A Diamond-Dybvig Model in which the Level of Deposits is Endogenous

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Introduction

- We consider the original Diamond-Dybvig (1983) model with a single technology. Withdrawals must satisfy a sequential service constraint, and the banking contract allows for partial suspension of convertibility. The new feature is that the level of deposits is endogenous.
- Now a contract specifies the deposit level and the sequence of withdrawals in period 1.

Introduction

- A few previous papers assume that consumers deposit their entire endowment, but are careful about modeling the pre-deposit game, and the fact that a non-zero propensity to run could affect the optimal contract. See Peck and Shell (2003) and Shell and Zhang (MD forthcoming).
- Our results are for the best no-run contract. The next revision will consider equilibria with a non-zero propensity to run.

Introduction

- A few previous papers model the level of deposits as endogenous, but depart somewhat from the original DD model.
  - In Ennis and Keister (2003), banks can invest in storage and capital, and consumers can store income that is not deposited. Partial suspension is not allowed. Multiple generations and economic growth is studied.
  - In Peck and Shell (2010) there are two technologies, liquid and illiquid. Utility includes an indivisible consumption opportunity (early or late), and all consumers care about left-over consumption. In the separated system, the bank is restricted from investing in the illiquid technology.
  - Shell and Zhang (IJET forthcoming) consider the environment of Peck and Shell (2010), but are careful to model the predeposit game.

Introduction: Preview of Results

- Let x^* denote the allocation that solves the bank's problem when consumers deposit their entire endowment, $d = 1$, and the propensity to run is zero. Proposition 1 shows that, when IC does not bind at the optimal no-run contract, the solution is indeterminate. There exists $d^* < 1$ such that for all $d > d^*$, there is an equilibrium with deposit d yielding the allocation x^*.
- Intuitively, consumers could be investing outside the bank, but the bank's withdrawal amounts augment the liquidated outside investment of impatient consumers by exactly the amount needed to achieve x^*.
- The smaller the deposit, the "more tempted" a consumer is to withdraw early.

Introduction: Results Continued

- We prove (by example) that there are economies in which (i) allocation x^* can be uniquely implemented, with no run equilibrium in the post-deposit subgame, when consumers deposit their entire endowment, and (ii) the post-deposit subgame has a run equilibrium for d sufficiently close to d^*.
- The example shows that run equilibria are easy to construct, even with two consumers and patient and impatient utility functions having the same functional form.

The Model

- There are three time periods, N consumers, and a single investment technology. Each unit of consumption invested in period 0, by either a consumer or the bank, yields 1 unit if harvested in period 1, and $R > 1$ units if harvested in period 2.
- Endowment is normalized to 1. Denote consumption received in period 1 by x_1 and the consumption received in period 2 by x_2.
- An impatient consumer receives utility $u(x_1)$ and a patient consumer receives utility $u(x_1 + x_2)$. We assume $u'(x) > 0$ and $u''(x) < 0$. Most interesting when $-\frac{xu''(x)}{u'(x)} > 1$ holds.

Distribuition of Types

- The number of impatient consumers, denoted by α, is a random variable with probability distribution $f(\alpha)$.
- Impatience can be i.i.d. or correlated across consumers.
- It is assumed that the distribution of α, *conditional* on being patient, denoted by $f_p(\alpha)$, is the same for all consumers. From Bayes' rule, we have

$$f_p(\alpha) = \frac{(1 - \frac{\alpha}{N})f(\alpha)}{\sum_{a=0}^{N-1}(1 - \frac{a}{N})f(a)}.$$

Timing of the Game

- Bank chooses a contract, $\{d, c_1(z, d), c_2(\alpha, d)\}$, specifying deposit level, period 1 withdrawals as a function of position, and period 2 withdrawal as a function of number of period 1 withdrawals.
- Consumers either deposit d and invest $(1 - d)$ outside the bank, or refuse to deposit.
- Consumers observe their type, observe a sunspot, and simultaneously decide whether or not to withdraw in period 1.
- Those who do not withdraw in period 1 arrive in period 2 (and do not contact the bank before that).

Resource Constraints

$$c_1(N,d) = dN - \sum_{z=1}^{N-1} c_1(z,d), \tag{1}$$

$$c_2(\alpha,d) = \frac{[dN - \sum_{z=1}^{\alpha} c_1(\alpha,d)]R}{N-\alpha}. \tag{2}$$

Overall Consumption

- An impatient consumer withdrawing c_1 from the bank in period 1 receives utility, $u(1-d+c_1)$.
- A patient consumer withdrawing c_1 from the bank in period 1 will store this consumption until period 2 and harvest her outside investment in period 2, thereby receiving utility, $u((1-d)R+c_1)$.
- A patient consumer withdrawing c_2 from the bank in period 2 will receive utility, $u((1-d)R+c_2)$.
- It will be convenient to use the notation, $x_1(z,d) \equiv 1-d+c_1(z,d)$ to denote the overall consumption of an impatient consumer who withdraws in position z in period 1 and $x_2(\alpha,d) \equiv (1-d)R+c_2(\alpha,d)$ to denote the overall consumption of a patient consumer who withdraws in period 2 when the number of consumers withdrawing in period 1 is α.

Welfare

The welfare (bank's objective) associated with a contract, C, is

$$\begin{aligned}\widehat{W}(C) &= \sum_{\alpha=0}^{N-1} f(\alpha)\left[\sum_{z=1}^{\alpha} u(x_1(z,d)) + (N-\alpha)u(x_2(\alpha,d))\right] \\ &+ f(N)\left[\sum_{z=1}^{N} u(x_1(z,d))\right].\end{aligned}$$

Incentive Compatibility

The incentive compatibility constraint for a patient consumer is given by

$$\sum_{\alpha=0}^{N-1} f_p(\alpha)\left[\frac{1}{1+\alpha}\sum_{z=1}^{\alpha+1} u((1-d)(R-1)+x_1(z,d))\right] \tag{3}$$
$$\leq \sum_{\alpha=0}^{N-1} f_p(\alpha)u(x_2(\alpha,d)).$$

The reason for the term, $(1-d)(R-1)$, in (3) is that a patient consumer who withdraws in period 1 receives this additional consumption because her outside investment is harvested in period 2 rather than period 1.

Optimal No-Run Contract

The bank's optimal contract, assuming the propensity to run is zero, solves

$$\begin{aligned} & \max \widehat{W}(C) \\ & \text{subject to} \\ & \text{resource, IC constraints} \\ c_1(z,d) \geq & \ 0 \quad \text{for all } z \end{aligned} \tag{4}$$

Proposition 1

- If we fix $d = 1$ and solve (4) for the optimal withdrawal schedule, we have the solution to the planner's problem studied in the previous literature in which consumers deposit their entire endowment. See, for example, Peck and Shell (2003) and Shell and Zhang (MD, forthcoming). Denote the correponding constrained efficient allocation by $x^* = \{x_1^*(z)|_{z=1}^{N}, x_2^*(\alpha)|_{\alpha=0}^{N-1}\}$.
- Proposition 1 shows that any allocation achievable with a given deposit level is achievable with a higher deposit level. Also, if IC is not binding with $d = 1$, there is an interval of deposit levels achieving x^*.

Proposition 1 continued

- **Proposition 1:** *Suppose that, for deposit level d', there is a contract satisfying the constraints in (4) yielding the allocation $x = \{x_1(z)|_{z=1}^{N}, x_2(\alpha)|_{\alpha=0}^{N-1}\}$. Then, for all deposit levels $d'' > d'$, there is a contract satisfying the constraints in (4) yielding the same allocation x. Also, if*

$$\sum_{\alpha=0}^{N-1} f_p(\alpha)\left[\frac{1}{1+\alpha}\sum_{z=1}^{\alpha+1} u(x_1^*(z))\right] < \sum_{\alpha=0}^{N-1} f_p(\alpha)u(x_2^*(\alpha)) \quad (5)$$

holds (i.e., IC is not binding when consumers deposit their entire endowment), then there exists $d^* < 1$ such that, for all $d > d^*$, there is a contract satisfying the constraints in (4) yielding the allocation x^*.

- A patient consumer is defined to be “more tempted” to join a run with deposit level d' than with deposit level d'' if the utility difference, between withdrawing in period 1 and withdrawing in period 2 (when all other consumers withdraw in period 1), is greater with deposit level d' than with deposit level d''.
- **Proposition 2:** *Consider an economy in which incentive compatibility is not binding at x^* when consumers deposit their entire endowment, so (5) holds. Then for all d', d'' such that $d^* < d' < d''$ holds, a patient consumer is “more tempted” to join a run with deposit level d' than with deposit level d''. Whenever the post-deposit subgame has a run equilibrium with deposit level d'', then the post-deposit subgame has a run equilibrium with deposit level d'.*

The utility advantage of joining a run is

$$\frac{1}{N}\sum_{z=1}^{N} u((1-d)(R-1)+x_1^*(z)) \\ -u(x_2^*(N-1)). \tag{6}$$

An Example

- Consider the CRRA utility function with risk aversion parameter, 2, given by

$$u(x) = -\frac{1}{x}.$$

- $N = 2$, and the probability of being impatient is i.i.d. with probability π.
- The allocation, x^*, as characterized by $x_1^*(1)$, is the solution to

$$\max_x \pi^2[u(x) + u(2-x)] + 2\pi(1-\pi)[u(x) + u((2-x)R)]$$
$$+2(1-\pi)^2 u(R)$$

subject to

$$\pi[\frac{u(x)}{2} + \frac{u(2-x)}{2}] + (1-\pi)u(x) \leq \pi u((2-x)R + (1-\pi)u(R). \tag{7}$$

Example Continued

- Once we have x^*, we can compute the values of d above which (i) IC is satisfied (d^*_{IC}), (ii) non-negativity of withdrawals is satisfied (d^*_{NN}), and (iii) the subgame does not have a run equilibrium (d^*_{NR}).
- For $\pi = \frac{1}{2}$ and $R = 3$, we have

$$
\begin{aligned}
x_1^*(1) &= 1.145898 \\
x_1^*(2) &= 0.854102 \\
x_2^*(0) &= 3 \\
x_2^*(1) &= 2.562306.
\end{aligned}
$$

Example Continued

- The value of d above which incentive compatibility does not bind is $d^*_{IC} = 0.1514666$, and the value d above which non-negativity does not bind is $d^*_{NN} = 0.145898$, so we have $d^* = 0.1514666$. Given $d > d^*$, the contract achieving the allocation x^* is given by

$$\begin{aligned} c_1^*(1,d) &= 0.145898 + d \\ c_1^*(2,d) &= -0.145898 + d \\ c_2^*(0,d) &= 3d \\ c_2^*(1,d) &= -0.437694 + 3d. \end{aligned}$$

  We can compute $d^*_{NR} = 0.2147067$, so the contract achieving the allocation x^* in the no-run equilibrium also has a run equilibrium, for deposit levels $d \in (0.1514666, 0.2147067)$, and does not have a run equilibrium for $d \in (0.2147067, 1)$.

Concluding Remarks

- Our results indicate that the banking system is more "fragile" when optimal no-run contracts with lower deposits are selected, but can bank runs occur on the equilibrium path?
- We are revising the paper to include a small return advantage for investments outside the banking system $(R + \varepsilon)$. We will show that, when the propensity to run, s, is small, ε is small, and $\frac{s}{\varepsilon}$ is small, then we have the following: Consider x^*, based on $s = 0$ and $\varepsilon = 0$, and suppose that we have $d^* < d^*_{NR} < 1$ (as in our example). Then the optimal contract, taking into account s and ε, must involve a positive probability of bank runs on the equilibrium path.