Economics 4905  
Financial Fragility and the Macroeconomy  
Cornell University, Fall 2018  
Prelim 1 Solutions  
Monday, October 1, 2017, 2:55PM to 4:10PM  
G26 Uris Hall

Instructions: This prelim is designed to take 60 minutes, but you have 75 minutes to write your answers. Answer each of the 4 questions. Do not seek, take, nor give advice from any source, animate or inanimate. Do not use calculators. There is no need to simplify numerical answers. Place all personal items - including books, paper, and computers - in a place determined by the proctors.

1 Question 1 (15 minutes)

Let $\omega = (\omega_1, \ldots, \omega_h, \ldots, \omega_n)$ be the vector of chocolate endowments and $\tau = (\tau_1, \ldots, \tau_h, \ldots, \tau_n)$ be the vector of dollar taxes. $P_m$ is the chocolate price of dollars. $P_m$ is the set of equilibrium money prices. The equilibrium allocation is denoted by $x = (x_1, \ldots, x_h, \ldots, x_n)$.

(a) Define balanced tax policy.

(b) Define bonafide tax policy.

(c) What is the relationship between balanced tax policies and bonafide tax policies.

Solutions:

(a) A tax policy is balanced if taxes exactly offset subsidies, or $\sum_{h=1}^{n} \tau_h = 0$.

(b) A tax policy is bonafide if there is at least one competitive equilibrium in which money is not worthless, or $P_m > 0$.

(c) They are equivalent.

2 Question 2 (15 minutes)

Using the symbols from Question 1, let $n = 4$ and $\omega = (900, 800, 700, 600)$. For each of the following calculate $P_m$ and the set of equilibrium allocations.

(a) $\tau = (5, 4, -1, -3)$

(b) $\tau = (3, 2, -1, -4)$

(c) $\tau = (0, 0, 0, 0)$
Solutions:

(a) First check if the tax policy is balanced:

$$\sum_{h=1}^{4} \tau_h = 5 + 4 + (-1) + (-3) = 5 \neq 0$$

The tax policy is not balanced and hence not bonafide. The set of equilibrium price of money is

$$\mathcal{P}^m = \{0\}$$

The set of equilibrium allocations is

$$x = \{(900, 800, 700, 600)\}$$

(b) First check if the tax policy is balanced:

$$\sum_{h=1}^{4} \tau_h = 3 + 2 + (-1) + (-4) = 0$$

The tax policy is balanced, hence there exists some equilibrium price of money $P^m > 0$. The equilibrium price of money $P^m$ has to satisfy the following conditions:

$$900 - 3P^m > 0$$
$$800 - 2P^m > 0$$

which are equivalent to

$$P^m < 300$$
$$P^m < 400$$

The set of equilibrium price of money is

$$\mathcal{P}^m = [0, 300)$$

The set of equilibrium allocations is

$$x = \{(900 - 3P^m, 800 - 2P^m, 700 + P^m, 600 + 4P^m)|P^m \in [0, 300)\}$$

(c) First check if the tax policy is balanced:

$$\sum_{h=1}^{4} \tau_h = 0 + 0 + 0 + 0 = 0$$

The tax policy is balanced. Since no consumer pays taxes, the equilibrium price of money $P^m$ can be any non-negative real number. The set of equilibrium price of money is

$$\mathcal{P}^m = [0, \infty)$$

Since there is no taxes, the set of equilibrium allocations is

$$x = \{(900, 800, 700, 600)\}$$
3 Question 3 (15 minutes)

As before, \( \omega = (900, 800, 700, 600) \). There are 2 monies, Red (\( R \)) and Blue (\( B \)). The units are \$R \) and \$B \). In each of the following cases, calculate the exchange rate between the 2 monies. Also, for each case calculate the set of equilibrium allocations, \( x \).

(a) \( \tau^R = (2, 1, 0, -1) \), \( \tau^B = (5, 0, -4, -3) \).

(b) \( \tau^R = (5, 4, -4, -5) \), \( \tau^B = (1, 1, -1, -1) \).

Solutions:

(a) First check if the exchange rate is determinate

\[
\sum_{h=1}^{4} \tau^R_h = 2 + 1 + 0 + (-1) = 2 \neq 0
\]

\[
\sum_{h=1}^{4} \tau^B_h = 5 + 0 + (-4) + (-3) = -2 \neq 0
\]

The exchange rate is

\[
\frac{P^R}{P^B} = -\frac{\sum_{h=1}^{4} \tau^B_h}{\sum_{h=1}^{4} \tau^R_h} = -\frac{-2}{2} = 1
\]

which is \$R1 to \$B1. The equilibrium allocation is

\[
x = (900 - 2P^R - 5P^B, 800 - P^R, 700 + 4P^B, 600 + P^R + 3P^B)
\]

\[
= (900 - 7P^R, 800 - P^R, 700 + 4P^R, 600 + 4P^R)
\]

The set of equilibrium price of Red money \( P^R \) has to satisfy the following conditions

\[
900 - 7P^R > 0
\]

\[
800 - P^R > 0
\]

which are equivalent to

\[
P^R < \frac{900}{7}
\]

\[
P^R < 800
\]

The set of equilibrium allocations is

\[
x = \left\{(900 - 7P^R, 800 - P^R, 700 + 4P^R, 600 + 4P^R) | P^R \in \left[0, \frac{900}{7}\right]\right\}
\]

(b) First check if the exchange rate is determinate

\[
\sum_{h=1}^{4} \tau^R_h = 5 + 4 + (-4) + (-5) = 0
\]

\[
\sum_{h=1}^{4} \tau^B_h = 1 + 1 + (-) + (-1) = 0
\]
The exchange rate is indeterminate. The exchange rate can be any non-negative real number.

\[ \frac{P_R}{P_B} \in [0, \infty) \]

The equilibrium allocation is

\[ x = (900 - 5P_R - P_B, 800 - 4P_R - P_B, 700 + 4P_R + P_B, 600 + 5P_R + P_B) \]

The set of equilibrium prices \((P_R, P_B)\) has to satisfy the following conditions:

\[ 900 - 5P_R - P_B > 0 \]
\[ 800 - 4P_R - P_B > 0 \]

The set of equilibrium allocations is then

\[ x = \{(900 - 5P_R - P_B, 800 - 4P_R - P_B, 700 + 4P_R + P_B, 600 + 5P_R + P_B) | P_R \geq 0, P_B \geq 0, 900 - 5P_R - P_B > 0, 800 - 4P_R - P_B > 0\} \]

### 4 Question 4 (15 minutes)

Mary’s utility function is

\[ \pi(A)u(x(A)) + \pi(B)u(x(B)) \]

where \(u(x) = \log(x)\), \(\pi(B) = 1 - \pi(A)\), \(\pi(A) \geq 0\), \(\pi(B) \geq 0\).

(a) Is Mary risk averse, risk neutral, or risk loving?

(b) Let \(x(A) = 900\), \(x(B) = 100\), \(\pi(A) = 0.90\), \(\pi(B) = 0.10\).

(i) Calculate Mary’s expected payoff \(E = \pi(A)x(A) + \pi(B)x(B)\).

(ii) Calculate \(u(E)\). How does \(u(E)\) compare to \(\pi(A)u(x(A)) + \pi(B)u(x(B))\)?

(iii) Should Mary buy fair-odds insurance or should she decline the insurance?

#### Solutions:

(a) Take the second derivative of \(u(x)\).

\[ u'(x) = \frac{1}{x} \]
\[ u''(x) = -\frac{1}{x^2} < 0 \]

Since the second derivative is negative for any \(x\), Mary is risk averse.

(b) (i)

\[ E = \pi(A)x(A) + \pi(B)x(B) \]
\[ = 0.90(900) + 0.10(100) \]
\[ = 820 \]
(ii) 

\[ u(E) = \log(820) \approx 6.7093 \]

\[ \pi(A)u(x(A)) + \pi(B)u(x(B)) = 0.9 \log(900) + 0.1 \log(100) \]
\[ \approx 6.5827 \]

From the numbers, it is clear that \( u(E) > \pi(A)u(x(A)) + \pi(B)u(x(B)) \). Even without the numbers, it can be inferred from the risk aversion that \( u(E) > \pi(A)u(x(A)) + \pi(B)u(x(B)) \) since a risk averse consumer always prefer the to receive the expected payoff with certainty than having the bet.

(iii) Since \( u(E) > \pi(A)u(x(A)) + \pi(B)u(x(B)) \), Mary should buy the fair-odds insurance.

This part is not required but we can calculate the maximum price that Mary is willing to pay for the fair-odds insurance (risk premium). Let \( P \) be the maximum price that Mary is willing to pay. \( P \) solves the following equation.

\[ u(E - P) = \pi(A)u(x(A)) + \pi(B)u(x(B)) \]
\[ \log(820 - P) = 0.9 \log(900) + 0.1 \log(100) \]
\[ P = 820 - 900^{0.9}100^{0.1} \approx 97.5326 \]