Economics 4905
Financial Fragility and the Macroeconomy
Cornell University, Fall 2018
Prelim 2
Monday, October 31, 2018, 2:55PM to 4:10PM
G26 Uris Hall

Instructions: This prelim is designed to take 60 minutes, but you have 75 minutes to write your answers. Answer each of the 2 questions. Do not seek, take, nor give advice from any source, animate or inanimate. Do not use calculators. Show your work. There is no need to simplify numerical answers. Place all personal items - including books, paper, and computers - in a place determined by the proctors.

1 Question 1, Banking (15 minutes)

The probability of being impatient is $\lambda = 0.4$. The type (patient or impatient) is realized in period 1 and it is private information. The utility function is:

$$u(c) = \log(c),$$

where c is consumption. Each individual has one unit of endowment in period 0. There is costless storage. If the endowment $\omega = 1$ is invested in period 0 and is harvested in period 1, the output is 1. If harvested late, the output is $R = 4$. Assume that the banking industry is free-entry. The bank contract is defined by (d_1, d_2) where d_t is the payment to the withdrawal in period t, $t = 1, 2$.

a) What is the depositor's *ex-ante* expected utility W as a function of c_1, consumption in period 1, and c_2, consumption in period 2?

   Solutions:
   $$W(c_1, c_2) = 0.6\log(c_1) + 0.4\log(c_2)$$

b) Show that the depositor prefers consumption smoothing.

   Solutions:
   Take the second derivative of the utility function:
   $$u'(c) = \frac{1}{c} \qquad u''(c) = -\frac{1}{c^2} < 0$$
   Hence, $u(c)$ is a strictly concave. A concave functions lies above its chords (Jensen's inequality):
   $$u(\lambda c_1 + (1-\lambda)c_2) > \lambda u(c_1) + (1-\lambda)u(c_2)$$
   for $c_1 \neq c_2$. That is she prefers $(\bar{c}, \bar{c})$ to (c_1, c_2) where $\bar{c} = \lambda c_1 + (1-\lambda)c_2$.

c) What is the bank's resource constraint RC? Write this down precisely and explain it in words.

Solutions:

$$\lambda d_2 \leq (1 - \lambda d_1)R$$
$$0.4 d_2 \leq 4(1 - 0.6 d_1)$$

d) What is the incentive problem? Write down the incentive constraint IC precisely, and explain it in words.

Solutions:

$$d_1 \leq d_2$$

e) Suppose that the optimal contract is given by $d_1^* = 1$. Is there a run equilibrium to this "optimal contract"? If yes, what is the value of the ex-ante utility in the run equilibrium, W_{run}?

Solutions:
There will not be a run.

2 Question 2, Overlapping Generations (45 minutes)

- 2 period lives
- 1 commodity per period, $\ell = 1$
- Stationary environment
- 1 person per generation

The utility functions are:

$$u_0(x_0^1) = 7 \log x_0^1$$
$$u_t(x_t^t, x_t^{t+1}) = \log x_t^t + 7 \log x_t^{t+1} \text{ for } t = 1, 2, \ldots$$

where x_s^t is the demand of generation s at period t. The endowments are 5 units for each period of a person's life:

$$\omega_0^1 = \omega_t^t = \omega_t^{t+1} = 5 \text{ for } t = 1, 2, \ldots$$

Define the excess demand/supply:

$$z^t = \omega_t^t - x_t^t$$
$$z^{t+1} = x_t^{t+1} - \omega_t^{t+1}$$

Let m_s^t be the money transfer to generation s at period t. The values are:

$$m_0^1 = 1$$
$$m_s^t = 0 \quad \text{otherwise}$$

a) For Mr t ($t = 1, 2, \dots$), write the equation for the indifference curve (IC) through the endowment and sketch the IC curve.

Solutions:

$$\log x_t^t + 7 \log x_t^{t+1} = \log 5 + 7 \log 5 = 8 \log 5$$

b) What is the slope of the IC at the endowment?

Solutions:
Taking the derivative of the utility function with respect to x_t^t and x_t^{t+1}:

$$\frac{\partial u(x_t^t, x_t^{t+1})}{\partial x_t^t} = \frac{1}{x_t^t} \qquad \frac{\partial u(x_t^t, x_t^{t+1})}{\partial x_t^{t+1}} = \frac{7}{x_t^{t+1}}$$

The slope of the IC at the endowment is:

$$\frac{\frac{\partial u(5,5)}{\partial x_t^t}}{\frac{\partial u(5,5)}{\partial x_t^{t+1}}} = \frac{1/5}{7/5} = \frac{1}{7}$$

c) What is the interest rate r at the endowment?

Solutions:
The interest rate is:

$$1 + r = \frac{1}{7}$$
$$r = -\frac{6}{7}$$

d) Is the economy Samuelson or Ricardo? Why?

Solutions:
The economy is Samuelson because $r < 0$.

e) Derive the equation of the offer curve (OC).

Solutions:
Using the equation from Problem Set 4:

$$z^{t+1} = \frac{\alpha \omega_t^{t+1} z^t}{\beta \omega_t^t - (\alpha + \beta) z^t}$$

we have

$$z^{t+1} = \frac{(1)(5) z^t}{(7)(5) - (1 + 7) z^t} = \frac{5 z^t}{35 - 8 z^t}$$

f) Draw and label the reflected, translated OC in the first quadrant.

Solutions:
Refer to lecture for diagram.

g) What are the stationary solutions?

Solutions:
Set $z^t = z^{t+1} = \bar{z}$:

$$\bar{z} = \frac{5\bar{z}}{35 - 8\bar{z}}$$

Solving for $\bar{z}$ gives

$$\bar{z} = 0 \text{ or } 15/4$$

h) What is the maximum equilibrium price of money $\bar{P}^m$?

Solutions:
Since $m_0^1 = 1$, the maximum equilibrium price of money is

$$\bar{P}^m = \frac{15}{4}$$

i) Do the full dynamic analysis.

Solutions:
If $0 < P^m < \frac{15}{4}$, then z^t is declining, and the bubble fades away through inflation. $z = 0$ is a stable steady state, in which money is worthless $P^m = 0$. $z = 15/4$ is an unstable steady state. If $z > 15/4$, hyperdeflation ensues and the bubble bursts in finite time.

j) Assume that $P^m = 3$. What are the values of z_0^1, z_1^1 and z_1^2?

Solutions:
Since $m_0^1 = 1$, the initial old's excess demand is

$$z_0^1 = 3$$

Using market clearing condition, the initial old's excess demand has to be equal to Mr 1's excess supply at $t = 1$.

$$z_1^1 = z_0^1 = 3$$

Using the OC, Mr 1's excess demand at $t = 2$ is

$$z_1^2 = \frac{5z_1^1}{35 - 8z_1^1} = \frac{5(3)}{35 - 8(3)} = \frac{15}{11}$$

k) Do the full welfare analysis.

Solutions:
When $P^m = 15/4$, the allocation is Pareto optiaml. Otherwise, it is not.