Available online at www.sciencedirect.com

SciVerse ScienceDirect

Journal of Economic Theory 147 (2012) 397–399

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

David Cass: Economic theorist

In this issue of *JET*, we celebrate Dave Cass's remarkable scientific career with two symposia: (1) "Economic Growth" organized by Daron Acemoglu, and (2) "General Equilibrium" organized by Yves Balasko and John Geanakoplos. Dividing the issue into two parts should not obscure the fact that Dave recognized only one economic science, one that includes growth, general equilibrium (GE), and more. Dave might begin the analysis of a problem in growth theory using the simplest (macro-style) model, but he invariably followed up in proper GE style with a general model that allows for many goods. Sometimes the results from the simple model would be confirmed, sometimes not. For example, in [6] it is shown that the analysis of optimal growth in the 1-sector model [2] is not robust to generalizations of the technology.

Full disclosure: Dave and I were good friends from grad school. At Stanford, I introduced Dave to Serra House, and Dave introduced me to the Uzawa seminar. Dave's first job was at Cowles. Mine was at MIT. We visited each other regularly for the seminars that we initiated in New Haven and Cambridge. We were Penn colleagues for more than 15 years. Dave and I worked separately and together on growth theory. We teamed up on overlapping-generations (OG) and sunspot equilibrium (SSE), which we then saw as a single project. We worked closely on several other subjects. Dave was a charter member of the *JET* editorial board.

Dave's first two publications were drawn from his dissertation on optimal growth. The *RES* article [2] is Dave's most cited. The technology is 1-sector, neoclassical. The time-horizon is infinite. The planner maximizes the integral over time of discounted utilities. Utility is an increasing, strictly concave function of per-capita consumption. A sufficiently well-endowed economy initially consumes all of its gross output. By assuming positive discounting, Dave generalizes Ramsey [11] while also side-stepping technical problems arising from the no-discounting case. If the discount rate is positive, then the Cass planning problem is well defined. Like the Solow [14] descriptive economy, the Cass planning economy is globally stable: the capital–labor ratio tends to its modified-golden rule (MGR) value, defined (by Dave) as the capital–labor ratio at which the interest rate is equal to the labor growth rate *plus* the discount rate. Hence the MGR is a generalization of the Golden Rule (GR) of Phelps [10]. Dave's companion article [3] in *Econometrica* is equally important. The model is the same as in [2] except that the planning horizon is finite. The discount rate can be positive, zero, or negative for the finite-horizon case. On the terminal date, a target capital–labor ratio must be met or exceeded. The optimal trajectory of the capital–labor ratio exhibits a Dorfman–Samuelson–Solow [8] turnpike property: as the time-horizon is increased, the capital–labor ratio is near its MGR value most of the time. The constraint on the terminal capital–labor ratio is the basis for the necessary transversality condition: either

0022-0531/$ – see front matter
doi:10.1016/j.jet.2012.02.002

the target is met with equality or the shadow price of investment is zero. The absence of natural transversality conditions in infinite horizon economies is central to understanding over-saving and other inter-temporal inefficiencies.

The GR capital–labor ratio can be used to identify programs that are short-run efficient but long-run inefficient. In the 1-sector neoclassical model, the only source of this phenomenon is over-saving. If the capital–labor ratio is bounded above its GR value, then production is long-run inefficient in the sense there is a feasible program providing at least as much consumption at every time while providing strictly more consumption over some period; this is the Phelps–Koopmans theorem [10]. If the capital–labor ratio is bounded below its GR value, the program is efficient. With only these two sufficient conditions, many situations are left uncovered.

In the *JET* article [4], which in my opinion is even more remarkable than his better-known works on optimal growth, Dave derives a complete characterization of infinite-horizon production inefficiency (and hence of production efficiency). This characterization, now referred to as the Cass criterion, is in terms of the inter-temporal prices that support short-run efficient plans, i.e. plans that are efficient for every finite horizon. The Cass criterion can be thought of as the transversality condition that separates finite-efficient programs into those that are infinite-efficient and those that are not. It refines and extends the Phelps–Koopmans sufficient condition. It must be noted that Malinvaud in his *Econometrica* article [9] was fully aware of the distinction between short-run optimality and long-run optimality. The failure of Pareto-optimality in Samuelson's overlapping-generations (OG) economy is solely due to the infinite horizon; see [8] on the role of double-infinity (of agents and dated commodities) in OG models. It turns out that the Cass criterion extends to infinite-horizon OG economies: it divides short-run Pareto-optimal allocations into allocations that are fully Pareto optimal and those that are not; see [1].

Dave and I introduced the concept of sunspot equilibrium (SSE) as a means for exploring the connection between individual rationality (including rational expectations) and social rationality. Can fully rational agents coordinate their actions on the realization of some purely extrinsic random variable (nicknamed "sunspots"), a variable that is assumed to have no effect on economic fundamentals? Is all of observed economic volatility due to intrinsic uncertainty and/or individual irrationality? In the *JPE* paper [7], we show that sunspots can matter in rational-expectations competitive economies in which some agents are restricted from participating in financial markets, as would naturally be the case in OG economies. In other papers, Dave and I have shown the prevalence of proper sunspot equilibrium in a variety of rational expectations models; see [13]. Dave's missing-markets paper [5] is a classic. There is a two-period economy in which all saving and borrowing is in terms of money. There are two sunspot states. If, instead of the unique savings medium, there were two properly chosen Arrow securities (or perfect forward markets), then sunspots would not matter; i.e., the economy would be immune from sunspot effects. But with only the single money, there is typically a continuum of sunspot equilibria. This is a major paper in the sunspots literature. It is also the grandfather of the literature on GEI: general equilibrium with incomplete markets. In [5], the missing markets are for the Arrow securities.

This brief preface is necessarily incomplete and quite possibly idiosyncratic. For the complete collection of David Cass's published works, I recommend the collection edited by Steve Spear [15]. Steve has done a good job, for which he deserves our thanks.

Dave Cass was an economic theorist's economic theorist. Indeed Dave defined himself by his economics research. *JET*'s mission is to further the science of economic theory. For these reasons, this preface focuses on Dave Cass's scientific contributions. Remembering Dave as a classmate, friend, co-author, collaborator, colleague, teacher, mentor, and drinking buddy is for another venue.

References

[1] Y. Balasko, K. Shell, The overlapping-generations model I: the case of pure exchange without money, J. Econ. Theory 23 (1980) 281–306.
[2] D. Cass, Optimum growth in an aggregative model of capital accumulation, Rev. Econ. Stud. 37 (1965) 233–240.
[3] D. Cass, A turnpike theorem, Econometrica 34 (1966) 833–850.
[4] D. Cass, On capital overaccumulation in the aggregative, neoclassical model of economic growth, J. Econ. Theory 4 (1972) 224–240.
[5] D. Cass, Sunspots and incomplete financial markets: The leading example, in: G. Feiwel (Ed.), The Economics of Imperfect Competition: Joan Robinson and Beyond, Macmillan, London, 1989.
[6] D. Cass, K. Shell, The structure and stability of competitive dynamical systems, J. Econ. Theory 12 (1976) 31–70.
[7] D. Cass, K. Shell, Do sunspots matter? J. Polit. Economy 91 (1983) 193–227.
[8] R. Dorfman, P.A. Samuelson, R.M. Solow, Linear Programming and Economic Analysis, McGraw–Hill, 1958 (Chapter 12).
[9] E. Malinvaud, Capital accumulation and efficient allocation of resources, Econometrica 21 (1953) 233–268.
[10] E.S. Phelps, Second essay on the golden rule of accumulation, Amer. Econ. Rev. 55 (1965) 793–814.
[11] F.P. Ramsey, A mathematical theory of saving, Econ. J. 38 (1928) 543–559.
[12] K. Shell, Notes on the economics of infinity, J. Polit. Economy 79 (1971) 1002–1011.
[13] K. Shell, Sunspot equilibrium, in: L.E. Blume, S.N. Durlauf (Eds.), The New Palgrave: A Dictionary of Economics, 2nd edition, Palgrave Macmillan, 2008.
[14] R.M. Solow, A contribution to the theory of economic growth, Quart. J. Econ. 70 (1954) 65–94.
[15] S.E. Spear, The Collected Scientific Work of David Cass, Emerald Press, 2011.

Karl Shell
Cornell University, United States
E-mail address: karl.shell@cornell.edu