36th Anniversary of the Classic Diamond-Dybvig *JPE* paper

DD Revolution in Finance:

- intermediation
- bank runs on depository institutions
- fragility of other financial institutions

Extensions to Macro, etc.

- beliefs about beliefs of others
- asymmetric information
- contracts, mechanisms
- fragility
- GE without Walras

DD Revolution: Best Contract versus Best Run-Proof Contract*

Karl Shell — Cornell University

Yu Zhang — Xiamen University

Slides for DD 36 Conference

Friday, March 29, 2019

Olin Business School

Washington University in St. Louis

*Extract from the draft: "The Diamond-Dybvig Revolution: Extensions Based on the Original DD Environment" by Shell and Zhang

Risk tolerance

- street crossing
- bridge building
- engineers versus economists
- insurance deductibles

- For the individuals for whom contract is designed
  - less risk is not always better
  - zero risk, even if feasible, is not always best
- For society
  - above 2 bullets apply
  - but if private banks are too risky because of externalities, we still need to model individual bank and depositor behavior.
  - Friedman, Kotlikoff

Extend the basic DD (JPE) environment

- continuum of consumers (potential depositors)
- Only feasible contract is the simple deposit contract. Partial suspension of convertibility is not allowed. In a break from DD, **there is no deposit insurance**.
- no aggregate uncertainty.
- expected utility maximization as consequence of free-entry banking
- generalize depositor beliefs
- REE

Why allow for bank runs?

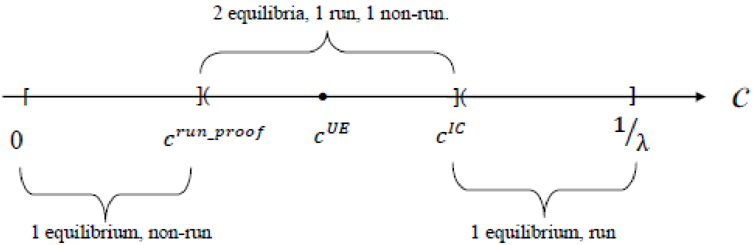
- consumers might tolerate risk
- especially so for non-bank applications
- if this risk is not socially desirable, we need to test risk-reducing social actions based on a model of risky private behavior
- runs are historical facts

- Large, excellent literature on run-proof mechanisms, e.g.
  - DD
  - Wallace
  - Green-Lin
- Peck-Shell (JPE)

  ⋆ pre-deposit game, in which individuals choose whether or not to deposit
  - tests whether run-proof mechanisms generalize. See also Ennis-Keister

Post-deposit game

- game-theory style reasoning
  - analyze **post** before **pre**
  - include off-equilibrium behavior
- Using DD notation.
  - c is withdrawal in period 1.
  - small c is conservative, large c is aggressive.
  - $c^{run-proof} = 1$.
  - $c^{IC} = \frac{R}{(1-\lambda)+\lambda R}$.



Post-Deposit Game

Pre-deposit game

- The pre-deposit game is a game between the bank and the consumers (while the post-deposit game is game among depositors)
- Consumers
  - coordinate on the same sunspot signal. Contrast with Gu.
  - beliefs dependent on contract c :

$$s(c) = \begin{cases} 0, \text{ if } c \in [0, c^{run\_proof}] \\ \widehat{s}(c), \text{ if } c \in (c^{run\_proof}, c^{IC}] \\ 1, \text{ if } c \in (c^{IC}, 1/\lambda]. \end{cases}$$

  - generalization of 1-step consumer beliefs in Peck-Shell in the spirit of Ennis-Keister

Pre-deposit game

- Bank
  - chooses $c(s)$ to max EU given consumer beliefs, $s(c)$

Equilibrium

- Following Ennis-Keister
  - REE is the fixed point of the pair $(s(c), c(s))$, where $s(c)$ is the depositor run probability function and $c(s)$ is the bank's EU-maximizing contract.
- Let $s_0(c)$ be the maximum value of s beyond which it is no longer optimal for the bank to tolerate runs under contract c.
- Define $\overline{s_0}$ by $\overline{s_0} = \max_c(s_0(c))$.

1-step beliefs (Peck-Shell):

- $\widehat{s}(c) = s_1 \in (0, 1)$
- low interaction assumption

Proposition (1-step):

- If $s_1 \in (0, \overline{s_0})$, unique REE is $(s_1, c(s_1))$.
  - s_1 is an equilibrium belief.
- If $s_1 > \overline{s_0}$, the unique REE is $(0, c^{run-proof})$.
  - s_1 is an off-equilibrium belief.
- If $s_1 = \overline{s_0}$, there are 2 equilibria: $(\overline{s_0}, c(\overline{s_0}))$ and $(0, c^{run-proof})$.

Example (1-step)

- $u(c) = \frac{(c+1)^{1-\theta}}{1-\theta} + 1$, where $\theta = 3$. $R = 2, \lambda = 0.3$. c^{run_proof} =1, c^{IC} =1.538 and c^{UE} =1.227. We have $\overline{s_0} = 0.0177$. We see that s_1 is an off-equilibrium belief if $s_1 \geq 0.0177$.
- If, for example, $s_1 = 0.0089$, then the REE is $(0.0089, 1.1982)$. Then s_1 is an equilibrium belief.

Comparative Statistics (1-step)

- Because the IC does not bind, c is strictly decreasing in s_1. Compare with PS and Shell-Zhang, in which the IC binds in some cases, and does not bind in other cases.
- Since the IC does not bind, the SSE in the pre-deposit game is never a mere randomization over the equilibria from the post-deposit game.

Generalizing from 1-step $\widehat{s}(c)$ to multiple steps:

$$\widehat{s}(c) = \begin{cases} 0,\ \text{if } c \in [0, c^{run\_proof}] \\ \\ s_1,\ \text{if } c \in (c^{run\_proof}, c^1] \\ \\ s_2,\ \text{if } c \in (c^1, c^{IC}] \\ \\ 1,\ \text{if } c \in (c^{IC}, 1/\lambda], \end{cases}$$

where $0 < s_1 < s_2 < 1$.

Example (2-step)

- Use the parameter values from the previous example. Let $\widehat{s}(c)$ be a multiple-step function with $s_1 = 0.0053$, $s_2 = 0.0107$ and $c^1 = 1.083$. s_1 and s_2 are equilibrium run beliefs. The corresponding equilibrium contracts are $c^1 = 1.083$ and $c^2 = 1.192$.
- The two REE are $(0.0053, 1.083)$ and $(0.0107, 1.192)$.
- The bank is indifferent between these 2 equilibria. The second one is riskier, but it provides more c to compensate exactly for the extra risk.

- $\widehat{s}(c)$ is continuous and strictly increasing in c :
  - REE exists
  - if, in addition, $\widehat{s}(c)$ is smooth then REE is unique
  - An example (built from our 2-step example) shows that if $\widehat{s}(c)$ is kinked, then there can be multiple REE even if $\widehat{s}(c)$ is continuous and strictly increasing.