Economics 4905
Financial Fragility and the Macroeconomy
Cornell University, Fall 2018
Prelim 1 Solutions
Monday, October 1, 2017, 2:55PM to 4:10PM
G26 Uris Hall

Instructions: This prelim is designed to take 60 minutes, but you have 75 minutes to write your answers. Answer each of the 4 questions. Do not seek, take, nor give advice from any source, animate or inanimate. Do not use calculators. There is no need to simplify numerical answers. Place all personal items - including books, paper, and computers - in a place determined by the proctors.

1 Question 1 (15 minutes)

Let $\omega = (\omega_1, \ldots, \omega_h, \ldots, \omega_n)$ be the vector of chocolate endowments and $\tau = (\tau_1, \ldots, \tau_h, \ldots, \tau_n)$ be the vector of dollar taxes. P^m is the chocolate price of dollars. $\mathcal{P}^m$ is the set of equilibrium money prices. The equilibrium allocation is denoted by $x = (x_1, \ldots, x_h, \ldots, x_n)$.

(a) Define balanced tax policy.

(b) Define bonafide tax policy.

(c) What is the relationship between balanced tax policies and bonafide tax policies.

Solutions:

(a) A tax policy is balanced if taxes exactly offset subsidies, or $\sum_{h=1}^{n} \tau_h = 0$.

(b) A tax policy is bonafide if there is at least one competitive equilibrium in which money is not worthless, or $P^m > 0$.

(c) They are equivalent.

2 Question 2 (15 minutes)

Using the symbols from Question 1, let $n = 4$ and $\omega = (900, 800, 700, 600)$. For each of the following calculate $\mathcal{P}^m$ and the set of equilibrium allocations.

(a) $\tau = (5, 4, -1, -3)$

(b) $\tau = (3, 2, -1, -4)$

(c) $\tau = (0, 0, 0, 0)$

Solutions:

(a) First check if the tax policy is balanced:

$$\sum_{h=1}^{4} \tau_h = 5 + 4 + (-1) + (-3) = 5 \neq 0$$

The tax policy is not balanced and hence not bonafide. The set of equilibrium price of money is

$$\mathcal{P}^m = \{0\}$$

The set of equilibrium allocations is

$$x = \{(900, 800, 700, 600)\}$$

(b) First check if the tax policy is balanced:

$$\sum_{h=1}^{4} \tau_h = 3 + 2 + (-1) + (-4) = 0$$

The tax policy is balanced, hence there exists some equilibrium price of money $P^m > 0$. The equilibrium price of money P^m has to satisfy the following conditions:

$$900 - 3P^m > 0$$
$$800 - 2P^m > 0$$

which are equivalent to

$$P^m < 300$$
$$P^m < 400$$

The set of equilibrium price of money is

$$\mathcal{P}^m = [0, 300)$$

The set of equilibrium allocations is

$$x = \{(900 - 3P^m, 800 - 2P^m, 700 + P^m, 600 + 4P^m) | P^m \in [0, 300)\}$$

(c) First check if the tax policy is balanced:

$$\sum_{h=1}^{4} \tau_h = 0 + 0 + 0 + 0 = 0$$

The tax policy is balanced. Since no consumer pays taxes, the equilibrium price of money P^m can be any non-negative real number. The set of equilibrium price of money is

$$\mathcal{P}^m = [0, \infty)$$

Since there is no taxes, the set of equilibrium allocations is

$$x = \{(900, 800, 700, 600)\}$$

3 Question 3 (15 minutes)

As before, $\omega = (900, 800, 700, 600)$. There are 2 monies, Red (R) and Blue (B). The units are $R\$$ and $B\$$. In each of the following cases, calculate the exchange rate between the 2 monies. Also, for each case calculate the **set** of equilibrium allocations, x.

(a) $\tau^R = (2, 1, 0, -1)$, $\tau^B = (5, 0, -4, -3)$.

(b) $\tau^R = (5, 4, -4, -5)$, $\tau^B = (1, 1, -1, -1)$.

Solutions:

(a) First check if the exchange rate is determinate

$$\sum_{h=1}^{4} \tau_h^R = 2 + 1 + 0 + (-1) = 2 \neq 0$$
$$\sum_{h=1}^{4} \tau_h^B = 5 + 0 + (-4) + (-3) = -2 \neq 0$$

The exchange rate is

$$\frac{P^R}{P^B} = -\frac{\sum_{h=1}^{4} \tau_h^B}{\sum_{h=1}^{4} \tau_h^R} = -\frac{-2}{2} = 1$$

which is R\$1 to B\$1. The equilibrium allocation is

$$\begin{aligned} x &= (900 - 2P^R - 5P^B, 800 - P^R, 700 + 4P^B, 600 + P^R + 3P^B) \\ &= (900 - 7P^R, 800 - P^R, 700 + 4P^R, 600 + 4P^R) \end{aligned}$$

The set of equilibrium price of Red money P^R has to satisfy the following conditions

$$900 - 7P^R > 0$$
$$800 - P^R > 0$$

which are equivalent to

$$P^R < \frac{900}{7}$$
$$P^R < 800$$

The set of equilibrium allocations is

$$x = \left\{(900 - 7P^R, 800 - P^R, 700 + 4P^R, 600 + 4P^R) | P^R \in \left[0, \frac{900}{7}\right)\right\}$$

(b) First check if the exchange rate is determinate

$$\sum_{h=1}^{4} \tau_h^R = 5 + 4 + (-4) + (-5) = 0$$
$$\sum_{h=1}^{4} \tau_h^B = 1 + 1 + (-) + (-1) = 0$$

The exchange rate is indeterminate. The exchange rate can be any non-negative real number.

$$\frac{P^R}{P^B} \in [0, \infty)$$

The equilibrium allocation is

$$x = (900 - 5P^R - P^B, 800 - 4P^R - P^B, 700 + 4P^R + P^B, 600 + 5P^R + P^B)$$

The set of equilibrium prices (P^R, P^B) has to satisfy the following conditions:

$$\begin{aligned} 900 - 5P^R - P^B &> 0 \\ 800 - 4P^R - P^B &> 0 \end{aligned}$$

The set of equilibrium allocations is then

$$\begin{gathered} x = \{(900 - 5P^R - P^B, 800 - 4P^R - P^B, 700 + 4P^R + P^B, 600 + 5P^R + P^B) | \\ P^R \geq 0, P^B \geq 0, 900 - 5P^R - P^B > 0, 800 - 4P^R - P^B > 0\} \end{gathered}$$

4 Question 4 (15 minutes)

Mary's utility function is

$$\pi(A)u(x(A)) + \pi(B)u(x(B)),$$

where $u(x) = \log(x)$, $\pi(B) = 1 - \pi(A)$, $\pi(A) \geq 0$, $\pi(B) \geq 0$.

(a) Is Mary risk averse, risk neutral, or risk loving?

(b) Let $x(A) = 900$, $x(B) = 100$, $\pi(A) = 0.90$, $\pi(B) = 0.10$.

  (i) Calculate Mary's expected payoff $E = \pi(A)x(A) + \pi(B)x(B)$.

  (ii) Calculate $u(E)$. How does $u(E)$ compare to $\pi(A)u(x(A)) + \pi(B)u(x(B))$?

  (iii) Should Mary buy fair-odds insurance or should she decline the insurance?

Solutions:

(a) Take the second derivative of $u(x)$.

$$\begin{aligned} u'(x) &= \frac{1}{x} \\ u''(x) &= -\frac{1}{x^2} < 0 \end{aligned}$$

Since the second derivative is negative for any x, Mary is risk averse.

(b) (i)

$$\begin{aligned} E &= \pi(A)x(A) + \pi(B)x(B) \\ &= 0.90(900) + 0.10(100) \\ &= 820 \end{aligned} \tag{1}$$

(ii)

$$u(E) = \log(820) \approx 6.7093$$

$$\begin{aligned}\pi(A)u(x(A)) + \pi(B)u(x(B)) &= 0.9\log(900) + 0.1\log(100) \\ &\approx 6.5827\end{aligned}$$

From the numbers, it is clear that $u(E) > \pi(A)u(x(A)) + \pi(B)u(x(B))$. Even without the numbers, it can be inferred from the risk aversion that $u(E) > \pi(A)u(x(A)) + \pi(B)u(x(B))$ since a risk averse consumer always prefer the to receive the expected payoff with certainty than having the bet.

(iii) Since $u(E) > \pi(A)u(x(A)) + \pi(B)u(x(B))$, Mary should buy the fair-odds insurance.

This part is not required but we can calculate the maximum price that Mary is willing to pay for the fair-odds insurance (risk premium). Let P be the maximum price that Mary is willing to pay. P solves the following equation.

$$\begin{aligned}u(E-P) &= \pi(A)u(x(A)) + \pi(B)u(x(B)) \\ \log(820-P) &= 0.9\log(900) + 0.1\log(100) \\ P &= 820 - 900^{0.9}100^{0.1} \approx 97.5326\end{aligned}$$