36th Anniversary of the Classic Diamond-Dybvig JPE paper
DD Revolution in Finance:

- intermediation
- bank runs on depository institutions
- fragility of other financial institutions
Extensions to Macro, etc.

- beliefs about beliefs of others
- asymmetric information
- contracts, mechanisms
- fragility
- GE without Walras
DD Revolution: Best Contract versus Best Run-Proof Contract

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*Extract from the draft: “The Diamond-Dybvig Revolution: Extensions Based on the Original DD Environment” by Shell and Zhang
Risk tolerance

- street crossing
- bridge building
- engineers versus economists
- insurance deductibles
For the individuals for whom contract is designed

- less risk is not always better
- zero risk, even if feasible, is not always best

For society

- above 2 bullets apply
- but if private banks are too risky because of externalities, we still need to model individual bank and depositor behavior.
- Friedman, Kotlikoff
Extend the basic DD (JPE) environment

- continuum of consumers (potential depositors)
- Only feasible contract is the simple deposit contract. Partial suspension of convertibility is not allowed. In a break from DD, there is no deposit insurance.
- no aggregate uncertainty.
- expected utility maximization as consequence of free-entry banking
- generalize depositor beliefs
- REE
Why allow for bank runs?

- consumers might tolerate risk
- especially so for non-bank applications
- if this risk is not socially desirable, we need to test risk-reducing social actions based on a model of risky private behavior
- runs are historical facts
Large, excellent literature on run-proof mechanisms, e.g.

- DD
- Wallace
- Green-Lin

Peck-Shell (JPE)

* pre-deposit game, in which individuals choose whether or not to deposit

* tests whether run-proof mechanisms generalize. See also Ennis-Keister
Post-deposit game

- game-theory style reasoning
  - analyze post before pre
  - include off-equilibrium behavior
- Using DD notation.
  - $c$ is withdrawal in period 1.
  - small $c$ is conservative, large $c$ is aggressive.
  - $c^{\text{run-proof}} = 1$.
  - $c^{\text{IC}} = \frac{R}{(1-\lambda)+\lambda R}$. 
Post-Deposit Game

- 2 equilibria, 1 run, 1 non-run.
- 1 equilibrium, non-run
- 1 equilibrium, run

$C^{\text{run-proof}}$ $C^{UE}$ $C^{IC}$ $\frac{1}{\lambda}$
The pre-deposit game is a game between the bank and the consumers (while the post-deposit game is game among depositors).

Consumers

- coordinate on the same sunspot signal. Contrast with Gu.
- beliefs dependent on contract $c$:

$$s(c) = \begin{cases} 
0, & \text{if } c \in [0, c^{\text{run-proof}}] \\
\hat{s}(c), & \text{if } c \in (c^{\text{run-proof}}, c^{IC}] \\
1, & \text{if } c \in (c^{IC}, 1/\lambda].
\end{cases}$$

- generalization of 1-step consumer beliefs in Peck-Shell in the spirit of Ennis-Keister
Pre-deposit game

- Bank
  - chooses $c(s)$ to max EU given consumer beliefs, $s(c)$
Equilibrium

- Following Ennis-Keister
  - REE is the fixed point of the pair \((s(c), c(s))\), where \(s(c)\) is the depositor run probability function and \(c(s)\) is the bank’s EU-maximizing contract.
  - Let \(s_0(c)\) be the maximum value of \(s\) beyond which it is no longer optimal for the bank to tolerate runs under contract \(c\).
  - Define \(\overline{s_0}\) by \(\overline{s_0} = \max_c(s_0(c))\).
1-step beliefs (Peck-Shell):

- $\hat{s}(c) = s_1 \in (0, 1)$
- low interaction assumption

Proposition (1-step):

- If $s_1 \in (0, s_0)$, unique REE is $(s_1, c(s_1))$.
  - $s_1$ is an equilibrium belief.
- If $s_1 > s_0$, the unique REE is $(0, c^{\text{run-proof}})$.
  - $s_1$ is an off-equilibrium belief.
- If $s_1 = s_0$, there are 2 equilibria: $(s_0, c(s_0))$ and $(0, c^{\text{run-proof}})$. 
Example (1-step)

- $u(c) = \frac{(c+1)^{1-\theta}}{1-\theta} + 1$, where $\theta = 3$. $R = 2$, $\lambda = 0.3$. $c_{\text{run proof}} = 1$, $c^{IC} = 1.538$ and $c^{UE} = 1.227$. We have $\bar{s}_0 = 0.0177$. We see that $s_1$ is an off-equilibrium belief if $s_1 \geq 0.0177$.

- If, for example, $s_1 = 0.0089$, then the REE is $(0.0089, 1.1982)$. Then $s_1$ is an equilibrium belief.
Because the IC does not bind, $c$ is strictly decreasing in $s_1$. Compare with PS and Shell-Zhang, in which the IC binds in some cases, and does not bind in other cases.

Since the IC does not bind, the SSE in the pre-deposit game is never a mere randomization over the equilibria from the post-deposit game.
Generalizing from 1-step $\hat{s}(c)$ to multiple steps:

$$
\hat{s}(c) = \begin{cases} 
0, & \text{if } c \in [0, c^{\text{run-proof}}] \\
 s_1, & \text{if } c \in (c^{\text{run-proof}}, c^1] \\
 s_2, & \text{if } c \in (c^1, c^{IC}] \\
 1, & \text{if } c \in (c^{IC}, 1/\lambda],
\end{cases}
$$

where $0 < s_1 < s_2 < 1$. 
Example (2-step)

- Use the parameter values from the previous example. Let $\hat{s}(c)$ be a multiple-step function with $s_1 = 0.0053$, $s_2 = 0.0107$ and $c^1 = 1.083$. $s_1$ and $s_2$ are equilibrium run beliefs. The corresponding equilibrium contracts are $c^1 = 1.083$ and $c^2 = 1.192$.

- The two REE are $(0.0053, 1.083)$ and $(0.0107, 1.192)$.

- The bank is indifferent between these 2 equilibria. The second one is riskier, but it provides more $c$ to compensate exactly for the extra risk.
\( \hat{s}(c) \) is continuous and strictly increasing in \( c \):

- REE exists
- if, in addition, \( \hat{s}(c) \) is smooth then REE is unique
- An example (built from our 2-step example) shows that if \( \hat{s}(c) \) is kinked, then there can be multiple REE even if \( \hat{s}(c) \) is continuous and strictly increasing.