Sunspot Equilibrium

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How does one explain the randomness which we see in the economy? Part of it can be traced to the randomness in the physical world which is transmitted through the economic fundamentals (such as endowments, technology and preferences). The weather provides an example. The randomness in rainfall causes randomness in crop yields which in turn generates randomness in agricultural outputs and agricultural prices. Since rainfall affects the economic fundamentals (in particular, it affects agricultural technology), it is said to be an intrinsic variable. Hence, uncertainty about rainfall is also an example of *intrinsic uncertainty* (see Cass and Shell, 1983, p. 194). The classic Arrow–Debreu extension of the general-equilibrium model to include uncertainty has long been the basis for analysing intrinsic uncertainty (see, e.g., Debreu, 1959, ch. 7).

Not all economic randomness can be explained in this way. Even if the fundamental parameters were non-random, economic outcomes would generally be random. This is because the economy is a social system composed of individual economic actors who are uncertain about each other's behaviour. In seeking to optimize his own actions, each participant in the market economy must attempt to predict the actions of the other participants. It is a complicated matter. Mr. A, in forecasting the market strategy of Mr. B, must forecast Mr. B's forecasts of the forecasts of others including that of Mr. A himself. And so on. Since market participants are not certain about the actions of others, they are uncertain about economic outcomes. Businessmen, for example, do not know what others will bid for their products, they do not know whether potential rivals will decide to enter or decide to hold back, they are uncertain about the inflation rate, and so forth.

Uncertainty of this sort is referred to as *market uncertainty* (see Peck and Shell, 1985). It is either created by the market economy or it is adopted from outside the economy as a means of coordinating the plans of the individual market participants. Market uncertainty is not transmitted through the fundamentals. It is, therefore, an instance of *extrinsic uncertainty*.

The interdependence of beliefs, even of 'rational' beliefs, is a central theme in the *General Theory*; see Keynes (1936, ch. 12). Keynes postulates that it is possible to encounter self-justifying expectations, beliefs which are individually rational but which may lead to socially irrational outcomes. The possible interdependence of individually rational beliefs is the central theme of the Townsend (1983) paper and the Frydman–Phelps (1983) volume. Nevertheless, it is fair to say that the formal modelling of market uncertainty has until recently lagged behind the modelling of uncertainty which is transmitted to the economy through its fundamental parameters. The recent work on 'Sunspot Equilibrium' introduced by Cass and Shell, reported in Shell (1977) and Cass–Shell (1983), is meant to provide a rigorous basis for the theory of market uncertainty. The Cass–Shell 'sunspots' are highly stylized. Contrary to fact and contrary to Jevons (1884), it is assumed that the sunspots represent purely extrinsic uncertainty: the economic fundamentals are assumed to be unaffected by the level of sunspot activity.

Can the level of sunspot activity affect the allocation of resources in a market economy? It has been known for some time that if probability beliefs (about sunspot activity) differ across individuals, then sunspots can matter. Consider the two-consumer, two-state, one-good, competitive exchange economy. Draw the usual Edgeworth box. Measure good consumption in state α on the horizontal. Measure good consumption in state β on the vertical. Because uncertainty is purely extrinsic, the box is a square: aggregate resources are independent of the state of nature. Also, because uncertainty is purely extrinsic, the endowment vector lies on the diagonal: individual endowments are independent of the state of nature. Assume that the consumers possess von Neumann–Morgenstern utility functions. Competitive equilibrium always exists. There are two cases: (1) The consumers have the same probability beliefs about the occurrence of states α and β. Indifference curve tangency, and hence contingent claims competitive equilibrium, occurs only on the diagonal. Sunspots do not matter. (2) The consumers have differing beliefs about the probabilities of α and β. Indifference curves will not be tangent on the diagonal. A contingent-claims competitive equilibrium will exist off the diagonal. Sunspots must matter.

There is a sense, however, in which the above sunspot equilibrium is unstable. Assume that the differences in probability beliefs are solely because of differences in information: the consumers share common prior beliefs, but because of differing information they have different posterior beliefs. The contingent-claims prices, however, reveal information. Indeed, in this example, the only competitive equilibrium in which individuals do not revise their beliefs from market information is based on common probability beliefs. Hence, we are especially interested in the special case where beliefs are commonly held. This might be thought of as the strong rational-expectations case.

Indeed, the original research on sunspot equilibrium was inspired by and in reaction to the rational-expectations macroeconomics literature as exemplified by Robert Lucas's (1972) classic paper in the *Journal of Economic Theory*. The Lucas paper was well received in some circles, while it was heavily criticized in others. Most of the critics took issue with the assumptions of individual rationality and perfect markets. Others, rather few in number at the time, were willing to ask whether or not the conclusions of the rational-expectations school follow from the assumptions. Does it follow that passive or simple 'monetary' rules are necessarily best? More generally, if individuals are rational and the government is nonerratic, will the social outcome be nonerratic?

Lucas gave us a formal model to shoot at. His model is based on the overlapping-generations model of Samuelson (1958), in which time is treated seriously and there is room for government debt (see Cass–Shell, 1980). In my Malinvaud lectures (Shell, 1977), I present an example of an overlapping-generations economy in which sunspots affect the allocation of resources solely because individuals believe that sunspot activity affects the price level. Their beliefs are rational: any single individual believing otherwise would be worse off. In the particular example, the best government policy is perpetually active and exhibits high variance. There is a continuum of perfect-foresight (nonsunspot) equilibria parametrized by the initial price of money and a vast multiplicity of sunspot equilibria partly parametrized by beliefs about the effects of sunspots. (The Shell (1977) model is in at least one way borderline: utility functions are linear. However, David Cass and I had presented similar results based on a non-linear overlapping-generations model at a Mathematical Social Science Board seminar in 1975.)

What features of this model allow for the existence of sunspot equilibria? The Shell (1977) model includes many of the salient features of decentralized, dynamic economies: Government debt is denominated in nominal (i.e. money) units. The time horizon is infinite. Market participation is restricted by natural lifetimes; that is, individuals cannot trade in markets which meet when they are not alive. Too much is included in the dynamic model of Shell (1977) to permit one to isolate 'the' source of sunspot equilibria.

Cass and Shell (1983) focus on only one of these aspects, the natural restrictions imposed on market participation. The model is finite. There is no government debt. Some individuals ('the old') can insure against the effects of sunspots; some individuals ('the young') cannot. If there were no restricted individuals ('no young'), there would be no sunspot equilibria. If there were no unrestricted individuals, a sunspot equilibrium would only be a randomization over nonsunspot equilibria. Otherwise, the typical sunspot equilibrium is not a mere lottery over nonsunspot equilibria. The set of equilibria has been expanded in a fundamental way: the classical Walrasian (nonsunspot) equilibria are only a subset of the set of equilibria. The new equilibria, the sunspot equilibria are never Pareto-optimal.

Cass and Shell (1983, Appendix) provide an example in which there is only one nonsunspot equilibrium but in which there is at least one sunspot equilibrium. The sunspot equilibrium cannot in this case be a randomization over nonsunspot equilibria, since there is only one nonsunspot equilibrium. What goes on in this simple example? Of course, the restricted consumers cannot transfer income across states of nature. The unrestricted consumers believe that relative prices will differ from one state to another. The unrestricted consumers have tastes which differ: in particular, intrastate indifference curves differ and rates of risk aversion differ. Hence the unrestricted consumers may find it advantageous to transfer income across states of nature. Consequently, when conditions are right consumer beliefs in a sunspot equilibrium outcome are validated.

I showed in my overlapping-generations paper (Shell, 1971) that the set of perfect-foresight equilibria is unaffected by the natural restrictions on market participation. (In particular, the possible inoptimality of perfect-foresight competitive equilibria in the overlapping-generations model is *not* due to restricted participation. It is due to the 'double-infinity' of (dated) commodities and (dated) consumers.) Hence, the restriction on market participation which naturally arises in dynamic economies, while not a source of the inoptimality of some nonsunspot equilibria (the 'Samuelson' cases), is *a* source of the existence of sunspot equilibria, which are always Pareto inoptimal. Is restricted market participation the only source of sunspot equilibria in rational-expectations economies? The answer is no! Indeed, absence of sunspot equilibria seems to be the exception rather than the rule. If Pareto optimality is assured, then strong rational-expectations equilibria (based on shared beliefs) are not affected by sunspots. The so-called Philadelphia Pholk 'Theorem' is the assertion: in each 'class' of models in which Pareto-optimal allocations are not guaranteed, one can find an example of sunspot equilibrium. The 'proof' is based on several examples put together by Cass and me and our co-authors. We deviate from the preconditions for Pareto-optimality in only one aspect per example. Tested deviations giving rise to the existence of sunspot equilibria are: incomplete markets, externalities, imperfect competition, and the double-infinity of consumers and commodities (but with imagined unrestricted market participation). In this last case, sunspots can be a partial substitute for money. Sunspots offer the possibility of improved (but never Pareto-optimal) coordination. In general, sunspot equilibria are at best optimal in only a weak sense in which consumers are labelled in the conventional way but are also differentiated by the history of the prenatal states of nature (see Cass–Shell, 1983, pp. 215–18).

It is fair to say that the existence (indeed the prevalence) of proper sunspot outcomes came as a big surprise to many rational-expectations equilibrium theorists. Game theorists, on the other hand, long ago accepted the naturalness of stochastic solutions to nonstochastic games. Consider the well-known notion of mixed strategy or Aumann's (1974, 1985) generalization, correlated strategy. Mixed-strategy equilibria and, more generally, correlated equilibria are examples in which extrinsic uncertainty matters to the outcomes and payoffs of games. The possibility of asymmetric information is what makes correlated equilibrium an interesting generalization of Nash equilibrium.

Peck and Shell (1985) analyse market uncertainty in an imperfect-competition model. The particular model chosen is that of the *market game* due to Shapley and Shubik (1977). Any other model of imperfect competition might have served as well for analysing market uncertainty. The market-game model is, however, a perfect stage for comparing sunspot equilibrium (originally applied to competitive *market* models) and correlated equilibrium (originally applied to matrix *games*).

Peck and Shell establish the following: In the market game, there exists a proper (non-degenerate) correlated equilibrium if and only if the endowments are not Pareto-optimal. For correlated equilibrium the uncertainty device is outside the rules of the game. If the device becomes part of the rules of the game, we create from the market game the 'securities game', an imperfect-competition analogue of the Arrow (1964) securities model. Every correlated equilibrium allocation to the market game is also a pure-strategy Nash equilibrium allocation to the securities game. Proper correlated equilibria to the market game are sunspot equilibria to the securities game. Because the securities game allows for across-state transfers, some sunspot equilibrium allocations are not correlated equilibrium allocations (see Peck and Shell, 1985). Assuming common priors and common knowledge, we know that the set of correlated equilibrium allocations is equivalent to the set of Bayes-rational equilibrium allocations (see Peck and Shell, 1985, which follows Aumann, 1985).

Here, a subset of the sunspot equilibria arise as sophisticated solutions to simple games. The observed uncertainty is the rational consequence of the uncertainty that one player has about the moves of the others. All sunspot equilibria could be considered as simple solutions to sophisticated games. In the sophisticated games, securities are traded. These securities are intended to insure against disturbances caused by randomness in the natural world, even though the effect of this randomness on economic fundamentals is negligible. For examples of correlated equilibria and related sunspot equilibria, see Maskin and Tirole (1985), Aumann, Peck and Shell (1985) and Peck and Shell (1985).

The original impetus for sunspot equilibrium comes from intertemporal economics (cf. Shell, 1977). While the importance of the sunspot-equilibrium notion and related notions of market uncertainty – such as correlated equilibrium, Bayes-rational equilibrium and speculative bubbles (see Tirole, 1985) – are quite general, much of the development of the sunspot model itself has been closely related to economic dynamics. Azariadis (1981) and Azariadis and Guesnerie (1986) go back to the simplest overlapping-generations model from macroeconomics with a stationary environment. Azariadis (1981) provides sufficient conditions for the existence of *stationary* stochastic business cycles based on sunspot activity. Azariadis and Guesnerie (1986) related the conditions for stationary sunspot cycles to the conditions for deterministic cycles. Spear (1985) challenges the view that the stationary sunspot cycles are 'likely' to be encountered when there is more than one commodity per period. Peck (1985) shows, however, that in simple overlapping generations models the existence of a continuum of nonsunspot equilibria (as 'often' arises in economies with taxes and transfers denominated in money units) implies the existence of (possibly nonstationary) sunspot equilibria. Peck's results do not depend on stationarity of the environment. Sunspot equilibria are not 'flukes'.

The connection between endogenous nonstochastic cycles and stationary sunspot equilibria is currently receiving substantial attention. It is too early to review this promising field. The interested reader should turn to the *Journal of Economic Theory* symposium issue (October 1986) on 'Nonlinear Economic Dynamics' edited by Jean-Michel Grandmont. There is a fair sampling of papers on these topics and related topics. The symposium issue also contains several references.

Sunspot equilibrium represents an example of the more general phenomenon, symmetry-breaking, in which symmetric problems have asymmetric solutions. See Balasko (1983) but expect to hear more from him on the subject of symmetry-breaking in economics.

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