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Robustness of sunspot equilibria★

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Summary. A sunspot equilibrium (SSE) is based on some extrinsic randomizing device (RD). We analyze the *robustness* of SSE. (1) We say that an SSE allocation is *robust to refinements* if it is also an SSE allocation based on any refinement of its RD. (2) We introduce two core concepts for analyzing the *robustness* of SSE *in the face of cooperative-coalition formation*. In the first, the blocking allocations are based on the RD that defines the SSE. In the second (stronger) core concept, coalitions select their own RDs. For the convex economy with restricted market participation, SSE allocations are robust under each of the definitions and the cores converge on replication of the economy to the set of SSE allocations. For the economy with an indivisible good, SSE allocations are *not* always robust. We provide examples of each of the following: (i) an SSE allocation that is not robust to refinement, (ii) an SSE allocation that is in neither core, (iii) an SSE allocation that is in the first core, but not in the second, and (iv) a core that does not converge upon replication to the set of SSE allocations.

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1 Introduction

The sunspot equilibrium notion introduced by Cass and Shell[1] has been applied to a wide class of economic models. The power of the sunspots approach is based on the fact that all volatility in these models is by defi-

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[1] Reported in Shell [16] and Cass and Shell [5].

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nition excess volatility. This suggests that sunspot equilibrium theory can be of use in analyzing "market uncertainty" or "endogeneous uncertainty"– the uncertainty created by the market economy as a part of the coordination of economic activity.

In the original sunspots papers, the extrinsic randomizing device is taken as one of the parameters defining the economy. In this way, the original SSE theory is a specialization of the existing general-equilibrium theory with uncertainty. In some of the later sunspots papers,[2] an SSE is defined to be an RD and a corresponding allocation that *jointly* solve the equilibrium problem – given the economic fundamentals such as endowments, preferences, and technologies. It is this (second) more powerful way of looking at SSE that is more congenial to the idea that sunspots represent market uncertainty, because the market then "chooses"a coordination device rather than having one imposed on it. This approach follows the one taken in game theory, where mixed-strategy equilibrium and more generally correlated equilibrium are stochastic *solutions* to nonstochastic games.

Under the first approach to extrinsic uncertainty, a sunspot equilibrium (SSE) allocation is based on a randomizing device (RD) – defined by a state space, an information structure, and probabilities over the events – that is used for coordinating the plans of individual economic actors. In any given economic model, there are usually many RDs that can serve as a basis for coordination. It would be useful to know how sensitive are the equilibrium outcomes to the specification of the RD and whether there are any meaningful economic restrictions on the RD. Under the second approach to extrinsic uncertainty, in which the RD is part of the solution, it is crucial to know which RDs can emerge as the outcome of the equilibrium process. The goal would be to reduce the set of SSEs to those that are "robust".

We introduce two basic notions of *robustness* of SSE allocations. In the first, we examine whether an existing SSE allocation for fixed RD "survives" as an equilibrium allocation when the RD is refined. By refinement, we mean using a finer partition for defining events. This comparative statics exercise, in which the RD is varied in this special way, can also be used to determine whether there is a maximal amount of information (concerning sunspot activity) that is consistent with a given SSE allocation. We restrict attention to variations in the RD which are refinements of it so that the equilibrium allocations can be measurable with respect to the different RDs. This robustness exercise is relevant for each of the two approaches to defining SSE, since it addresses the question of existence of SSE as we take finer partitions

[2] The paper by Peck and Shell [14] is about SSE in non-cooperative market games. Two related SSE concepts are used there. In one, the RD is a part of the rules of the game (i.e., the fundamentals of the game); in the other, the RD is part of the solution to the game. Solving jointly for the RD and prices as the solution to the SSE problem is advocated in Shell and Wright [17]. Spear [18] constructs SSE by simultaneously conditioning actions of the agents on each other.

of the underlying continuous sunspot activity.[3] This exercise is also useful in understanding the limiting behavior in going from a discrete state space to a continuous representation of sunspot activity.

The second basic notion of robustness leads us to two different concepts of the core. Unlike the first concept of robustness, which is based on competitive behavior, the second assesses robustness from the co-operative point of view. If extrinsic uncertainty represents "market uncertainty" or "endogenous uncertainty", then a natural question is whether the economic agents can take cooperative actions to 'reduce' this volatility. For our first core, we assume that the blocking coalitions take the RD as given. This corresponds to the first interpretation of SSE in which the RD is a parameter of the economy. For our second core, we allow the coalitions to choose their own RDs on which to base the blocking allocations. This allows us to directly determine which RDs could be "chosen" through cooperative action. If an SSE allocation (and the RD on which it is based) is in each of these cores, then we can say that the effect of the extrinsic uncertainty on the economy is robust.

We consider two different economies which have appeared in the sunspots literature – the convex restricted-market-participation economy (Cass and Shell [5]) and the economy with an indivisible consumption good (Shell and Wright [17]). These two economies yield different results. In the convex economy with restricted-market-participation, an SSE based on a RD is robust in the face of every refinement of that RD. In this economy, the SSE is also robust to blocking by coalitions using any (possibly coalition-specific) randomizing device. There are no restrictions on the RD in this economy: there is an SSE for each RD. However, in the economy with indivisible goods, some SSEs are *not* robust to refinements in the RD. Also in this economy, coalitions can sometimes block SSE allocations while using as the coordinating device the same RD used to define the SSE. In some other cases, blocking of the SSE with coordination restricted to the same RD is not possible, but blocking of the SSE is possible if coalitions have other RDs available for generating their blocking allocations. Hence, in some cases, whether the RD is treated as exogenous or endogenous will lead to different results. For the model with indivisibilities, one can derive some simple (numerical) restrictions on the RD that are necessary for the existence of an SSE.[4]

In the economy with restricted market participation, we have two sets of agents: the "old" who can insure against sunspot activity, and the "young" who cannot, since they are "born" after the realization of sunspot activity. An alternative interpretation of these restrictions on market participation is

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[3] Préchac [15] studies invariance to refinement in an incomplete-markets economy with *intrinsic* uncertainty. His assumptions about restrictions on trading across refined states – and hence his results – differ substantially from ours.

[4] See Proposition 1 in Shell and Wright [17], which can be seen as providing (severe) conditions on the RD necessary for existence SSE.

that there are two sets of agents: the "uninformed" (or unrestricted) and the "informed" (or restricted). Through this interpretation, our first core is related to the *core with differential information* (Wilson [20]) and specifically to the *private core* (Yannelis [21]), in which the blocking allocations must be measurable with respect to the private information of the coalition members (see also Allen [1] and Koutsougeras and Yannelis [11]). In the sunspots economy, there is no intrinsic uncertainty. In this model, the only information at issue is about the realization of the sunspot variable. Hence our first core is a specialization (to extrinsic uncertainty) of the private core. Our second core is *not* related to the core with differential-information; indeed, it seems to be new. (In Garratt and Qin [9], work that follows ours, their "lottery core" is very similar to our second core concept.) Furthermore, the focus of the literature on the differential-information core – namely, existence and information sharing – is quite different from ours.

We also analyze core convergence in the two economies. We establish convergence of each core in the restricted-market-participation economy; we provide examples of non-convergence in the indivisible-good economy.

The plan of the paper is as follows. In Section 2, we provide the description of the economic fundamentals and the extrinsic RD. In the next two sections, we define our robustness notions. In Section 5, we apply these notions to the convex restricted-market-participation economy. In Section 6, we apply the robustness notions to the (non-convex) indivisible-good economy. In Section 7, we examine the convergence of the cores to the SSE.

2 The model

We analyze the pure-exchange economy. The *economic fundamentals* are the endowments and the preferences.

We begin with the certainty economy; the extension to include extrinsic uncertainty follows naturally. There are ℓ commodities indexed by i and n consumers indexed by h. The commodity space is $X \subset \mathbb{R}^{\ell}$. Consumption of consumer h, x_h, must lie in his consumption set $X_h \subset X$. Preferences are represented by the utility function $u_h(x_h)$, i.e.,

$$u_h : X_h \to \mathbb{R}.$$

The endowments of consumer h lie in the commodity space, i.e., we have

$$\omega_h \in X.$$

In order to allow for the possibility of stochastic allocations, we introduce the probability space, or *randomizing device* (RD),

$$(S, \Sigma, \pi),$$

where S is a set of states s, Σ is a σ-algebra of subsets called events, and π is the probability measure. Without any apparent loss of generality, we fix S as the unit interval, i.e., $S = [0, 1]$. Hence, when we talk about changing the randomizing device, it should be interpreted as changes in the σ-algebra (which defines events) over the fixed state space S. In a part of the analysis, we will consider a finite partition of the state space S. We will denote the partition as $\mathscr{A}$.

Uncertainty is said to be *extrinsic* if it does not affect the economic fundamentals (see Cass and Shell [5, p. 196]). We assume that the randomizing device (S, Σ, π) represents extrinsic uncertainty. It is a "commonly observed" randomizing device. We do not treat here the general case in which the "information structure" Σ might depend on h, although we come close in our concepts of the core.[5]

With extrinsic uncertainty, the commodity space is the set of Σ-measurable functions, $x : S \rightarrow X$, bounded in the essential supremum norm. Let this space be denoted by $L_\infty(S, \Sigma, \pi)$.[6] Prices, $\tilde{p}$, are thus measurable functions bounded in the L_1 norm with the following interpretation: For any set $A \in \Sigma$, $\int_A p^i(s)\,\pi\,(ds)$ is the cost of one unit of good i to be delivered when event A occurs. The consumption set, Z_h, is the set of functions such that $x_h(s) \in X_h$, for all s. The state-symmetric utility function over the new consumption set is denoted as $v_h(x_h(s))$. The endowment of consumer h is a constant map $\omega_h : S \rightarrow X$ with $\omega_h(s) = \omega_h$ for each $s \in S$ and $h = 1, \ldots, n$.

We denote this economy (with extrinsic uncertainty) by

$$\mathscr{E}\left[(u_h, \omega_h)_{h=1,\ldots,n}, (S, \Sigma, \pi)\right].$$

2.1 Definition

A *feasible allocation* for the economy $\mathscr{E}\left[(u_h, \omega_h)_{h=1,\ldots,n}, (S, \Sigma, \pi)\right]$ is a collection of Σ-measurable functions, $x_h(s)$, $x_h(s) \in Z_h, h = 1, \ldots, n$, such that for each $s \in S$

$$\sum_h x_h\,(s) \leq \sum_h \omega_h.$$

Consumers maximize their relevant utilities subject to their budget constraints. The form of this maximization problem will depend on the particular model we examine.

2.2 Definition

A *sunspot equilibrium (SSE)* for the economy $\mathscr{E}[(u_h, \omega_h)_{h=1,\ldots,n}, (S, \Sigma, \pi)]$ is a combination of the allocations, the randomizing device, and a non-negative pricing function, $\tilde{p}^*(\cdot)$, normalized such that $\int \sum_i \tilde{p}^{i*}\,(s)\,\pi\,(ds) = 1$, where

[5] See Peck and Shell [14] for the analysis of *asymmetric-information* sunspot equilibria.

[6] See the discussion in Bewley [4], Duffie [7], Stokey and Lucas [19], and in footnote 10 of Shell and Wright [17].

(i) $x_h^*(\tilde{p}^*(\cdot))$ solves the consumer maximization problem

and

(ii) $\Sigma_h x_h^*(\tilde{p}^*(s)) \leq \Sigma_h \omega_h$.

We denote the set of SSE allocations for the economy $\mathscr{E}\left[(u_h, \omega_h)_{h=1,\ldots,n}, (S, \Sigma, \pi)\right]$ by

$$E\left[(u_h, \omega_h)_{h=1,\ldots,n}, (S, \Sigma, \pi)\right].$$

3 Robustness to refinements of the randomizing device

We begin with the formal definition of refinement.

3.1 Definition

A randomizing device $\left(S, \widehat{\Sigma}, \widehat{\pi}\right)$ is said to be a *refinement* of the randomizing device (S, Σ, π) if we have

(i) $\Sigma \subset \widehat{\Sigma}$, i.e., every Σ-measurable set is $\widehat{\Sigma}$-measurable,

and

(ii) If A is a Σ-measurable set with measure $\pi(A)$, then $\widehat{\pi}(A) = \pi(A)$.

The set of refinements of (S, Σ, π) is denoted by $Ref(S, \Sigma, \pi)$, and hence if $(S, \widehat{\Sigma}, \widehat{\pi})$ is a refinement of (S, Σ, π), we have

$$\left(S, \widehat{\Sigma}, \widehat{\pi}\right) \in Ref(S, \Sigma, \pi).$$

In defining refinement, we are taking Σ to be a sub σ-algebra of $\widehat{\Sigma}$. If S is a finite set, then our definition coincides with the usual definition (see e.g. Duffie [7, p. 83]). Next, we provide two examples of refinements.

3.2 Example

Let (S, Σ, π) be defined by the partition $\mathscr{A} = \{\alpha, \beta\}$ with $\alpha \cap \beta = \emptyset$, $\alpha \cup \beta = [0, 1]$, and $\pi(\alpha) + \pi(\beta) = 1$. Define $\left(S, \widehat{\Sigma}, \widehat{\pi}\right)$ by $\hat{\mathscr{A}} = \{\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3\}$ with $\alpha_1 \cup \alpha_2 = \alpha$, $\alpha_1 \cap \alpha_2 = \emptyset$, $\beta_1 \cup \beta_2 \cup \beta_3 = \beta$, $\beta_1 \cap \beta_2 = \emptyset, \beta_1 \cap \beta_3 = \emptyset$, $\beta_2 \cap \beta_3 = \emptyset$, and $\widehat{\pi}(\alpha_1) + \widehat{\pi}(\alpha_2) = \pi(\alpha)$, $\widehat{\pi}(\beta_1) + \widehat{\pi}(\beta_2) + \widehat{\pi}(\beta_3) = \pi(\beta)$. Then we have $\left(S, \widehat{\Sigma}, \widehat{\pi}\right) \in Ref(S, \Sigma, \pi)$. □

3.3 Example

Let (S, Σ, π) be defined by $\Sigma = \{S, \emptyset\}$. Consider a partition $\hat{\mathscr{A}} = \{A_1, A_2, \ldots, A_r\}$, and $A_i \cap A_j = \emptyset$ if $i \neq j$, $\cup A_i = S$, and $\widehat{\pi}(A_1) + \widehat{\pi}(A_2) + \ldots + \widehat{\pi}(A_r) = 1$. Then $\left(S, \widehat{\Sigma}, \widehat{\pi}\right) \in Ref(S, \Sigma, \pi)$. □

3.4 Definition

The equilibrium allocations of the economy $\mathscr{E}\left[(u_h, \omega_h)_{h=1,\ldots,n}, (S, \Sigma, \pi)\right]$ are said to be *robust to refinement* if for every randomizing device (S, Σ, π) and each $(S, \widehat{\Sigma}, \widehat{\pi}) \in Ref(S, \Sigma, \pi)$, we have

$$E\left[(u_h, \omega_h)_{h=1,\ldots,n}, (S, \Sigma, \pi)\right] \subset E\left[(u_h, \omega_h)_{h=1,\ldots,n}, \left(S, \widehat{\Sigma}, \widehat{\pi}\right)\right].$$

Thus, in an economy that is robust to refinements, we never "lose" SSE by refining the RD (but we might "introduce" new SSE allocations by refining the RD.).

Next, we provide an illustration of robustness to refinement.

3.5 Illustration

Consider an economy in which the initial randomizing device and the refinement are given in Example (3.2). Let $\left\{(x_h^*(\alpha),\ x_h^*(\beta))_{h=1,\ldots,n}\right\}$ be an equilibrium allocation in the economy based on the initial randomizing device. If, in the economy based on the refined randomizing device, there is an equilibrium allocation $\left\{(x_h(\alpha_1), x_h(\alpha_2), x_h(\beta_1), x_h(\beta_2), x_h(\beta_3))_{h=1,\ldots,n}\right\}$ with the property that

$$x_h(\alpha_1) = x_h(\alpha_2) = x_h^*(\alpha) \text{ and } x_h(\beta_1) = x_h(\beta_2) = x_h(\beta_3) = x_h^*(\beta),$$

then the SSE $\left\{(x_h^*(\alpha), x_h^*(\beta))\right\}$ is robust to this refinement. If each SSE allocation is robust to every refinement, then the economy is robust to refinements. □

4 Core robustness

Sunspot equilibria are meant to capture "market uncertainty," in which individuals are uncertain about the beliefs and acts of others.[7] Coalitions of individuals could possibly form to mitigate the effects of this uncertainty. For example, consumers might group together in insurance cooperatives, gambling clubs, and so forth. Would the existence of such institutions affect the sunspot equilibria? To answer this type of question leads us to study the relationship between SSE allocations and core allocations.

The classical approach to defining the core for an economy with uncertainty is to fix the RD as part of the economic fundamentals. In the spirit of Arrow-Debreu, the randomizing device defines the commodity space. We denote by $Core(S, \Sigma, \pi)$ the set of allocations that cannot be blocked in this (Arrow-Debreu) commodity space.

For our purposes, the classical core concept is too weak. It does not take advantage of the fact that the uncertainty is extrinsic. A sunspot allocation could be thought of by the consumer as a conventional lottery. Under this interpretation, the allocation is composed of bundles of commodities *and* the

[7] See Peck and Shell [14].

probabilities at which they will be delivered. In forming coalitions, consumers are free to select a randomizing device different from the one generating the prevailing SSE. The set of unblocked allocations in which coalitions are free to choose their own randomizing device is denoted by *Core*(S). The formal definitions of these two sets follow.

4.1 Definition

An allocation $\{(x_h^*(s))_{h=1,\ldots,n}\}$ is in $Core(S, \Sigma, \pi)$ if there does not exist a coalition C which can improve upon the allocation. A coalition C can improve upon an allocation $\{(x_h^*(s))_{h=1,\ldots,n}\}$ if there exists another Σ-measurable allocation $\{(x_h(s))_{h=1,\ldots,n}\}$ with

(i) $\Sigma_{g\in C}\, x_g(s) \leq \Sigma_{g\in C}\, \omega_g$,

and

(ii) $v_g(x_g(s)) \geq v_g(x_g^*(s))$ for $g \in C$,

where in (ii) the utility v_g is evaluated relative to (S, Σ, π) and at least one inequality is strict.

4.2 Definition

An extended allocation, i.e., $\{(x_h^*(s))_{h=1,\ldots,n}\}$ *and* the randomizing device (S, Σ^*, π^*) is in *Core*(S) if there does not exist a coalition C which can improve upon the extended allocation. A coalition C can improve upon an extended allocation $\{(x_h^*(s))_{h=1,\ldots,n}\}$ *and* (S, Σ^*, π^*) if there exists another extended allocation, $\{(x_h(s))_{h=1,\ldots,n}\}$ *and* (S, Σ, π), such that $\{(x_g(s))_{g\in C}\}$ are Σ-measurable and

(i) $\Sigma_{g\in C}\, x_g(s) \leq \Sigma_{g\in C}\, \omega_g$,

(ii) $\left(S, \widehat{\Sigma}, \widehat{\pi}\right)$ is a probability space,

and

(iii) $v_g(x_g(s)) \geq v_g(x_g^*(s))$ for $g \in C$,

where in (iii) the utility v_g is evaluated relative to the respective RDs and at least one inequality is strict.

The following lemma formalizes the fact that the second core concept is the stronger one.

4.3 Lemma

We have the inclusion

$$Core(S) \subset Core(S, \Sigma, \pi)\,.$$

Proof. Immediate from Definitions (4.1)–(4.2). □

5 Restricted market participation

In the restricted market participation economy of Cass and Shell [5], the set of consumers H is divided into two groups, G^0 and G^1. The group G^0 can be interpreted as the ''old'' generation born before the realization of uncertainty. This group can buy and sell on the contingent-claims market which meets before the realization of uncertainty. These consumers maximize expected utility subject to a single budget constraint by choosing a function $x_h(s)$ that

$$\text{Maximizes } v_h(x_h) = \int_s u_h(x_h(s))\pi(ds)$$
$$\text{subject to } \int_s \hat{p}(s) \cdot x_h(s)\,\pi(ds) \leq \int_s \hat{p}(s) \cdot \omega_h \pi(ds).$$

If s has a density function $\varphi(s)$, then this maximization problem can be rewritten as

$$\text{Maximizes } v_h(x_h) = \int_s u_h(x_h(s))\varphi(s)ds$$
$$\text{subject to } \int_s p(s) \cdot x_h(s)ds \leq \int_s p(s) \cdot \omega_h ds,$$

where $p(s) \equiv \tilde{p}(s)\varphi(s)$ and the ith component, $p^i(s)$, of $p(s)$ is the price of good i in state s. If the partition of the set of states is finite, we have the familiar budget constraint for unrestricted consumers

$$\sum_s p(s)\ .\ x_h(s) \leq \sum_s p(s) \cdot \omega_h,$$

where $p(s) = \tilde{p}(s)\ \pi\ (s)$.[8]

The second group, G^1, can be interpreted as the ''young'' generation born after the realization of the uncertainty. The ''young'' are restricted from participating on the contingent-commodity market.[9] They maximize spot-market, *ex-post* utilities subject to state-by-state budget constraints, i.e., they choose $x_h(s)$ to solve

$$\text{Maximize } u_h(x_h(s))$$
$$\text{subject to } \tilde{p}(s)\ .\ x_h(s) \leq \tilde{p}(s) \cdot \omega_h$$

for each s.

The standard regularity assumptions are adopted for the restricted-market-participation, certainty economy: the consumption set, X_h, is $\mathbb{R}^{\ell}_{+}$, the utility functions u_h are twice continuously differentiable, differentially mo-

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[8] There is some abuse of notation here since s is also being used to denote the generic element of the partition $\mathscr{A}$.

[9] Consumers from G^0 can also trade on spot markets. If rational expectations are assumed, this will not alter the analysis, see Cass and Shell [5, Proposition 2].

notonic and strictly concave, the endowments ω_h lie in the interior of the consumption set, i.e., we have $\omega_h \in \mathbb{R}^\ell_{++}$, and the closure of each indifference surface $\{x_h \in \mathbb{R}^\ell_{++} \mid u_h(x_h) = \bar{u}\}$ lies in $\mathbb{R}^\ell_{++}$. These assumptions extend in a standard way to the economy with extrinsic uncertainty.

In the next two theorems, we establish that the restricted-market-participation economy is robust to refinements of the RD. The first theorem allows for only finite RDs. The second theorem extends the result to include continuous RDs.

5.1 Theorem

In the restricted-market-participation economy based on a finite randomizing device, the equilibria are robust to refinement, i.e., we have

$$E\Big[(u_h, \omega_h)_{h=1,\dots,n}, (S, \Sigma, \pi)\Big] \subset E\Big[(u_h, \omega_h)_{h=1,\dots,n}, (S, \widehat{\Sigma}, \widehat{\pi})\Big].$$

Proof
Let the partition $\mathscr{A}$ of the state space be finite and the distribution over it be given by some probability measure π, $\pi(s) > 0$, $\Sigma_{s\in\mathscr{A}}\pi(s) = 1$. For each unrestricted consumer, $h \in G^0$, each event $s \in \mathscr{A}$ and each $i = 1, \dots, n$, we have the first-order condition,

$$\pi(s)\frac{\partial u_h(x_h^*(s))}{\partial x_h^i(s)} = \lambda_h p^{i*}(s)$$

and the budget constraint

$$\sum_{s\in\mathscr{A}} p^*(s) \cdot x_h^*(s) = \sum_{s\in\mathscr{A}} p^*(s) \cdot \omega_h.$$

For each restricted consumer, $h \in G^1$, each event $s \in \mathscr{A}$ and each good $i \in \{1, \dots, n\}$, we have the first-order condition

$$\frac{\partial u_h(x_h^*(s))}{\partial x_h^i(s)} = \lambda_h(s) p^{i*}(s)$$

and the budget constraint

$$p^*(s) \cdot x_h^*(s) = p^*(s) \cdot \omega_h.$$

These first-order conditions are necessary and sufficient because of the "convexity" of the maximization problems; hence, these equations characterize the set of SSEs for the RD defined above.

Without loss of generality, consider a refinement of the RD where each event s is partitioned into $t = 1, \dots, T$ sub-events, s_t, with probabilities $\pi(s_t) > 0$, $\Sigma_t \pi(s_t) = \pi(s)$, $s = 1, \dots, \#\mathscr{A}$. Consider the prices, $\hat{p}^i(s_t) = (\pi(s_t)/\pi(s))p^{i*}(s)$, $s = 1, \dots, \#\mathscr{A}$, $t = 1, \dots, T$. It is easy to check that at these prices, the allocation defined by $x_h(s_t) = x_h^*(s)$, $t = 1, \dots, T$, $h \in G_0 \cup G_1$, satisfy the equations that characterize the set of SSEs for the refined RD.□

In the general case, with a continuous probability space, a similar argument applies. As in the finite dimensional case, the first-order conditions together with the market-clearing conditions are sufficient for an equilibrium, see Mas-Colell and Zame [12]. In the continuous case, the first-order conditions are similar to those in Theorem 5.1 after changing the summation signs to integrals.

5.2 Theorem

In the restricted-market-participation economy, the equilibria are robust to refinement.

Proof

Proceed exactly as in the proof of Theorem 5.1, except that the definition of the prices is different. Note that the Kuhn-Tucker theorem continues to apply in the larger commodity space under the assumption that we use only RDs for which there is a continuous probability density function. Let the density functions over the two state spaces are $\varphi(s)$ and $\widehat{\varphi}(s)$ respectively and define $\hat{p}(s)$ by $\hat{p}(s) = (\widehat{\varphi}(s)/\varphi(s))p(s)$. □

5.3 Corollary

The restricted-market-participation economy is robust to refinement.

Proof. From Theorems 5.1 and 5.2, we have

$$E\Big[(u_h, \omega_h)_{h=1,\dots,n}, (S, \Sigma, \pi)\Big] \subset \bigcap_{(S,\hat{\Sigma},\hat{\pi}) \in \, Ref\,(S,\Sigma,\pi)} E\Big[(u_h, \omega_h)_{h=1,\dots,n}, (S, \widehat{\Sigma}, \widehat{\pi})\Big]. \quad \square$$

In the next two remarks, we compare our results to related results in Cass and Shell [5].

5.4 Remark

In Cass and Shell [5, Proposition 4], it is shown that a certainty equilibrium always reappears as a (degenerate) SSE. In our terminology [see Example (3.3.)], this reduces to

$$E\Big[(u_h, \omega_h)_{h=1,\dots,n}, (S, \{S, \emptyset\}, \pi)\Big] \subset \bigcap_{(S,\hat{\Sigma},\hat{\pi}) \in \, Ref(S,\Sigma,\pi)} E\Big[(u_h, \omega_h)_{h=1,\dots,n}, (S, \widehat{\Sigma}, \widehat{\pi})\Big].$$

Our Theorem 5.2 is thus a generalization of this Cass-Shell result.

5.5 Remark

The Cass-Shell immunity theorem [5, Proposition 3] states that when there are no restricted consumers, i.e., $G^1 = \emptyset$ and $G^0 = H$, sunspots cannot matter. An immediate consequence of this result is that we have

$$E\left[(u_h, \omega_h)_{h=1,\dots,n}, (S, \{S, \emptyset\}, \pi)\right] = E\left[(u_h, \omega_h)_{h=1,\dots,n}, (S, \widehat{\Sigma}, \widehat{\pi})\right]$$

for any $(S, \widehat{\Sigma}, \widehat{\pi}) \in Ref(S, \Sigma, \pi)$ whenever we have $G^1 = \emptyset$. For the *special case* of unrestricted market participation, Theorem 5.2 is weaker than the Cass-Shell immunity theorem.

This completes our analysis of the robustness to refinements of the restricted-market-participation economy. We turn next to the analysis of robustness to cooperative-coalition formation in the same economy. For this purpose, it will be convenient to transform the restricted-market-participation model into a quasi-Walrasian model.[10] With each consumer who is restricted from participating on the contingent-claims market, we associate in the transformed model one quasi-Walrasian consumer for each state of nature. These consumers consume only in their "own" state, and have non-zero endowments only in that state. The essential thing is that they have a single budget constraint; formally we are now in an Arrow-Debreu economy in which some of the consumers have consumption sets that are a strict subset of the full commodity space. However, the results on existence, efficiency, and the core in the standard case do not hinge on the consumption sets being the entire commodity space as long as the economy exhibits either resource-relatedness or irreducibility.[11] The presence of unrestricted consumers guarantees resource-relatedness and irreducibility. Formally, with each consumer $h \in G^1$ and each state $s \in S$, associate the quasi-Walrasian consumer hs. The consumption set of consumer hs is the set of bounded Σ-measurable functions such that $x_{hs}(s') \in X_h$, for $s' = s$ and 0 otherwise. The utility function and endowments are now derived appropriately for these consumers. The utility functions satisfy $v_{hs}(x_{hs}) = u_h(x_h(s))$; the endowments satisfy $\omega_{hs}(s') = \omega_h$ if $s = s'$ and 0 otherwise.

In the next definition and the next theorem, we take the RD as fixed.

5.6 Definition

In the restricted-market-participation economy, an allocation $\{(x_h^*(s))_{h=1,\dots,n}\}$ is in $Core(S, \Sigma, \pi)$ if there does not exist a coalition C which can improve upon the allocation. A coalition C can improve upon an allocation $\{(x_h^*(s))_{h=1,\dots,n}\}$ if there exists another Σ-measurable allocation $\{(x_h(s))_{h=1,\dots,n}\}$ with

(i) $\Sigma_{g\in C}\, x_g(s) = \Sigma_{g\in C}\, \omega_g$,

(ii) $\int_s u_g(x_h(s))\pi(ds) \geq \int_s u_g(x_g^*(s))\ \pi(ds)$ for $g \in G^0 \cap C$,

[10] See Balasko, Cass, and Shell [3] for the background on the role of quasi-Walrasian economies in the analysis of restricted-participation economies.

[11] See McKenzie [13] on irreducibility and Arrow and Hahn [2] on the closely related concept of resource-relatedness.

and

(iii) $u_g(x_g(s)) \geq u_g(x_g^*(s))$ for $g \in G^1 \cap C$,

with at least one strict inequality in (ii) or (iii).

5.7 Theorem

In the restricted-market-participation economy every SSE allocation based on the RD (S, Σ, π) is in $Core(S, \Sigma, \pi)$, i.e., we have

$$E\Big[(u_h, \omega_h)_{h=1,\ldots,n}, (S, \Sigma, \pi)\Big] \subset Core(S, \Sigma, \pi).$$

Proof. A standard argument will show that a competitive equilibrium allocation is in the core of the economy with quasi-Walrasian consumers. This will correspond to the core with some restricted consumers defined in (5.6).

□

Note that allocations in $Core(S, \Sigma, \pi)$ are Pareto-optimal with respect to the quasi-Walrasian consumers but not necessarily with respect to the natural consumers. See Cass and Shell [5, p. 217].

In what follows, the coalitions are able to choose their own RD. In this case, the RD is also a part of the (extended) allocation. We can think of RDs chosen this way as *endogenous randomizing devices.*

5.8 Definition

An extended allocation, i.e., $\{(x_h^*(s))_{h=1,\ldots,n}\}$ *and* the randomizing device (S, Σ^*, π^*), is in $Core(S)$ if there does not exist a coalition C which can improve upon the extended allocation. A coalition C can improve upon an extended allocation $\{(x_h^*(s))_{h=1,\ldots,n}\}$ *and* (S, Σ^*, π^*) if there exists another extended allocation $\{(x_h(s))_{h=1,\ldots,n}\}$ and $(S, \Sigma, \pi,)$ such that $\{(x_g(s))_{g\in C}\}$ are Σ-measurable and

(i) $\Sigma_{g\in C}\, x_g(s) = \Sigma_{g\in C}\, \omega_g$,

(ii) (S, Σ, π) is a probability space,

(iii) $\displaystyle\int_s u_g(x_g(s))\pi(ds) \geq \int_s u_g(x_g^*(s))\pi^*(ds)$ for $g \in G^0 \cap C$,

and

(iv) $u_g(x_g(s)) \geq u_g(x_g^*(s))$ for $g \in G^1 \cap C$,

with at least one strict inequality in (iii) or (iv).

The next theorem shows that every SSE from the restricted-market-participation economy is in $Core(S)$. We sketch the proof for the case in which the partition of S is finite; it will become apparent how this can be extended to the continuous state space.

5.9 Theorem

Let (S, Σ, π) be a finite RD. Consider an SSE allocation based on (S, Σ, π). This extended allocation is in *Core*(S).

Sketch of the proof

(i) From a utility viewpoint, what is important for the consumers is the probability of getting specified bundles. Let us say, that there are K elements in the partition $\mathscr{A}$ of S, $s_1, \ldots, s_k, \ldots, s_K$, with probabilities, $\pi(s_1), \ldots, \pi(s_k), \ldots, \pi(s_K)$, and $\pi(s_k) > 0$ for $k = 1, \ldots, K$. Consider an SSE allocation in this economy, $(x_h(s_1), \ldots, x_h(s_k), \ldots, x_h(s_K))$. This equilibrium allocation is equivalent to an equilibrium in the economy with a continuum of states, [0,1], and with quasi-Walrasian consumers. In the new economy, now with a continuum of states and hence commodities, the allocations are probability measures. The corresponding equilibrium in the new economy is as follows. With probability $\pi(s_k)$ consumer $h \in G^0$ receives allocation $x_h(s_k)$. Consumer hs_k with $h \in G^1$ receives $x_h(s_k)$ with probability $\pi(s_k)$ and receives the bundle 0 with probability $1 - \pi(s_k)$.
(ii) Theorem (5.2) implies that this new allocation will be an equilibrium allocation in the new economy.
(iii) *Any* sunspot allocation for *any* randomizing device can be represented in the same fashion.
(iv) The new economy (with quasi-Walrasian consumers and a continuum of states) is an Arrow-Debreu economy and hence every sunspot equilibrium will be in the core.
(v) Thus, with suitable interpretation, every SSE of the economy with the fixed RD (S, Σ, π) is also in the core of the economy where S is given by a uniform density over the interval [0,1]. In other words, the original equilibrium allocation cannot be blocked by an extended allocation, either with a finite or continuous probability device. □

One of the implications of this theorem is that, if the consumers are initially using some non-degenerate sunspot device, then no coalition will be able to block the allocation contingent on that RD by trying to replace or even "get rid of" the extrinsic uncertainty. While the initial outcome may not be *ex-ante* Pareto efficient, i.e., in the *ex-ante* sense everyone can be made better off by having an "average" allocation constant across the different states, it will still be *dynamically* Pareto efficient (Cass and Shell [5, Proposition 6]). Some quasi-Walrasian consumer hs gets an allocation $x_h(s)$ that is better than the "average" allocation for consumer h, $\bar{x}_h$, i.e., for some h and some s, we have $u_h(x_h(s)) > u_h(\bar{x}_h)$.

The same argument applies if the initial RD is continuous. In the relevant infinite-dimensional commodity space, the result that every competitive equilibria is in the core is still true (see Bewley [4]).

6 Consumption indivisibilities

We have established that the SSE from the convex restricted-market-participation economy are robust to refinements in the RD and robust to the formation of cooperative coalitions, even when coalitions can introduce their own RDs. These results do not extend to all sunspots economies. We show in this section that things are quite different in the (nonconvex) indivisible-good economy of Shell and Wright [17].

In the indivisible-good economy, the certainty consumption sets, $X_h \subset \mathbb{R}_+$, consist of two points, $X_h = \{0, 1\}$, while the endowments $\omega_h \in (0, 1]$ need not lie in X_h. The identical preferences are given by a monotonic utility function normalized so that

$$u_h(0) = 0 \text{ and } u_h(1) = b > 0.$$

It is assumed that every consumer has access to the contingent-claims market, i.e., we have

$$H = G^0 \text{ and } G^1 = \emptyset.$$

By introducing the randomizing device (S, Σ, π), the model is extended in the natural way to allow for SSE allocations and contingent claims prices.

In the following, we provide examples of the failure of robustness and comment on their importance.

6.1 Example (In the indivisible-good economy, an SSE which is not robust to refinements of the RD.)

Consider an economy with a continuum of non-atomic consumers. There are two types, $t = 1, 2$, of consumers each with unit mass. Let the density of each type be uniform on the unit interval. Each consumer of type t has an endowment of ω_t. Assume further that we have $\Sigma_t \omega_t = 1$; in particular, assume that we have $\omega_1 = 8/9$ and $\omega_2 = 1/9$.

First, consider the case of only one state of nature (no extrinsic uncertainty). The unique equilibrium allocation is $(x_1^*, x_2^*) = (0, 0)$.

Now partition the event into two mutually exclusive sub-events, $\mathscr{A} = \{\alpha, \beta\}$. Assume that the probabilities are equal, i.e., $\pi(\alpha) = \pi(\beta) = 1/2$. Then there is a unique (up to a relabelling of the sub-events) SSE in which the allocation is given by

$$[x_1(\alpha),\ x_1(\beta)] = (1, 0) \text{ and } [x_2(\alpha),\ x_2(\beta)] = (0, 1)$$

and the prices satisfy $\hat{p}(\alpha) = \hat{p}(\beta) = 1/2$.

Now consider a further refinement of the RD to the unit interval with uniform density. Then, up to a relabelling, there is a unique sunspot equilibrium with the following properties

$$\text{(i) } p(s) = 1 \ a.s.$$
and
$$\text{(ii) } \text{prob}(x_t(s) = 1) = \omega_t.$$

(This is the content of Proposition 3 in Shell and Wright [17].) Thus the equilibrium allocation has the type-1 consumers getting one unit with probability 8/9 as opposed to probability 1/2 in the earlier equilibrium. □

The example we have given relies on consumption indivisibilities. They make for simple examples, but they are not essential. An example is given in Goenka and Shell [10], in which there are no non-convexities on the consumption side, but there are indivisibilities on the production side (one input – labor – is useful to the firm only in discrete *individual-specific* amounts) that shows that the certainty economy has no equilibrium but that adding sunspot states restores existence in a way similar to that in Example 6.1.

Now we turn to the analysis of robustness to cooperative-coalition formation in the indivisible-good economy. Examples 6.2 and 6.3 show that it is possible for a coalition to benefit by replacing the existing RD by another one. Example 6.3 shows that a given allocation may be in the core relative to the ruling RD but not in the core without restructions on the RD.

6.2 Example (An SSE allocation in the indivisible-good economy in neither core.)

Consider an economy with two types of consumers, and 10 consumers of each type. The endowments of the two types are given by

$$\omega_1 = 9/10 \text{ and } \omega_2 = 1/10.$$

If we look at the RD defined by the partition $\mathscr{A} = \{\alpha, \beta\}$, $\pi(\alpha) = \pi(\beta) = 1/2$, then the following SSE allocation is not in the core based on this RD:

$$\left(x^*_{1,n}(\alpha), x^*_{1,n}(\beta)\right) = (1, 0) \text{ and } \left(x^*_{2,n}(\alpha), x^*_{2,n}(\beta)\right) = (0, 1),$$

where $x^*_{t,n}$ is the allocation of the *nth* consumer of type t, $t = 1, 2$, and $n = 1, \ldots, 10$. This allocation is blocked by any coalition of 10 type-1 consumers. The blocking allocation is given by

$$(x_{1,n}(\alpha), x_{1,n}(\beta)) = (1, 1) \text{ for } n = 1, \ldots, 8,$$
$$(x_{1,9}(\alpha), x_{1,9}(\beta)) = (1, 0),$$
and
$$(x_{1,10}(\alpha), x_{1,10}(\beta)) = (0, 1).$$

Next, let the potential coalitions choose their own RDs. A coalition of the 10 type-1 consumers can block the SSE allocation. The RD employed by the coalition consists of 10 equiprobable events. The allocation for the coalition is given by the 10×10 matrix A, where the element a_{hs} is the allocation of consumer h in event s,

$$A = (a_{hs}) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}.$$

In this allocation, each consumer in the blocking coalition is *strictly* better off since he gets $x_{1,n}(s) = 1$ with probability 9/10 (rather than probability 1/2 as he does in the SSE allocation). □

The second part of Example 6.2 is largely redundant in light of Lemma 4.3, except that the blocking coalition is *strictly* better off in the second part.

6.3 Example (An SSE allocation in the indivisible-good economy in Core(S, Σ, π), but not in Core(S).)

Consider an economy with two types of consumers, and 4 consumers of each type. The endowments of the two types are given by

$$\omega_1 = 2/3 \text{ and } \omega_2 = 1/3.$$

If we look at the RD defined by the partition $\mathscr{A} = \{\alpha, \beta\}$, $\pi(\alpha) = 1/2$ and $\pi(\beta) = 1/2$, then the following SSE allocation is in the core based on this RD:

$$(x_{1,n}(\alpha), x_{1,n}(\beta)) = (1, 0) \text{ and } (x_{2,n}(\alpha), x_{2,n}(\beta)) = (0, 1),$$

where $x_{t,n}$ is the allocation of the nth consumer of type t, $t = 1, 2$ and $n = 1, \ldots, 4$.

Next allow coalitions to choose their own RDs in proposing blocking allocations. The allocation above is not in *Core*(S): it can be blocked by a coalition of 4 consumers of type 2 and 1 consumer of type 1, and the following allocation: each consumer in the blocking coalition receives one unit of the good with probability 3/5. □

7 Core convergence

In this section, we show that as the convex restricted-market-participation economy is replicated, *Core*(S) and *Core*(S, Σ, π) shrink to the set of SSE allocations. Since *Core*$(S) \subset$ *Core*(S, Σ, π), convergence of the latter implies convergence of the former. The proof of this follows in a straightforward way from of Debreu and Scarf [6, Theorem 3]. We know from Theorem 5.8 that an SSE allocation is in *Core*(S, Σ, π). If the equal treatment property holds and the assumptions of Debreu and Scarf are satisfied, their theorem would be applicable. To show that Debreu-Scarf is applicable, we again use the quasi-Walrasian consumers trick.

The partition $\mathscr{A}$ of S is assumed to be finite. The regularity assumptions made on the characteristics of the economy in Section 5 are maintained. In the quasi-Walrasian economy, with every consumer $h \in G^1$ and $s \in \mathscr{A}$ we associate the quasi-Walrasian consumer hs. Consider replica economies. In the v-th fold replication, there are $H' = \#G^0 + (\#\mathscr{A})(\#G^1)$ types of quasi-Walrasian consumers, with v consumers of each type. Thus, there are $H'v$ consumers in the v-fold replica economy. We will denote a consumer by the pair (h, q) with $h \in H'$ and $q \in \{1, \ldots, v\}$. Each of the v consumers of a given type has the same consumption set, preferences, and endowment.

7.1 Lemma

For the restricted-market-participation model with the randomizing device (S, Σ, π), an allocation in $Core(S, \Sigma, \pi)$ assigns the same consumption to each consumer of the same type.

Proof. This follows the same argument as in Debreu and Scarf [6] after we replace the restricted consumers with their quasi-Walrasian counterparts. Since each quasi-Walrasian consumer of type hs is treated equally, this will imply that each consumer of type $h \in G^l$ is also treated equally. □

7.2 Lemma

For the restricted-market-participation economy with the randomizing device (S, Σ, π), if an allocation $\{(x_h)_{h=1,\ldots,n}\}$ is in $Core(S, \Sigma, \pi)$, then in the quasi-Walrasian economy we have

$$x_{hs}(s') = 0 \text{ for } s' \neq s \text{ and } h \in G^1.$$

Proof. This follows from the definition of the quasi-Walrasian consumer, who does not gain utility from consumption outside "his" state. □

We next formally state and prove the core-convergence result.

7.3 Theorem

If the allocation $\{(x_{h,q})_{h=1,\ldots,n,\ q=1,\ldots,v}\}$ is in $Core(S, \Sigma, \pi)$ for all v, then it is an SSE allocation.

Proof. Consider the economy with quasi-Walrasian consumers. Note that an allocation in $Core(S, \Sigma, \pi)$ is in the (ordinary) core of the quasi-Walrasian economy. Let Γ_h be the set of all $z \in X_h$ such that $z + \omega_h \succ_h x_h$, and let Γ = convex hull$(\cup_h \Gamma_h)$. Using the same argument as in Debreu and Scarf [6], we can show that $0 \notin \Gamma$. Thus, there exists a hyperplane with normal p such that $p \cdot z \geq 0$ for all $z \in \Gamma$. Hence p is a compensated-equilibrium price vector for the constructed economy and $p \cdot x_h = p \cdot \omega_h$ for all h. To establish that we have found a competitive-equilibrium allocation for the constructed economy, we must show that x_h maximizes the preferences of the hth consumer. This follows from the fact that the endowments lie in the interior of the consumption set. Note that for $h \in G^1$ we have

$$\omega_{hs} = (0, \ldots, 0, \omega_h, 0, \ldots, 0)$$

where ω_h is the s-th component of ω_{hs}. But we also have

$$X_{hs} = (0, \ldots, 0, X_h, 0, \ldots, 0)$$

where X_h is the s-th component of X_{hs}, for $h \in G^1$.

If the allocation is a competitive-equilibrium allocation in the economy with quasi-Walrasian consumers, it is also an SSE in the original economy. □

To show core equivalence with a continuous state space (where we have an infinite number of quasi-Walrasian consumers, and an infinite number of event-specific commodities), the results of Bewley [4] may have been thought to apply since we use the same commodity space that he does. However, here unrestricted consumers are atoms and Bewley's result does not apply. Secondly, the results of Gabszewicz and Mertens [8] do not apply because the unrestricted consumers are "too big" in their sense. We conjecture that core convergence also holds when the state space is continuous.

Example 6.2 provides an instance of non-convergence of the core to the set SSE allocations. In the following example, an SSE allocation in the core of the economy is no longer in the core after sufficient replication of the economy.

7.4 Example (In the indivisible-good economy, an SSE allocation which is in $Core(S, \Sigma, \pi)$ (resp. $Core(S)$) for replication ν, may not be in $Core(S, \Sigma, \pi)$ (resp. $Core(S)$) for replication ν′, where $\nu' > \nu$.)

Consider an economy with two types of consumers, and 1 consumer of each type. The endowments of the two types are given by

$$\omega_1 = 9/10 \text{ and } \omega_2 = 1/10.$$

If we look at the RD defined by the partition $\mathscr{A} = \{\alpha, \beta\}$ and $\pi(\alpha) = \pi(\beta) = 1/2$, then the following allocation is in the core based on this RD:

$$(x_1(\alpha),\ x_1(\beta)) = (1, 0) \text{ and } (x_2(\alpha),\ x_2(\beta)) = (0, 1).$$

This allocation is also in $Core(S)$. Consider a 10-fold replication of the economy. Then this SSE allocation is in neither of the two cores (see Example 6.2). □

Our core convergence result is different from that suggested in Koutsougeras and Yannelis [11]: the equal-treatment property fails in their private core. Our core convergence result depends on the special structure of the restricted-market-participation sunspots model. Our non-convergence example follows in part from the failure of the equal-treatment property in the non-convex (sunspots) environment.

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