1 Consumer Problem

Consider

\[
\max_{x^1_h, x^2_h} u_h(x^1_h, x^2_h) = A \log x^1_h + B \log x^2_h
\]

subject to \(p^1 x^1_h + p^2 x^2_h = p^1 \omega^1_h + p^2 \omega^2_h = w_h\)

Solution:

1. By substitution:

Rearranging the budget constraint gives

\[
x^2_h = \frac{w_h - p^1 x^1_h}{p^2}
\]

The consumer problem becomes

\[
\max_{x^1_h} A \log x^1_h + B \log \left( \frac{w_h - p^1 x^1_h}{p^2} \right)
\]

Taking the first-order derivative with respect to \(x^1_h\) and equating it to zero gives

\[
\frac{A}{x^1_h} - \frac{B p^1}{p^2} \frac{w_h - p^1 x^1_h}{p^2} = 0
\]

Solving for \(x^1_h\)

\[
x^1_h = \frac{A}{A + B} \frac{w_h}{p^1}
\]

Plugging this into the budget constraint to solve for \(x^2_h\)

\[
x^2_h = \frac{B}{A + B} \frac{w_h}{p^2}
\]

2. By Lagrangian:

Setting up the Lagrangian of the problem:

\[
\mathcal{L} = A \log x^1_h + B \log x^2_h + \lambda (w_h - p^1 x^1_h - p^2 x^2_h)
\]

Taking the derivative of the Lagrangian with respect to \(x^1_h, x^2_h\) and \(\lambda\) and set them to zero:

\[
\frac{\partial \mathcal{L}}{\partial x^1_h} = \frac{A}{x^1_h} - \lambda p^1 = 0
\]

\[
\frac{\partial \mathcal{L}}{\partial x^2_h} = \frac{B}{x^2_h} - \lambda p^2 = 0
\]

\[
\frac{\partial \mathcal{L}}{\partial \lambda} = w_h - p^1 x^1_h - p^2 x^2_h = 0
\]
The solution to the above system of equations is

\[ x_h^1 = \frac{A}{A + B} \frac{w_h}{p^1} \quad x_h^2 = \frac{B}{A + B} \frac{w_h}{p^2} \quad \lambda = \frac{A + B}{w_h} \]