Economic Theory 9, 169–178 (1997)

Economic Theory

When sunspots don't matter*

Aditya Goenka[1] and Karl Shell[2]

[1] Department of Economics, University of Essex, Colchester CO4 3SQ, UK
[2] Department of Economics, Cornell University, Ithaca, NY 14853-7601, USA

Received: July 17, 1995; revised version November 30, 1995

Summary. We show that a finite, competitive economy is *immune* to sunspots if *(i)* preferences are strictly convex, *(ii)* the set of feasible allocations is convex, and *(iii)* the contingent-claims market is perfect. The conditions *(i)–(ii)* cover some, but not all, economies with nonconvex technologies. Based on an indivisible-good example, we show that even economies with strictly convex preferences and full insurance are *not* in general immune from sunspots. We also show that *(1)* the sufficient conditions *(i)–(iii)* are not necessary for sunspot immunity and *(2) ex-ante* efficiency is not necessary for immunity from sunspots.

JEL Classification Numbers: D51, D84, E32.

1 Introduction

When is an economy immune to the effects of sunspot activity? That is, under what conditions is it guaranteed that the equilibrium allocation is independent of extrinsic uncertainty?

The first *Sunspot Immunity Theorem* (Cass and Shell [4, Proposition 3]; see also Balasko [2, Theorem 1]) covers the competitive, pure-exchange economy with strictly convex preferences and full insurance against the effects of sunspot activity. The Cass-Shell immunity result is easily extended to include convex production. Two notes (Cass and Polemarchakis [3] and Sorger [14]) address the possibility of immunity from sunspots in the economy with nonconvex production.

In the present note, we provide a set of (sufficient) conditions that guarantee sunspot immunity in finite, competitive economies with production.

* This paper is based on an earlier paper, "Indivisibilities in Production, and Sunspot Equilibrium," presented at the 1990 S.E.D.C. Meetings, Minneapolis-St. Paul, June 1990. The research support of NSF Grant SES-9012780, the Center for Analytic Economics, and the Thorne Fund is gratefully acknowledged.

Correspondence to: A. Goenka

Essentially, our conditions that together imply sunspot immunity (in finite, competitive economies) are:

1. Preferences are strictly convex.
2. The set of feasible allocations (attainable states in the terminology of Debreu [5]) is convex.
3. Full insurance against the effects of sunspots is available to each economic agent.

We also demonstrate through examples that:

1. Strict convexity of preferences and full insurance are *not* sufficient for sunspot immunity.
2. Our set of sufficient conditions are *not* necessary for sunspot immunity.
3. *Ex-ante* Pareto-optimality of the equilibrium allocations is *not* necessary for sunspot immunity.

In addition, we recall that *even if the set of feasible allocations is convex*:

1. Strict convexity of preferences is *not* sufficient for sunspot immunity. (See Cass and Shell [4].)
2. Full insurance is *not* sufficient for sunspot immunity. (See Shell and Wright [13] and Guesnerie and Laffont [8].)

There is a sense in which our sunspot immunization conditions are widely applicable. The form of the immunity theorem we state and prove employs *ex-ante* Pareto optimality, but our approach works in a wider class of economies. An example is given in the present paper of an economy with excise taxes. This approach has also been applied in models with rationing (Goenka [6]), models with public goods (Goenka [7]), and with incomplete markets (Préchac [10]). In these and other cases, the equilibria will satisfy an appropriate notion of constrained optimality. The corresponding constrained optimal allocations are necessarily symmetric across the states of nature.[1]

On the other hand, there is a sense in which our immunity result is weak. A wide variety of important cases in public policy analysis include "strong" nonconvexities. These include increasing returns in production, indivisible goods, matching problems, and income taxation.

In the next section, we present the model, state and prove the sunspot immunity theorem, and provide our examples.

2 Sunspots and immunity

Our model and notation is chosen to be as close as possible to that in Balasko [2] and Cass and Polemarchakis [3]. Let there be $s = 1 \ldots, T$ states of nature occurring with equal probability. Denote the set of states by S. In each state there are $\ell = 1, \ldots, L$ commodities. The commodity space is, thus, $X \subset \Re^{LT}$. Let G be the group consisting of bijections on S. It operates on the commodity

[1] Compare with the account of the "Philadelphia Pholk Theorem" in Shell [12].

space X as follows, $x^\sigma = x \circ \sigma : S \mapsto \Re^L$ with $\sigma: S \mapsto S$ and $x: S \mapsto \Re^L$, i.e., it is a permutation of states of nature (see Balasko [2]).

The consumers, $i = 1, \ldots, I$, are defined through their preferences, endowments and profit shares. The non-empty consumption set is denoted as $X_i \subset X$. X_i is bounded from below, closed, convex, and $x_i \in X_i$ if and only if $x_i^\sigma \in X_i$. The preferences over X_i are given by the weak preference ordering R_i. For all consumers $x_i R_i x_i'$ if and only if $x_i^\sigma R_i x_i'^\sigma$. The preferences are continuous and satisfy local non-satiation. The profit shares are given by $0 \leqq \theta_{ij} \leqq 1$, where θ_{ij} is consumer i's share of profit of firm j, with $\sum_i \theta_{ij} = 1$ and $\theta_{ij} = \theta_{ij}^\sigma$. The endowments of the consumers are given by $\omega_i \in X_i$, and $\omega_i = \omega_i^\sigma$. In what follows, when security markets are complete and unrestricted, θ_{ij} should be interpreted as a scalar rather than as an S-dimensional vector.

Firms, $j = 1, \ldots, J$, have the production set $Y_j \subset X$, such that $y_j \in Y_j$ if and only if $y_j^\sigma \in Y_j$. Y_j are closed for $j = 1, \ldots, J$; $0 \in Y_j$; $Y \cap (-Y) \subset \{0\}$, and $(-\Re^{LT}) \subset Y$, where $Y = \sum_j Y_j$.

The (symmetric) way we have defined the economy implies that there is no intrinsic uncertainty in this economy. The case in which the consumption sets and production sets are merely T-fold Cartesian product of the relevant set in the certainty economy is a special case of symmetry. Similarly, the von Neumann-Morgenstern preferences for the consumers constitute a special case of symmetric preferences. The private ownership economy with extrinsic uncertainty, $\mathscr{E}^S$, is thus given by

$$\mathscr{E}^S = \{Y_j, j = 1, \ldots, J; (X_i, R_i, \theta_i, \omega_i), i = 1, \ldots, I\}.$$

There will be complete markets for insurance against sunspots if each consumer faces a single contingent-claims budget constraint (see Cass and Shell [4]), and the producers maximize (ex-ante) profits at contingent-claims prices. The problem for the consumer i is to

$$(C) \text{ Choose } x_i \text{ to maximize } R_i \text{ over } \left\{ x_i \in X_i \,|\, p \cdot x_i \leqq p \cdot \omega_i + \sum_j \theta_{ij} \Pi_j \right\}$$

where Π_j is profit of firm j. The problem for firm j is to

$$(F) \text{ Choose } y_j \in Y_j \text{ to maximize } \Pi_j = p \cdot y_j.$$

Next, we define the set of market equilibria, M^S, and the set of attainable allocations (states), A^S (see Debreu [5]).

Definition 1
An allocation $\{(x_i), (y_j)\}$ of $\mathscr{E}^S$ is a *market equilibrium* if

$$\sum_i x_i - \sum_j y_j = \sum_i \omega_i.$$

The *set of market equilibria* is denoted as M^S.

Definition 2
An allocation $\{(x_i), (y_j)\}$ of $\mathscr{E}^S$ is *attainable* if

$$x_i \in X_i, y_j \in Y_j, \quad \text{and} \quad \sum_i x_i - \sum_j y_j = \sum_i \omega_i.$$

The *set of attainable allocations* of $\mathscr{E}^S$ is denoted as A^S. We have

$$A^S = \left(\prod_i X_i\right) \times \left(\prod_j Y_j\right) \cap M^S.$$

Definition 3

A *sunspot equilibrium* (*SSE*) of $\mathscr{E}^S$ is an $(I + J + 1)$-tuple of $\mathfrak{R}^{LT}$ such that $\{(x_i^*), (y_j^*), p^*\}$ satisfy

1. x_i^* is a greatest element of $\{x_i \in X_i | p^* \cdot x_i \leqq p^* \omega_i + \sum_j \theta_{ij} p^* y_j^*\}$ according to R_i.
2. y_j^* maximizes profit relative to p^* on Y_j.
3. $\sum_i x_i^* - \sum_j y_j^* = \sum_i \omega_i$.

Definition 4

Sunspots do not matter if

$$x_i^* = x_i^{*\sigma}, i = 1, \ldots, I.$$

The assumptions made on the preferences are standard, while those on the technology are similar to Debreu [5] except we do not assume convexity. The next two assumptions play important roles for our result.

Assumption A.1

R_i is strictly convex for $i = 1, \ldots, I$, i.e., if

$$x_i R_i x_i', \text{ then } (tx_i + (1-t)x_i') P_i x_i, \quad \text{for } 0 < t < 1,$$

where P_i is the derived strict preference relation.

Assumption A.2

The set of attainable allocations, A^S, is convex.

Assumption A.2 may be satisfied even if the aggregate production set or the individual productions sets are non-convex. It clearly obtains for the models in Cass and Shell [4] and Balasko [2]. In some of the examples in the present note, Assumption A.2 does not obtain.

Definition 5

An attainable allocation $\{(x_i^*), (y_j^*)\}$ is (*ex-ante*) *Pareto efficient* if there does not exist another allocation $\{(x_i), (y_j)\} \in A^S$, such that $x_i R_i x_i^*$ with at least one strict preference.

Theorem

If A.1. and A.2. are satisfied and there is full insurance, then sunspots do not matter.

Proof

The proof of the theorem proceeds in two steps.

Step 1. An SSE allocation must be (*ex-ante*) Pareto efficient. This proof is identical to that in Debreu [5].

Step 2. If an allocation, $\{(x_i^*),(y_j^*)\}$, is (*ex-ante*) Pareto efficient, it must be symmetric, i.e., $x_i^* = x_i^{*\sigma}$, $i=1,\ldots,I$, and $y_j^* = y_j^{*\sigma}, j=1,\ldots,J$. Suppose it is not. Then consider the symmetric allocation, $\{(x_i'),(y_j')\}$ defined by

$$x_i' = \frac{1}{\#G}\sum_{\sigma\in G} x_i^{*\sigma}, i=1,\ldots,I, \quad \text{and} \quad y_j' = \frac{1}{\#G}\sum_{\sigma\in G} y_j^{*\sigma}, j=1,\ldots,j.$$

Using that $\{(x_i^*),(y_j^*)\}$ was attainable and the convexity of A^S, the allocation $\{(x_i'),(y_j')\}$ is feasible, i.e., we have $\{(x_i'),(y_j')\}\in A^S$. This is shown as follows:

$$\begin{aligned}\sum_i x_i' - \sum_j y_j' &= \frac{1}{\#G}\left[\sum_i\left(\sum_{\sigma\in G} x_i^{*\sigma}\right) - \sum_j\left(\sum_{\sigma\in G} y_j^{*\sigma}\right)\right]\\ &= \frac{1}{\#G}\left[\sum_{\sigma\in G}\left(\sum_i x_i^{*\sigma} - \sum_j y_j^{*\sigma}\right)\right]\\ &= \frac{1}{\#G}\left[\sum_{\sigma\in G}\omega_i^\sigma\right] = \sum_i \omega_i,\end{aligned}$$

because of the convexity of A^S. From $x_i^* R_i x_i^{*\sigma}$ and the strict convexity of preferences it follows that $x_i' R_i x_i^*$. This is strict whenever $x_i^* \neq x_i^{*\sigma}$. □

Remark 1
We cannot claim that an SSE necessarily exists. The nonconvexity of the production sets may introduce discontinuities in the excess demand correspondence.

Remark 2
It is crucial in our proof that the symmetric dominating allocation is attainable in the original economy.

Remark 3
The usual (macroeconomic) interpretation of the economy in Example 1 (below) is that the consumers (workers) supply labor, x_i^2, continuously but only effective labor $\tilde{y}^2$ is useful in production: worker i's effort is productive only if *he* works full time. This is the indivisibility in labor which is often modelled as an indivisibility on the consumption side (see e.g. Hansen [9], Rogerson [11], and Shell and Wright [13]) for analytical convenience, even though the basic economic motivation is from the production side. Example 1 also provides a formal justification of the Hansen-Rogerson short cut.

Example 1
Strict convexity of preferences and complete insurance markets together are not sufficient for immunity against the effects by sunspots.

Let there be 2 extrinsic states $S=\{\alpha,\beta\}$, with probabilities $\pi(\alpha)=\pi(\beta)$. There are two identical consumers with von Neumann-Morgenstern preferences R_i over $X_i = (\Re_{++}\times[0,1])^2$, described by the utility function

$$V_i(x_i(s)) = \sum_S\left(\log x_i^1(s) - m x_i^2(s)\right), m>0.$$

The endowments of consumer i in state $s, \omega_i(s)$, are

$$\omega_i(s) = (0, 1, 1/2).$$

There is a single firm whose technology is described by

$$y^1(s) = \min\{\tilde{y}^2(s), y^3(s)\}, \tilde{y}^2(s) = \sum_h^{\infty} \chi_h(s), \quad \text{where } \chi_h(s) = \begin{cases} 1 & \text{if } x_h^2(s) = 1 \\ 0 & \text{otherwise.} \end{cases}$$

The firm maximizes profits, Π, which are given by

$$\Pi = \sum_s p(s)y^1(s) - p(s)y^2(s) - p(s)y^3(s), \quad \text{where } y^2(s) = \sum_h x_h^2(s).$$

In equilibrium, markets clear, i.e., we have

$$\sum_i x_i - y = \sum_i \omega_i.$$

Market clearing and profit maximization then yield

$$y^1(s) = y^2(s) = \tilde{y}^2(s) = y^3(s) = 1 \quad \text{and} \quad p^1(s) = p^2(s) + p^3(s) \quad \text{for } s = \alpha, \beta.$$

The relevant maximization problem for consumer i leads to the Lagrangian function

$$\sum_s \pi(s)(\log x_i^1(s) - m x_i^2(s)) + \lambda_i \left[\sum_s \left(p^2(s)x_i^2(s) + \tfrac{1}{2}p^3(s) - p^1(s)x_i^1(s) \right) \right] + \sum_s \left(\mu_i(s)x_i^2(s) \right) + \sum_s \left(\gamma_i(s)(1 - x_i^2(s)) \right).$$

Hence, the necessary and sufficient first-order conditions for consumer i are

$$\frac{\pi(s)}{x_i^1(s)} = \lambda_i p^1(s)$$

$$\pi(s)m = \lambda_i p^2(s) + \mu_i(s) - \gamma_i(s)$$

$$\sum_s \left(p^2(s)x_i^2(s) + \tfrac{1}{2}p^3(s) - p^1(s)x_i^1(s) \right) \geqq 0 \quad \lambda_i \geqq 0$$

$$\lambda_i \left[\sum_s \left(p^2(s)x_i^2(s) + \tfrac{1}{2}p^3(s) - p^1(s)x_i^1(s) \right) \right] = 0$$

$$\mu_i(s) \geqq 0, x_i^2(s) \geqq 0, \quad \mu_i(s)x_i^2(s) = 0$$

$$\gamma_i(s) \geqq 0, (1 - x_i^2(s)) \geqq 0, \quad \text{and} \quad \gamma_i(s)(1 - x_i^2(s)) = 0$$

for $s = \alpha, \beta$. Next we show that

$$x_i = (\tfrac{1}{2}, 1, 0; \tfrac{1}{2}, 0, 0)$$

and

$$x_j = (\tfrac{1}{2}, 0, 0; \tfrac{1}{2}, 1, 0), \quad i \neq j$$

defines an SSE allocation. Using the above and noting that $\mu_i(\alpha) = \gamma_i(\beta) = 0$, the first-order conditions reduce to

$$\lambda_i p^1(s) = 1 \quad \text{for } s = \alpha, \beta,$$
$$\lambda_i p^2(\alpha) - \gamma_i(\alpha) = \tfrac{1}{2}m,$$
$$\lambda_i p^2(\beta) + \mu_i(\beta) = \tfrac{1}{2}m,$$

and the budget constraint. Similarly for consumer j, we have

$$\lambda_j p^1(s) = 1 \quad \text{for } s = \alpha, \beta$$
$$\lambda_j p^2(\alpha) - \mu_j(\alpha) = \tfrac{1}{2}m,$$
$$\lambda_j p^2(\beta) + \gamma_j(\beta) = \tfrac{1}{2}m,$$

and his budget constraint. Consider the case where

$$p(\alpha) = p(\beta), p^1(s) = 1, p^2(s) = \frac{m}{2}, \quad \text{and} \quad \mu_i(\beta) = \gamma_i(\alpha) = \mu_j(\alpha) = \gamma_j(\beta) = 0.$$

This will be a candidate for a SSE for $m < 2$, since in this case the first-order conditions, the budget constraints, and the equation $p^2(s) + p^3(s) = 1$ are satisfied.

We next show that there is no symmetric allocation giving the same or greater level of utility to each consumer. To find an alternative, feasible allocation which is symmetric and to which each consumer is indifferent, it must be the case that consumer i works in both states, i.e.,

$$x_i^2(s) = 1 \quad \text{and} \quad x_j^2(s) = 0 \quad \text{for } s = \alpha, \beta.$$

For their *ex-ante* utilities to be equal we must have

$$\log\delta - m = \log(1 - \delta),$$

$$\text{or} \quad \log\left(\frac{\delta}{(1-\delta)}\right) = m,$$

$$\text{or} \quad \delta = \frac{e^m}{(1 + e^m)}.$$

Now for consumer i, comparing utilities in the sunspot asymmetric and symmetric allocation we have $\log\frac{1}{2} - \frac{1}{2}m > \log\left(\frac{e^m}{(1+e^m)}\right) - m = \log\left(\frac{1}{(1+e^m)}\right)$. Otherwise,

$$\log\left(\frac{1}{2}\right) - \frac{m}{2} \leqq \log\left(\frac{1}{(1+e^m)}\right)$$

$$\text{or} \quad \log\left(\frac{(1+e^m)}{2}\right) \leqq \frac{1}{2}m$$

$$\frac{1}{2} + \frac{1}{2}e^m \leqq e^{(m/2)}$$

which is not true at least in the neighborhood of $m = 1$. It is easy to check that the allocation where both consumers work in both states will be even worse. It can be easily checked that the preferences are strictly convex as the utility function which represents them is strictly quasi-concave. □

Remark 4

It is easy to extend Example 1 to the case where all the three goods are consumed. Change the technology to

$$y^1(s) = \min\{\tilde{y}^2(s), ky^3(s)\}, k > 1,$$

and the utility function over $X_i = (\Re_{++} \times [0,1]^2)^2$ to

$$V_i(x_i) = \sum_s \pi(s)(\log x_i^1(s) - mx_i^2(s) + n(x_i^3(s))),$$

where n_i is twice continuously differentiable, strictly concave and increasing with $n'(\frac{1}{2}(k-1)/k) = \frac{1}{2}(1 - \frac{1}{2}m)k$. The following constitutes a SSE

$$p(s) = \left(1, \frac{1}{2}m, \left(1 - \frac{1}{2}m\right)k\right),$$

$$1 = y^1(s) = \tilde{y}^2(s) = y^2(s),$$

$$y^3(s) = \frac{1}{k},$$

$$x_i = \left(\frac{1}{2}, 1, \frac{k-1}{2k}; \frac{1}{2}, 0, \frac{k-1}{2k}\right),$$

and

$$x_j = \left(\frac{1}{2}, 0, \frac{k-1}{2k}; \frac{1}{2}, 1, \frac{k-1}{2k}\right).$$

To see this, note that for zero profit we must have

$$p^2(s) + \frac{1}{k}p^3(s) = 1 \quad \text{or} \quad p^3(s) = (1 - p^2(s))k.$$

Looking at the first-order conditions, there are now two additional conditions (besides the change in the budget constraint)

$$\lambda_h = \frac{1}{p^3(s)}[\pi(s)n'(x_h^3(s))]$$

for $s = \alpha, \beta$, and $h = 1, 2$. It can be checked that the new prices satisfy the budget constraint and the first-order conditions for the equilibrium allocation for each of the consumers. □

Remark 5

This example cannot be modified to make the utility function strictly concave in the second commodity.

Suppose it were. Then take $m(x_i^2(s))$ to be a twice continuously differentiable, strictly convex (i.e., $-m(\cdot)$ strictly concave) and increasing function of $x_i^2(s)$. Then the first-order condition for consumer i implies that we have

$$\tfrac{1}{2}m'(1)+\gamma_i(\alpha)=\tfrac{1}{2}m'(0)-\mu_i(\beta).$$

However, strict convexity implies $m'(0)<m'(1)$, and $\gamma_i(\alpha)$ and $\mu_i(\beta)$ are both non-negative. Changing $p^2(\alpha)$ and $p^2(\beta)$ will not solve the problem because a similar condition for consumer j must be satisfied.

Example 2
The convextiy of A^S is not necessary for sunspot immunity.

Consider the same economy as in Example 1, but the technology is now given by $y^1(s)=\min\{\tilde{y}^2(s),y^3(s)\}$ where $\tilde{y}^2(s)=\text{integer } y^2(s)$. (Here the total labor is useful only in discrete amounts, but the firm does not care if individual workers work full time or part time.) The utility functions of the consumers are modified to be

$$V_i(x_i(s))=\sum_s \pi_s\big((x_i^1(s))^{1/2}-m(x_i^2(s))\big),$$

where $m(\cdot)$ is twice continuously differentiable, strictly increasing, strictly convex and satisfies $m'(\tfrac{1}{2})\in(0,2)$. In this economy there is a unique equilibrium, with the allocation

$$y^1(s)=y^2(s)=\tilde{y}^2(s)=y^3(s)=1, x_1=x_2=(\tfrac{1}{2},\tfrac{1}{2},0;\tfrac{1}{2},\tfrac{1}{2},0).$$

The prices are

$$p(s)=(1,\tfrac{1}{2}m'(\tfrac{1}{2}),1-\tfrac{1}{2}m'(\tfrac{1}{2})),\quad \text{for } s=\alpha,\beta.$$

The set A^S is not convex. Consider

$$((x_i)_i,y)=((\tfrac{1}{2},\tfrac{1}{2},0;\tfrac{1}{2},\tfrac{1}{2},0)_i,(1,1,1;1,1,1))\in A^S$$

and

$$((x_i^*)_i,y^*)=((0,1,0;0,1,0)_i,(0,0,1;0,0,1))\in A^S.$$

For no $\lambda\in(0,1)$ do we have

$$\lambda(x,y)+(1-\lambda)(x^*,y^*)\in A^S.$$ □

Example 3
In a strictly convex environment with unrestricted and complete security markets, Pareto efficiency of equilibrium is not necessary for sunspot immunization.

Let there be two extrinsic states of nature, $S=\{\alpha,\beta\}$, with equal probabilities. In each state there are two consumption goods. There are two consumers with identical preferences over $X_i=(\Re^2_{++})^2$ given by the von Neumann-Morgenstern utility functions

$$V_i(x_i(s))=\sum_s \pi(s)\big((x_i^1(s))^{1/2}(x_i^2(s))^{1/2}\big).$$

The endowments of the consumers are

$$\omega_1(s) = (1,0) \quad \text{and} \quad \omega_2(s) = (0,1)$$

for $s = \alpha, \beta$. Consumer 1 faces an *ad valorem* tax of $0 < t < 1$ on commodity 2 in each state, i.e., faces the price $(1 + t)p^2(s)$. Consumer 2 does not face the tax. The tax receipts are given as a lump-sum transfer to consumer 2 in each state. Since the Theorem 1 applies, the equilibria are necessarily symmetric. In fact, it can be calculated as follows. The unique equilibrium has

$$(p^1(s), p^2(s)) = \left(1, \frac{(2+t)}{(2+2t)}\right).$$

The allocations are

$$(x_1^1(s), x_1^2(s)) = \left(\frac{1}{2}, \frac{1}{(2+t)}\right), \quad (x_2^1(s), x_2^2(s)) = \left(\frac{1}{2}, \frac{(1+t)}{(2+t)}\right).$$

However, the outcome is not Pareto-efficient, neither *ex-ante* nor *ex-post*, since the marginal rates of substitution are not equal. □

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