Econ 4905: Lecture 11
Bank Runs: The Pre-Deposit Game

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Updates

- Mortgage update
- Reverse mortgages
- Tax updates
  - Recent change to itemized deduction
  - Recent change for SALT
  - Recent change to charitable giving for IRA
- My schedule on Wednesday afternoon
Introduction to Bank Runs

- Bryant (1980) and Diamond and Dybvig (1983): “bank runs” in the *post-deposit* game
  - multiple equilibria in the *post-deposit* game
- One cannot understand bank runs or the optimal contract without the full *pre-deposit* game
- Peck and Shell (2003): A *sunspot-driven* run can be an equilibrium in the *pre-deposit* game for sufficiently small run probability.
- We show how sunspot-driven run risk affects the optimal contract depending on the parameters.
The Model: Consumers

- 2 ex-ante identical vNM consumers and 3 periods: 0, 1 and 2.
- Endowments: \( y \)
- Preferences: \( u(c^1) \) and \( v(c^1 + c^2) \):  
  - impatient: \( u(x) = A \frac{(x)^{1-b}}{1-b} \), where \( A > 0 \) and \( b > 1 \).
  - patient: \( v(x) = \frac{(x)^{1-b}}{1-b} \).
- Types are uncorrelated (so we have aggregate uncertainty.): \( p \)
The Model: Technology

- **Storage:**

\[
\begin{array}{ccc}
  t = 0 & t = 1 & t = 2 \\
  -1 & 1 & \\
  -1 & 1 & 
\end{array}
\]

- **More Productive**

\[
\begin{array}{ccc}
  t = 0 & t = 1 & t = 2 \\
  -1 & 0 & R \\
\end{array}
\]
The Model

- Sequential service constraint (Wallace (1988))
- Suspension of convertibility.
- A depositor visits the bank only when he makes withdrawals.
- When a depositor makes his withdrawal decision, he does not know his position in the bank queue.
- If more than one depositor chooses to withdraw, a depositor’s position in the queue is random. Positions in the queue are equally probable.
- Aggregate uncertainty
Post-Deposit Game: Notation

- $c \in [0, 2y]$ is any feasible banking contract
- $\hat{c} \in [0, 2y]$ is the unconstrained optimal banking contract
- $c^* \in [0, 2y]$ is the constrained optimal banking contract
- Smaller $c$ is conservative; larger $c$ is fragile
Post-Deposit Game: $c^{early}$

- A patient depositor chooses early withdrawal when he expects the other depositor to also choose early withdrawal.

\[
\frac{v(c) + v(2y - c)}{2} > v[(2y - c)R]
\]

- Let $c^{early}$ be the value of $c$ such that the above inequality holds as an equality.
A patient depositor chooses late withdrawal when he expects the other depositor, if patient, to also choose late withdrawal. (ICC)

\[ pv[(2y - c)R] + (1 - p)v(yR) \geq p[v(c) + v(2y - c)]/2 + (1 - p)v(c). \]

Let \( c^{\text{wait}} \) be the value of \( c \) such that the above inequality holds as an equality.
Post-Deposit Game: “usual” values of the parameters

- $c^{early} < c^{wait}$ if and only if

\[
b < \min\{2, 1 + \ln 2 / \ln R\}
\]

The post-deposit game has two equilibria: one run and one non-run.

- Only the non-run equilibrium exists.
- Only the run equilibrium exists.
Pre-Deposit Game

- For the rest of the presentation, we focus on the "usual" values of $b$ and $R$.
- Whether bank runs occur in the pre-deposit game depends on whether the optimal contract $c^*$ belongs to the region of strategic complementarity (i.e., $c \in (c^{\text{early}}, c^{\text{wait}}]$).
- To characterize the optimal contract, we divide the problem into three cases depending on $\hat{c}$, the contract supporting the unconstrained efficient allocation.
  - $\hat{c} \leq c^{\text{early}}$ (Case 1)
  - $\hat{c} \in (c^{\text{early}}, c^{\text{wait}}]$ (Case 2)
  - $\hat{c} > c^{\text{wait}}$ (Case 3)
Impulse parameter $A$ and the 3 cases

- $\hat{c}$ is the $c$ in $[0, 2y]$ that maximizes

\[
\hat{W}(c) = \{ p^2 [u(c) + u(2y - c)] + 2p(1 - p)[u(c) + v((2y - c)R)] \\
+ 2(1 - p)^2 v(yR) \}.
\]

- $\hat{c} = \frac{2y}{\{ p/(2 - p) + 2(1 - p)/[(2 - p)AR^{b-1}]\}^{1/b} + 1}$.

- $\hat{c}(A)$ is an increasing function of $A$. 
Parameter A and the 3 Cases

- Neither $c^{\text{early}}$ nor $c^{\text{wait}}$ depends on $A$
Example

- The parameters are

\[ b = 1.01; \quad p = 0.5; \quad y = 3; \quad R = 1.5 \]

- We see that \( b \) and \( R \) satisfy the condition which makes the set of contracts permitting strategic complementarity non-empty. We have that \( c^{early} = 4.155955 \) and \( c^{wait} = 4.280878 \).

- \( A^{early} = 6.217686 \) and \( A^{wait} = 10.27799 \).

- If \( A \leq A^{early} \), we are in Case 1; If \( A^{early} < A \leq A^{wait} \), we are in Case 2; If \( A > A^{wait} \), we are in Case 3.
The Optimal Contract: Case 1

- Case 1: The *unconstrained efficient allocation* is DSIC, i.e.,
  \[ \hat{c} \leq c^{early}. \]

- It is straightforward to see that the optimal contract for the *pre-deposit* game supports the *unconstrained efficient allocation*
  \[ c^*(s) = \hat{c}. \]

  and that the optimal contract doesn’t tolerate runs.
The Optimal Contract: Case 2

- Case 2: The *unconstrained efficient allocation* is BIC but not DSIC, i.e., \( c^{\text{early}} < \widehat{c} \leq c^{\text{wait}} \).

- The optimal contract \( c^*(s) \) satisfies: (1) if \( s \) is larger than the threshold probability \( s_0 \), the optimal contract is run-proof and \( c^*(s) = c^{\text{early}} \). (2) if \( s \) is smaller than \( s_0 \), the optimal contract \( c^*(s) \) tolerates runs and it is a strictly decreasing function of \( s \).
The Optimal Contract: Case 2

- Using the same parameters as the previous example. Let $A = 8$. (We have seen that we are in Case 2 if $6.217686 < A \leq 10.27799$.)
- $c^*$ switches to the best run-proof contract (i.e. $c^{early}$) when $s > s_0 = 1.382358 \times 10^{-3}$.
Case 3: The *unconstrained efficient allocation* is not BIC, i.e.,
$c^{\text{wait}} < \hat{c}$. 

The optimal contract $c^*(s)$ satisfies: (1) If $s$ is larger than the
threshold probability $s_1$, we have $c^*(s) = c^{\text{early}}$ and the
optimal contract is run-proof. (2) If $s$ is smaller than $s_1$, the
optimal contract $c^*(s)$ tolerates runs and it is a weakly
decreasing function of $s$. Furthermore, we have $c^*(s) = c^{\text{wait}}$
for at least part of the run tolerating range of $s$. 
The Optimal Contract: Case 3

- Using the same parameters as in the previous example. Let \( A = 10.4 \). (We have seen that we are in Case 2 if \( A > 10.27799 \).)
- \( c^* \) switches to the best run-proof (i.e. \( c^{early} \)) when \( s > 4.524181 \times 10^{-3} \).
- ICC becomes non-binding when \( s \geq s_2 = 1.719643 \times 10^{-3} \).
The Optimal Contract: Case 3

- Let $A = 11$. (PS case)
- $c^*$ switches to the best run-proof (i.e. $c^{early}$) when $s > s_1 = 5.281242 \times 10^{-3}$. 

Figure 5. $c^*$ and $c$-No-ICC for $A=11$
The Optimal Contract

- $c^*$ versus $s$ and $A$
Summary and Concluding Remark

- In Cases 2 and 3, the optimal contract tolerates runs when the run probability is sufficiently small:

- In Case 2, the optimal contract adjusts continuously and becomes strictly more conservative as the run probabilities increases.

- The optimal allocation is never a mere randomization over the unconstrained efficient allocation and the corresponding run allocation from the post-deposit game. Hence this is also a contribution to the sunspots literature: another case in which SSE allocations are not mere randomizations over certainty allocations.
In Case 3, the ICC binds for small run-probabilities, which forces the contract to be more conservative than it would have been without the ICC. Hence, for Case 3, the optimal contract does not change with $s$ until the ICC no longer binds.
In Case 3, the ICC binds for small run-probabilities, which forces the contract to be more conservative than it would have been without the ICC. Hence, for Case 3, the optimal contract does not change with $s$ until the ICC no longer binds.

For small $s$, the optimal allocation is a randomization over the constrained efficient allocation and the corresponding run allocation from the post-deposit game.