three periods: $T = 0, 1, 2$

a single good

a continuum of agents with measure 1

Each agent is endowed with 1 unit of the good in period 0.
The Model: Asset Return

\[ T = 0 \quad T = 1 \quad T = 2 \]

\[
\begin{array}{c}
-1 \\
R
\end{array}
\begin{array}{c}
\{ 0 \\
1
\}
\begin{array}{c}
0 \\
0
\end{array}
\]
The Model: Preferences

- In period 0, all agents are identical.
- In period 1, some agents become “patient” and others become “impatient”. (private information)

\[
\begin{align*}
    & \quad \text{if impatient} \\
    \begin{cases}
        u(c_1) & \text{if impatient} \\
        u(c_1 + c_2) & \text{if patient}
    \end{cases}
\end{align*}
\]

- The probability of being impatient is $\lambda$ for each agent in period 0.
- $\lambda$ is also the measure (“fraction”) of impatient agents.
Autarky

- autarky:
  - utility of the impatient in period 1: $u(1)$
  - utility of the patient in period 2: $u(R)$
  - expected utility in period 0: $\lambda u(1) + (1 - \lambda) u(R)$

- $1 < R$
  - “insurance” against the liquidity shock is desirable.
Banks offers demand deposit contract \((d_1, d_2)\).

Agents

- make deposits in period 0.
- withdraw \(d_1\) in period 1.
- or withdraw \(d_2\) in period 2.

Free-entry banking sector: \((d_1, d_2)\) maximizes the depositor’s expected utility.
Optimal Deposit Contract

\[
\max_{d_1,d_2} \lambda u(d_1) + (1 - \lambda) u(d_2)
\]

s.t. \[\begin{array}{c}
(1 - \lambda) d_2 \\ \text{withdrawals in period 2}
\end{array}\] \leq \[\begin{array}{c}
(1 - \lambda d_1) R \\ \text{resources in period 2}
\end{array}\]  \quad (RC)

\[d_1 \leq d_2\]  \quad (IC)
Optimal Deposit Contract:

\[(1 - \lambda) d_2 = (1 - \lambda d_1) R\]

\[\text{slope} = -\frac{\lambda}{1 - \lambda} R\]
Optimal Deposit Contract:

\[ \lambda u(d_1) + (1 - \lambda) u(d_2) = \text{const} \]

slope = \(-\frac{\lambda}{1 - \lambda} \frac{u'(d_1^*)}{u'(d_2^*)} \)

\( (1 - \lambda)d_2 = (1 - \lambda d_1)R \)

\[ \frac{\lambda}{1 - \lambda} \frac{u'(d_1^*)}{u'(d_2^*)} = \frac{\lambda}{1 - \lambda} R \]

\( MRS \) \( MRT \)
What do banks do?

- \( u'(d_1^*) / u'(d_2^*) = R \)
- \( u'' < 0 \Rightarrow d_1^* < d_2^* \)
- CRRA: \( u(c) = \frac{c^{1-\gamma}}{1-\gamma} \)
  
  - \( u'(c) = c^{-\gamma} \Rightarrow u'(d_1) / u'(d_2) = (d_2 / d_1)^\gamma \)
  - if \( \gamma = 1 \Rightarrow d_1^* = 1, d_2^* = R \)
  - if \( \gamma > 1 \Rightarrow 1 < d_1^* < d_2^* < R \)
Why do bank runs occur?

- \( \gamma > 1 \implies 1 < d_1^* < d_2^* < R \)
- IC: \( d_1 \leq d_2 \)
- Expectation: Only the impatient depositors withdraw in period 1.
  - A patient depositor can \( \begin{cases} 
    \text{get } d_2^* & \text{if he withdraws in period 2} \\
    \text{get } d_1^* & \text{if he withdraws in period 1}
  \end{cases} \)
Why do bank runs occur?

- $\gamma > 1 \implies 1 < d_1^* < d_2^* < R$

- Expectation: All other depositors demand withdraw in period 1.

- A patient depositor can
  - get *nothing* if he withdraws in period 2
  - get $d_1^*$ w.p. $1/d_1^*$ if he withdraws in period 1

Yu Zhang (Cornell University)