Economics 4905
Financial Fragility and the Macroeconomy
Cornell University
Fall 2018
Practice Questions for Prelim 2

1. The Overlapping Generations Model

- 2 period lives
- 1 commodity per period, $\ell = 1$
- Stationary environment
- 1 person per generation

The utility functions are given as:

$$u_0(x_0^1) = x_0^1$$
$$u_t(x_t^t, x_t^{t+1}) = (x_t^t)^\alpha (x_t^{t+1})^\beta \text{ for } t = 1, 2, \ldots$$

The endowments are 1 unit for each person during each period she is alive:

$$\omega_0^1 = \omega_t^t = \omega_t^{t+1} = 1 \text{ for } t = 1, 2, \ldots$$

Define:

$$z^t = \omega_t^t - x_t^t$$
$$z^{t+1} = x_t^{t+1} - \omega_t^{t+1}$$

Case 1: $\alpha = 1, \beta = 10, m_0^1 = 1, m_s^t = 0$ otherwise
Case 2: $\alpha = 1, \beta = 0.2, m_0^1 = 1, m_s^t = 0$ otherwise

For each of the above cases, solve for the following:

a) The demand (x_t^t, x_t^{t+1})

b) The offer curve (OC)

c) The steady states

d) The set of equilibrium money prices, $\mathcal{P}^m$

e) The full dynamic analysis, including the stability of steady states

2. Diamond-Dybvig Bank

The probability of being impatient is $\lambda = 0.4$. The type (patient or impatient) is realized in period 1 and it is private information. The utility function is:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

where $\gamma = \frac{3}{2} > 0$. Each individual has one unit of endowment in period 0. There is costless storage. If the endowment $\omega = 1$ is invested in period 0 and is harvested in period 1, the return is 0. If harvested late, the return is 4. Assume that the banking industry is free-entry.

(a) What is the depositor's *ex-ante* expected utility W as a function of c_1 consumption in period 1, and c_2, consumption in period 2?

(b) Show that the depositor prefers consumption smoothing.

(c) What is the bank's resource constraint RC? Write this down precisely and explain in words.

(d) What is the incentive problem? Write down the incentive constraint IC precisely, and explain it in words.

(e) Solve for the optimal deposit contract for the post-deposit bank assuming that there is no run. (That is: Write down the optimal first-period payment d_1^* as a function of λ, R and γ)

(f) Is there a run equilibrium to this "optimal contract"?

3. The 2-Consumer, Pre-Deposit Bank

The utility function of the impatient agent is

$$u(x) = \frac{Ax^{1-\gamma}}{1-\gamma}$$

and the utility function of the patient agent is

$$v(x) = \frac{x^{1-\gamma}}{1-\gamma},$$

where $\gamma > 1$. The probability λ of being impatient is 50%. The parameter γ is 1.01. The endowment is y = 3. The rate of return on the asset if harvested late is 50%, i.e., R = 1.5. The probability of being first in line if 2 agents withdraw early is 50%.

(a) Solve for the numerical values of c^{early} and c^{wait}. Show that they are independent of the impulse parameter A.

(b) Write down the expression for the *ex-ante* expected utility of the depositor, W.

(c) Solve for $\hat{c}$, the value of c that maximizes W in the post-deposit game, as a function of A.

(d) Calculate the critical values A^{early} and A^{wait}.

(e) Let A = 7. Describe the optimal contract for the pre-deposit game, $c^*(s)$, as a function of s, the exogenous run probability.

3. Bank Runs

Consider the Diamond-Dybvig bank run model. The probability λ of being impatient is 50%. The utility function is:

$$u(c) = 10 - \frac{1}{(0.5)\sqrt{c}}.$$

The rate of return to the asset harvested late is 400%, i.e.,

$$R = 5.$$

(a) What is the depositor's *ex-ante* expected utility W as a function of c_1, consumption in period 1, and c_2, consumption in period 2?

(b) Show that the depositor prefers consumption smoothing.

(c) Why can't she insure on the market against liquidity shocks?

Assume that her endowment is 100 and that she deposits her entire endowment in the bank.

(d) What is her utility W in autarky?

(e) What is her utility W under perfect smoothing, i.e. when $c_1 = c_2$?

(f) What is the bank's resource constraint RC? Write this down precisely. Explain this in words.

(g) What is the incentive problem? Write this down precisely and explain in words the incentive constraint IC.

(h) Find the optimal deposit contract for this bank. What is W if there is no run?

(i) Why is there a run equilibrium for this bank?

(j) Calculate the following numerical values of *ex-ante* utility W and and rank them in numerical ascending order: $W_{autarky}$, $W_{perfect\ smoothing}$, $W_{no\ run}$, W_{run}.

(k) Assume that the run probability s is 2%. Will individuals deposit in this bank? That is, will they accept this banking contract? Explain.