z$ and q . c is fixed at 0.45

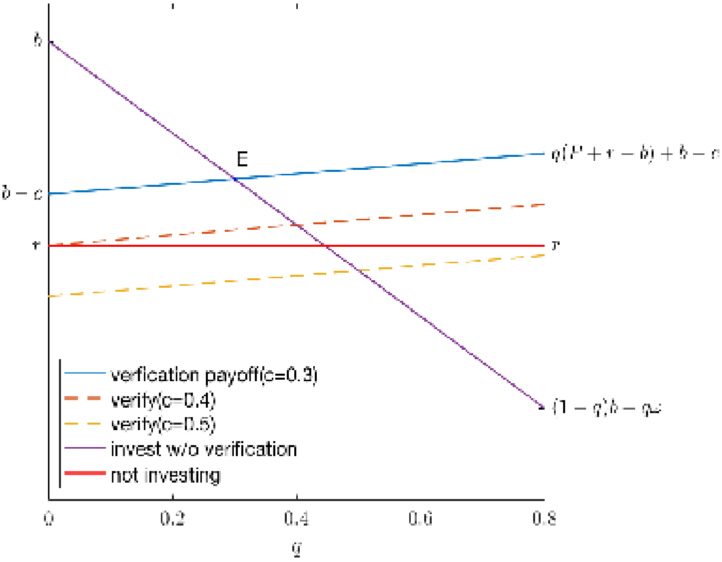

Figure: Lender's payoff conditional on $s = Z$ and q. P is fixed at 0.5.

Iso-welfare lines.—First-best payoff is 1.280 .....attained as $c \longrightarrow 0$, for any $P \Rightarrow$ iso-welfare lines move counterclockwise

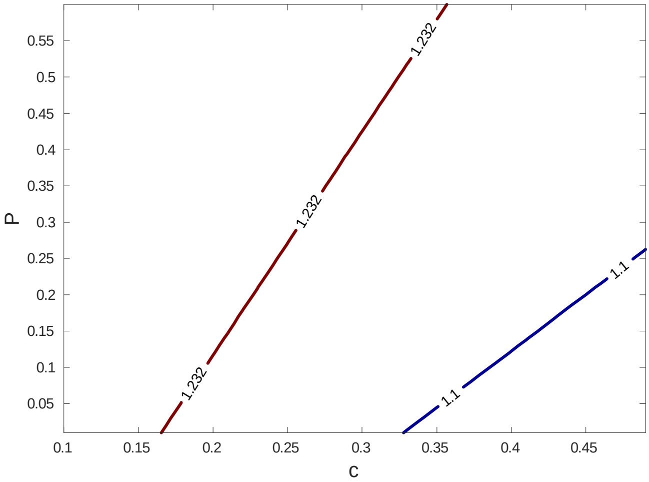

Figure: Iso-Welfare lines in (c, P) space

Comparative statics

1. As $c \longrightarrow 0$, $\bar{v}$ *is unchanged*.
2. The resources spent on verification, $c\bar{v} \longrightarrow 0$.
3. B's payoff stays unchanged but L's rises. This in contrast to KG

“Burned money” –when P is paid to a third party

to a regulator for example –
the slope of the “verify payoff” lines in Figs. 3 and 4 become flatter,
Equilibrium entails a larger value of q, lower welfare and more “excess lending

Model 2: Many states – Crawford Sobel 82

z is uniform on $[0,1]$, and the sender observes z. The action is k and the sender's bias is b. The payoffs of S and R are

$$\begin{aligned} U_S(z,k) &= -(k-z-b)^2, \text{ with } b < \frac{1}{4} \\ U_R(z,k) &= -(k-z)^2 \end{aligned}$$

CS equilibrium (no commitment and no verification), message is $z \in A_i := [a_i, a_{i+1}]$ where $A_0, A_1 ... A_N$ are a partition of $[0,1]$ such that $a_0 = 0, a_N = 1,\ a_i = \frac{i}{N} + 2bi(i-N)$ and

$$N \leq \frac{1}{2}\left[\left(1+\frac{2}{b}\right)^{1/2} - 1\right] \equiv N_{\max}$$

If R cannot verify,

$$k(A_i) = \frac{a_i + a_{i+1}}{2}$$

If R verifies,

$$k = z$$

and the payoffs are

$$u_{\mathrm{S}}(z, k, v, A) \equiv U_S(z, k) - I_{\{z \notin A\}} P v$$

$$u_{\mathrm{R}}(z, k, v, A) \equiv U_R(z, k) + \left(I_{\{z \notin A\}} P - c\right) v$$

The KG outcome = full revelation – 2009 wp

Verification eliminates the coarse CS equilibria.—We look for a function

$$N_{\min} = \phi(c).$$

The function ϕ will be decreasing

Q: As $c \to 0$ do we approach the KG outcome with full revelation?

For R to not want to verify, we have for $i = 1, 2, \ldots N-1$

$$\begin{aligned} E\left[u_{\mathrm{R}}(z, k, v\right. &= \left. 0)\right] \geqslant -c \quad \Rightarrow \\ a_{i+1} - a_i &\leqslant 2\sqrt{3c} \end{aligned} \tag{10}$$

To remove the least informative equilibrium, we need

$$c < \frac{1}{12} \tag{11}$$

we can remove the least informative eq.. More generally

$$N_{\min} = \frac{b + \sqrt{3c} - \sqrt{\left(b + \sqrt{3c}\right)^2 - 2b}}{2b}$$

If $N_{\min} > N_{\max}$, not yet solved for the equilib. – there has to be mixing by R.