Game Theory and Financial Institutions

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Outline for Today

- Introduction/review of static games
    - Strategy profiles, (mixed) nash equilibrium, prisoner’s dilemma
- Dynamic (sequential) games
    - Subgame Perfect Equilibrium, backwards induction
- Relationship to financial markets

What is a game?

- Set of players
    - Players p_1, p_2, p_3, ..., p_n
- Set of Strategies
    - Strategies s_1, s_2, s_3, ..., s_n
- Set of actions
    - Actions a_1, a_2, a_3, ..., a_n
- Set of payoffs
    - Determined by each players utility function
    - Inputs will be the actions that all the players play
    - Utility functions can (and often are) different for different players

Simple Example of a Game

		P2	
		A	B
P1	A	4,4	1,3
	B	3,1	0,0

Nash Equilibrium

Formally: x^* is a Nash Equilibrium if $u_i(x^*, x^*_{-i}) \geq u_i(x, x^*_{-i})$ for any strategy $x \neq x^*$

Informally: A strategy profile is a Nash Equilibrium if no player can do better by unilaterally changing their strategy. Essentially, with strategies of other players known and treated as if they were set in stone, if you cannot benefit by changing your strategy then you are in a nash equilibrium

Informally (again): a strategy profile is nash equilibrium if it is the best response to all other strategies of other players in that equilibrium

Example with N.E.

		P2	
		A	B
P1	A	$\underline{4},\underline{4}$	1,3
	B	3,1	0,0

Prisoners Dilemma

		P2	
		C	D
P1	C	-1,-1	-3,0
	D	0,-3	-2,-2

Prisoners Dilemma

		P2	
		C	D
P1	C	-1,-1	-3,0
	D	0,-3	<u>-2</u>,<u>-2</u>

Matching Pennies: a motivating example

		P2	
		H	T
P1	H	1, -1	-1,1
	T	-1, 1	1,-1

Mixed Strategies and Mixed Nash

A **pure strategy** is an unconditional, defined choice that a person makes in a situation or game.

Example: In Rock-Paper-Scissors,if a player would choose to only play scissors for each and every independent trial, regardless of the other player's strategy, choosing scissors would be the player's pure strategy.

A **mixed strategy** is an assignment of probability to all choices in the strategy set

Example: With Rock-Paper-Scissors, if a person's probability of employing each pure strategy is equal, then the probability distribution of the strategy set would be 1/3 for each option, or approximately 33%. In other words, a person using a mixed strategy incorporates more than one pure strategy into a game

A **Mixed Nash Equilibrium** strategy is a mixed strategy for which, in that equilibrium, no player can do better by changing their strategy

Matching Pennies and Mixed Nash

Clearly, there is no pure strategy Nash Equilibrium to this game. However this game has a mixed strategy Nash equilibrium.

(.5T + .5H , .5H + .5T)

P2

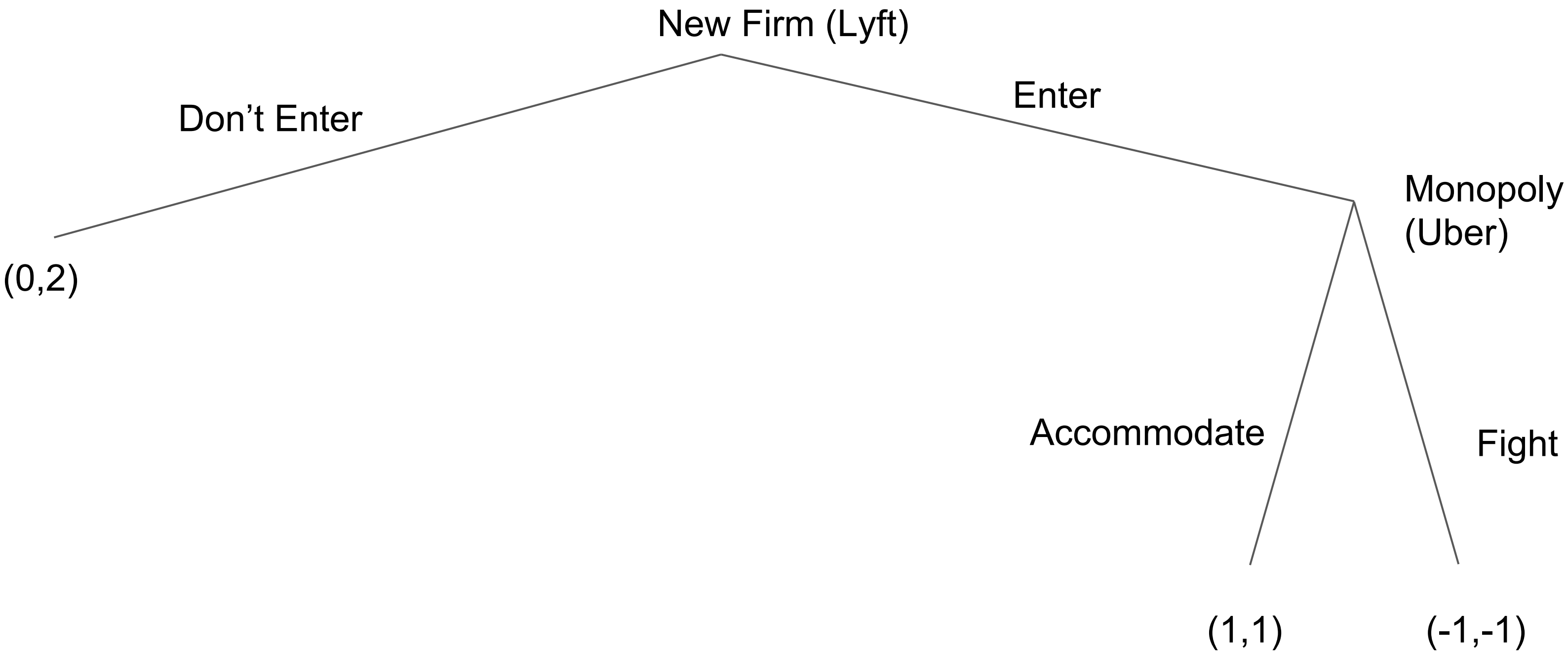
P1		H	T
	H	1, -1	-1,1
	T	-1, 1	1,-1

Static vs Dynamic Games

In a static game both players choose their strategy and act at the same time. In a dynamic game, players move at different times, which changes our analysis of the game.

Example: Chess, Market Entry Game

Market Entry Game



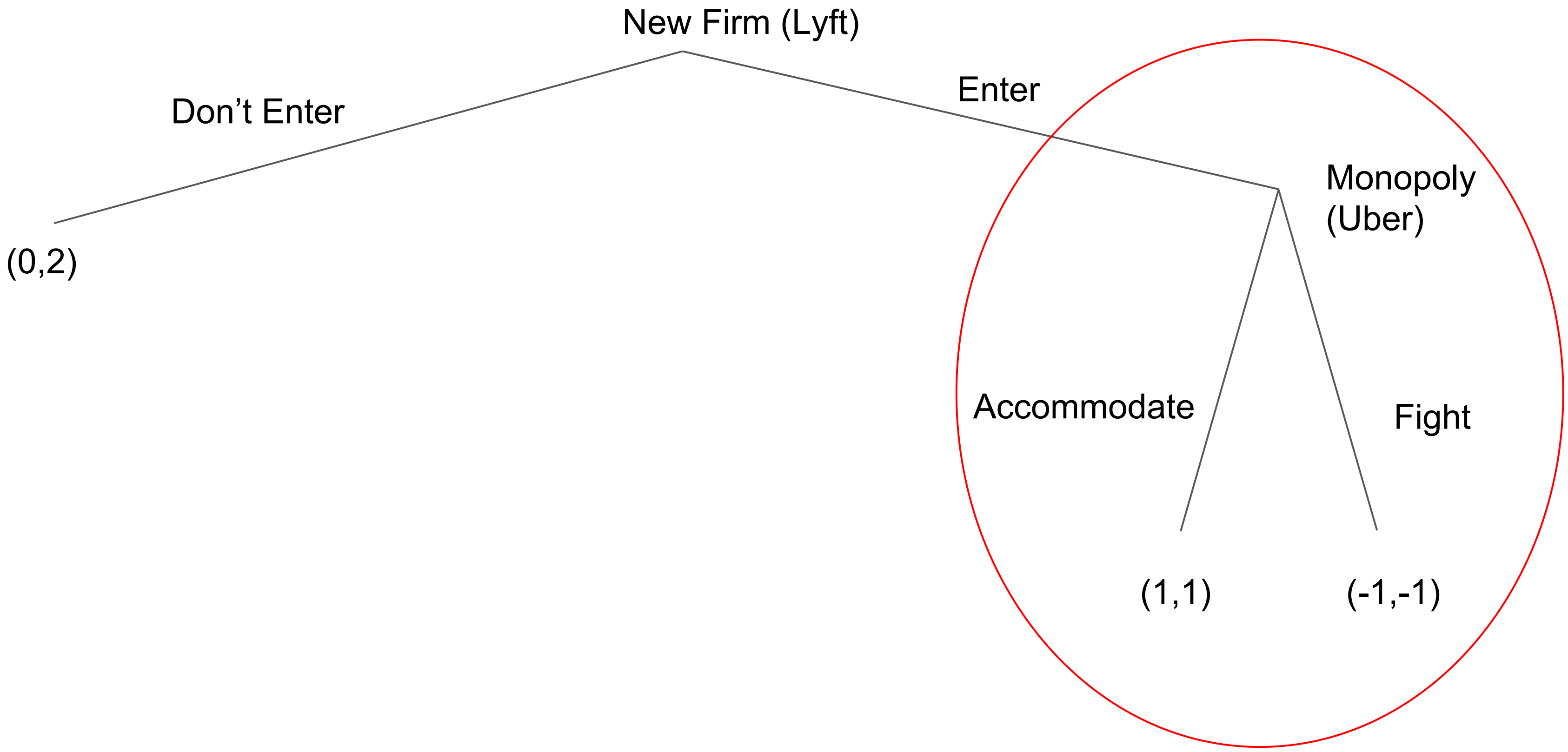
Subgame Perfect Equilibrium

Colloquially: An SPE is a nash equilibrium in a sequential game where each subgame is also in nash equilibrium

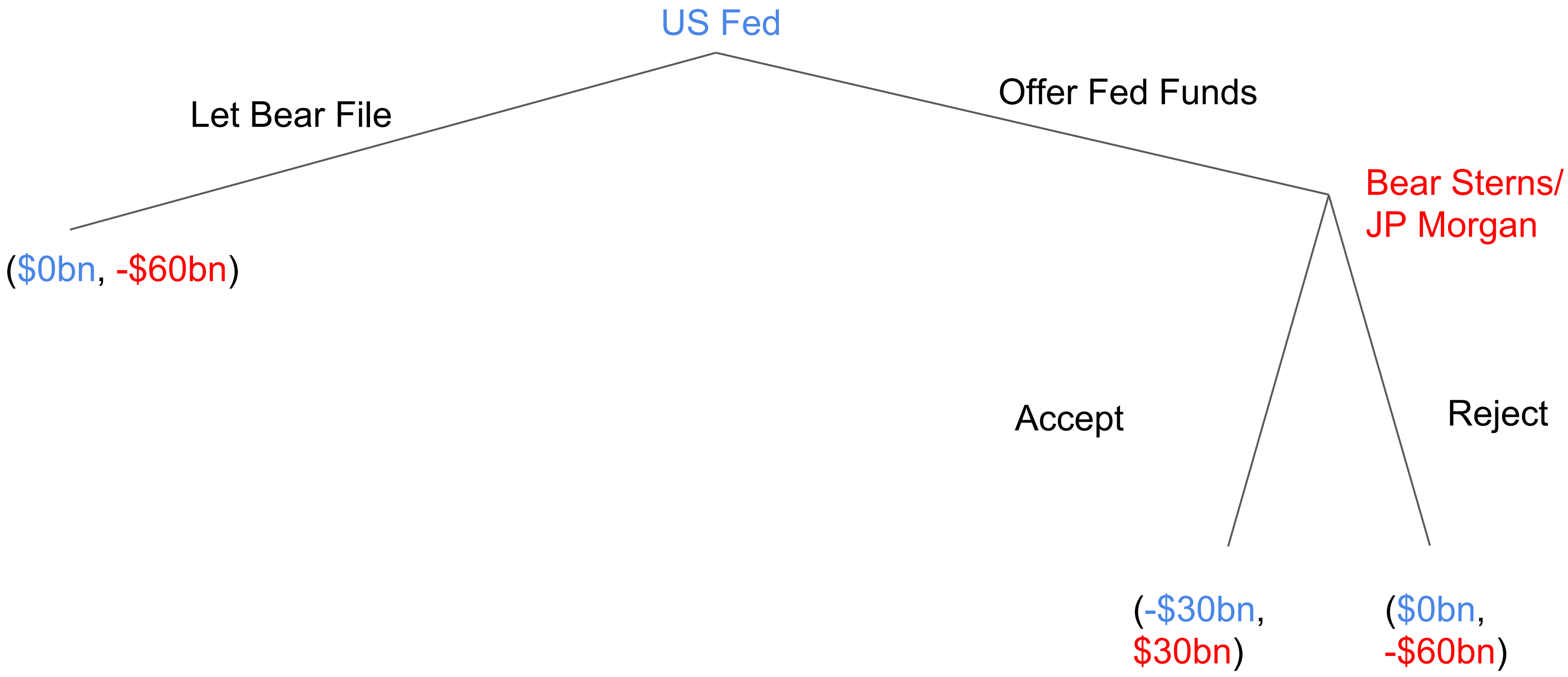
Subgame: A step in a dynamic game which could be treated independently as a game

Informally (and for the purposes of these examples), its any “node” where there are additional nodes below it

Market Entry Game



Federal Reserve Bailout Game



The Post-Deposit Game

- 2 players (depositors)
- Actions: Early or Late withdrawal
- Types (states of nature): patient (w.p. 1-p) or impatient (w.p. p)
- Payoffs: CRRA Utility functions
  - If impatient: $u(x) = A(x\ )^{1-b}/(1-b)$
  - If patient: $v(x) = (x\ )^{1-b}/(1-b)$

Formulation

	(E, E)	(L,E)
(E,E)	T_1, T_1	T_3,T_2
(L,E)	T_2,T_3	T_4,T_4

$$T_1 = (1-p)\frac{v(c)+v(2y-c)}{2} + p\frac{u(c)+u(2y-c)}{2},$$

$$T_2 = (1-p)v[(2y-c)R] + p\frac{u(c)+u(2y-c)}{2},$$

$$T_3 = (1-p)[(1-p)v(c) + p\tfrac{v(c)+v(2y-c)}{2}] \\ +p[(1-p)u(c) + p\tfrac{u(c)+u(2y-c)}{2}],$$

$$T_4 = (1-p)[(1-p)v(yR) + pv[(2y-c)R]] \\ +p[(1-p)u(c) + p\tfrac{u(c)+u(2y-c)}{2}].$$

Run Equilibrium is a (Bayesian) Nash Equilibrium

	(E, E)	(L,E)
(E,E)	T_1, T_1	T_3,T_2
(L,E)	T_2,T_3	T_4,T_4

$c \in [0, 2y]$ satisfies:

$[v(c) + v(2y-c)]/2 > v[(2y-c)R]$

i.e. $v(E,E) > v(L,E)$

i.e. $T_1 > T_2$

Non Run Equilibrium is a (Bayesian) Nash Equilibrium

	(E, E)	(L,E)
(E,E)	T_1, T_1	T_3,T_2
(L,E)	T_2,T_3	T_4,T_4

$c \in [0, 2y]$ satisfies:

$(1\text{-}p)v(yR) + pv[(2y\text{-}c)R] \geq$
$(1\text{-}p)v(c) + p[v(c) + v(2y\text{-}c)]/2$

i.e. $v(L,E) \geq v(E,E)$

i.e. $T_4 \geq T_3$

The Pre-Deposit Game

- 2 players (depositors)
- Endowments: y
- Strategies deposit contract c $\in [0, 2y]$
- Types (states of nature): patient (w.p. 1-p) or impatient (w.p. p)
- Payoffs: CRRA Utility functions
  - If impatient: $u(x) = A(x)^{1-b}/(1-b)$
  - If patient: $v(x) = (x)^{1-b}/(1-b)$

Solving for (Bayesian) Nash Equilibrium

As we saw in class, in an unconstrained efficient allocation, depositors maximize welfare i.e. choose c that maximizes welfare:

$W(c) = p^2[u(c)+u(2y-c)] + 2p(1-p)[u(c)+v((2y-c)R)] + 2(1-p)^2v(yR)$

Solving for c s.t. W(c) is maximized yields:

$C = 2y / \{p/(2-p) + 2(1-p) / [(2-p)AR^{b-1}]\}^{1/b} + 1$