CAE Working Paper #88-34

Asymmetric Information and Sunspot Equilibria: A Family of Simple Examples****

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July 1985
Revised October 1988

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****This paper is based on Rough Notes prepared 7/7/85 for an IMSSS all-day workshop held at Stanford, July 16, 1985. We thank Mordecai Kurz for his encouragement of this note and of its three authors. The current version is based on a presentation at the June 1987 advanced study institute "Incomplete Information and Bounded Rationality Decision Models," Anacapri, Italy. We thank Harold Kuhn for his patience and encouragement. We also thank NSF grant #SES-8606944 and the CAE at Cornell University for research support.

1. Extrinsic uncertainty in market economies and games.

Economic outcomes (prices and net trades) in market economies are random. Some of this randomness is based on uncertainty about economic fundamentals (endowments, production possibilities, and preferences). This uncertainty, which is exogenous to the economy but intrinsic to the fundamentals, is transmitted through the economy. The leading example is the weather. Rainfall affects crop yields. The market economy transmits uncertainty about rainfall into uncertainty about farm production and agricultural prices.

This is not the only source of economic randomness. Even if economic fundamentals were certain, economic outcomes would still be random. This is because economies are social organizations. Each economic actor is uncertain about the strategies of the others. Businesspeople, for example, are uncertain about the plans of their customers and rivals and of the government tax, monetary, and regulatory authorities. This type of economic randomness is generated by the market economy: it is thus endogenous to the economy, but extrinsic to the economic fundamentals.

Until recently, the modelling of economic randomness in competitive market economies was based on analogy to the rainfall model. Implicitly, it had been assumed that all economic randomness is intrinsic. The first explicit modelling of extrinsic uncertainty in competitive economies is the sunspot equilibrium model of Cass and Shell; see Shell (1977), Cass and Shell (1983), and Shell (1988). The sunspots of this model are highly stylized: they represent purely extrinsic uncertainty, i.e., sunspots do not affect the economic fundamentals. Sunspots are formally exogenous to the economy, but since economic actors choose which randomizing device to employ, it is also permissible to think of stylized sunspots as endogenous randomness: the uncertainty created by the economy, as opposed to intrinsic uncertainty, the randomness exogenous to the economy which is transmitted to economic outcomes through the economic fundamentals. <u>Sunspot equilibria are the stochastic outcomes of economies in which the fundamentals are nonstochastic</u>.

The sunspot model represents a break with the traditional approach to competitive economies. We now know that economies with completely rational economic actors are likely to generate economic uncertainty in addition to transmitting economic uncertainty.

Looked at from the perspective of game theory, sunspot equilibria are not so surprising. We are used to stochastic solutions to nonstochastic games. Mixed-strategy equilibrium is a commonplace concept. The solution concept correlated equilibrium is a generalization of both mixed-strategy equilibrium and randomizations over pure-strategy equilibria. See Aumann (1974, 1987).

2. <u>Asymmetric information</u>.

For a while, it seemed that the introduction of uncertainty into economic models would not require big changes in the analysis. It seemed that for general equilibrium and welfare with uncertainty one need only re-interpret Arrow and Debreu and for microeconomics with uncertainty one need only re-interpret Irving Fisher. These simple isomorphisms, however, do not apply to situations in which information is asymmetric (or, "incomplete.") See, e.g., Akerlof (1970) and Spence (1974). Even the early literature on sunspot equilibrium, which emphasized the role of extrinsic uncertainty in general equilibrium, was based on the symmetric or "complete" information case.

In the remainder of this section, we give a brief review of the role of extrinsic uncertainty in finite market economies. In this review, we focus on the absence or presence of asymmetric information. We note whether or not the new equilibria, in this case the sunspot equilibria, are outside the convex hull of the set of (old) certainty equilibria.

In a finite competitive economy with naturally restricted market participation, Cass and Shell (1983, Appendix) provide an example in which the nonsunspot equilibrium is unique and there is at least one sunspot equilibrium. Since the set of sunspot equilibria is disjoint from the set of nonsunspot equilibrium, is not in the convex hull of the set of the nonsunpot equilibria (here a singleton). This sunspot equilibrium is not a correlated equilibrium since there are income transfers across states of nature. Signals to the economic actors are perfectly correlated, i.e., information is symmetric.

Peck and Shell (1985) analyze sunspot equilibrium in market games. Signals are perfectly correlated. Some sunspot equilibrium allocations are correlated equilibrium allocations. Other sunspot equilibrium allocations are not correlated equilibrium allocations; in these cases, income is transfered across states of nature.

Maskin and Tirole (1987) provide the first successful example of sunspot equilibrium driven by asymmetric information and incomplete markets. In their example of a competitive economy, the nonsunspot equilibrium is unique and there is at least one sunspot equilibrium. The sunspot equilibrium is also a correlated equilibrium, but it is obviously not in the convex hull of the set of nonsunspot equilibria.

Peck and Shell (1988, Example 5.8) construct a correlated equilibrium in a market game which is driven by asymmetric information. The setting is general (the number of commodities is ℓ), but the example is "nonrobust" because of the bankruptcy rule.

Here we present a relatively simple family of market games. There are only two commodities. Bankruptcy is not a possibility in this simple setting. Indeed, there is no role for money and credit for spot (i.e., intrastate) trading. We construct sunspot equilibrium allocations which are also correlated equilibrium allocations. Signals are imperfectly correlated, i.e., information is asymmetric. The correlated equilibria are neither mixed-strategy equilibria nor randomizations over pure-strategy Nash equilibria. The limit of this family of market games is a competitive economy. In this economy, the nonsunpot equilibrium is unique. As in Maskin and Tirole (1987), the correlated equilibrium is outside the convex hull of the nonsunspot equilibrium allocations.

3. The example.

There are 2 types of consumers and $m > 1$ individuals of each type. An individual consumer is identified by his type $t = 1,2$ and the index $h = 1,\ldots,m$. There are two commodities. Each individual of type 1 is endowed with 10 units of commodity 1 and 0 units of commodity 2. Each individual of type 2 is endowed with 0 units of commodity 1 and 10 units of commodity 2. Hence we have

$$\omega_{1,h} = (\omega^1_{1,h}\,,\ \omega^2_{1,h}) = (10,\ 0)$$

and

$$\omega_{2,h} = (\omega^1_{2,h}\,,\ \omega^2_{2,h}) = (0,\ 10)$$

for $h = 1,\ldots,m$. Endowments are unaffected by uncertainty. Consumers of type 2 are risk-averse with identical utility functions given by

$$u_{2,h}(x^1_{2,h}\,,\ x^2_{2,h}) = -\left(\frac{1}{x^1_{2,h}} + \frac{1}{x^2_{2,h}}\right)$$

for $h = 1,\ldots,m$. Consumers of type 1 possess utility functions $u_{1,h}$ which are identical and satisfy the following restrictions on the marginal rates of substitution

$$-\frac{\partial u_{1,h}(8,5)/\partial x^1_{1,h}}{\partial u_{1,h}(8,5)/\partial x^2_{1,h}} = \frac{5-5m}{2m}$$

and

$$-\frac{\partial u_{1,h}(2,5)/\partial x^1_{1,h}}{\partial u_{1,h}(2,5)/\partial x^2_{1,h}} = \frac{5-5m}{8m}$$

for $h = 1,\ldots,m$. Clearly, we are <u>not</u> considering an m-replication of a given economy since preferences of the type 1 consumers depend on the parameter m.

Assume that there are 2 states of nature, α and β, denoting the level of sunspot activity. Assume that the probabilities of occurence, $\pi(\alpha)$ and $\pi(\beta)$, are given by

$$\pi(\alpha) = \frac{3m+5}{15(m-1)}$$

and

$$\pi(\beta) = 1 - \pi(\alpha) .$$

Consumers of type 1 can detect sunspot activity. Consumers of type 2 cannot. This is the basis in this example of asymmetric information or imperfectly correlated signals. In this example, the signal to a type 1 individual is perfectly correlated with the signal to another type 1 individual, but is uncorrelated with the (constant, nondiscriminating) signal to an individual of type 2.

The strategies of the consumers must be "measurable with respect to their information sets." The information set for a consumer of type 1 is the finest partition of the set $\{\alpha, \beta\}$. The information set for a consumer of type 2 is the coarsest partition of the set $\{\alpha, \beta\}$. Consumers of type 1 can adopt strategies which are contingent upon sunspot activity. Consumers of type 2 cannot.

It is assumed that in each state, α or β, a given consumer can be only on one side of the market. He may either offer (sell) a positive amount of a given good or bid for (buy) a positive amount of it, but not both. Let $q_{t,h}(s) \geq 0$ be the offer of commodity 1 by consumer h of type t in state s. Assume that offers must be in physical commodities; hence we have

$$q_{t,h}(s) \leq \omega^1_{t,h}$$

for $s = \alpha, \beta$. Let $b_{t,h}(s) \geq 0$ be the bid for commodity 1 by consumer h of type t in state s. Since the bids for commodity 1 must be in units of physical commodity 2, we have

$$b_{t,h}(s) \leq \omega^2_{t,h}$$

for $s = \alpha, \beta$. The strategy sets $S_{t,h}$ are thus given by

$$S_{t,h} = \{(b_{t,h}(s), q_{t,h}(s)) \text{ for } s = \alpha, \beta \mid 0 \leq b_{t,h}(s) \leq \omega^2_{t,h},$$
$$0 \leq q_{t,h}(s) \leq \omega^1_h, \; b_{t,h}(s)\, q_{t,h}(s) = 0 \text{ for } s = \alpha, \beta \text{ and the}$$
$$\text{strategy is measurable with respect to the consumer's information partition}\}$$

for $t = 1,2$ and $h = 1,\ldots,m$. Notice that $b_{t,h}(s)$ is both the bid for commodity 1 and the offer of commodity 2, while $q_{t,h}(s)$ is both the offer of commodity 1 and the bid for commodity 2. Hence, following the usual allocation rules in market games, we have

$$x^1_{t,h}(s) = \omega^1_{t,h} - q_{t,h}(s) + b_{t,h}(s) \frac{\sum_{v=1}^{2} \sum_{k=1}^{m} q_{v,k}(s)}{\sum_{v=1}^{2} \sum_{k=1}^{m} b_{v,k}(s)}$$

and

$$x^2_{t,h}(s) = \omega^2_{t,h} - b_{t,h}(s) + q_{t,h}(s) \frac{\sum_{v=1}^{2} \sum_{k=1}^{m} b_{v,k}(s)}{\sum_{v=1}^{2} \sum_{k=1}^{m} q_{v,k}(s)}$$

for $s = \alpha, \beta$, where $x^i_{t,h}(s)$ is consumption of good i, $i = 1, 2$, in state s by consumer h of type t. Notice that since there are only 2 commodities there is no intrastate need for money or credit. There could very well be a need for money and credit in transferring incomes across states of nature (see Peck and Shell (1985, 1988)), but we assume here that the state contingent market is either absent or closed.

We next formally state the claim that there is a sunspot equilibrium allocation for this example. Consumers of type 1 play random strategies. Consumers of type 2 play pure strategies because of the "incompleteness" of their information. Each player's allocation is stochastic in this solution.

Claim: For the above market game, the following strategies define a type-symmetric correlated equilibrium (and hence sunspot equilibrium).

$$q_{1,h}(\alpha) = 2 \qquad q_{2,h}(\alpha) = 0$$

$$q_{1,h}(\beta) = 8 \qquad q_{2,h}(\beta) = 0$$

$$b_{1,h}(\alpha) = 0 \qquad b_{2,h}(\alpha) = 5$$

$$b_{1,h}(\beta) = 0 \qquad b_{2,h}(\beta) = 5$$

for $h = 1,\ldots,m$. The corresponding consumption levels are given by

$$x_{1,h}(\alpha) = (x^1_{1,h}(\alpha),\ x^2_{1,h}(\alpha)) = (8,5)$$

$$x_{1,h}(\beta) = (x^1_{1,h}(\beta),\ x^2_{1,h}(\beta)) = (2,5)$$

$$x_{2,h}(\alpha) = (x^1_{2,h}(\alpha),\ x^2_{2,h}(\alpha)) = (2,5)$$

$$x_{2,h}(\beta) = (x^1_{2,h}(\beta),\ x^2_{2,h}(\beta)) = (8,5)$$

for $h = 1,\ldots,m$.

<u>Proof</u>: First, notice that the strategy of each consumer is measurable with respect to his information partition.

In state α, a consumer of type 1 faces a budget set in $(x^1,\ x^2)$ space,[1] the boundary of which is given by

$$x^2_1 = \frac{(50 - 5x^1_1)m}{10 - x^1_1 + 2(m-1)} .$$

At the point (8,5) this budget curve has slope

$$\frac{\partial x_1^2}{\partial x_1^1} = \frac{5 - 5m}{2m} ,$$

which also equals the marginal rate of substitution at that point as a consequence of utility maximization. That is, the budget curve is tangent to the indifference curve at (8,5), so the consumer is behaving optimally in state α.

In state β, the equation of the budget curve is

$$x_1^2 = \frac{(50 - 5x_1^1)m}{10 - x_1^1 + 8(m-1)} .$$

At the point (2,5), the budget curve is tangent to the indifference curve. It follows that each consumer of type 1 is choosing a utility-maximizing strategy.

For a consumer of type 2, the first-order conditions of expected-utility maximization are necessary and sufficient for an optimum. If the following equation holds, the claim is proved

$$\pi(\alpha)\left[\frac{\partial u_2(2,5)}{\partial x_2^1} \quad \frac{10(m-1)\ m}{25m^2}\right] + \pi(\beta)\left[\frac{\partial u_2(8,5)}{\partial x_2^1} \quad \frac{40m(m-1)}{25m^2}\right]$$

(1)

$$= \pi(\alpha)\ \frac{\partial u_2(2,5)}{\partial x_2^2} + \pi(\beta)\ \frac{\partial u_2(8,5)}{\partial x_2^2} .$$

The parameters of the economy have been chosen so that Equation (1) holds at the proposed allocation, so it is in fact a correlated equilibrium allocation and hence a sunspot equilibrium allocation. ■

4. The competitive economy.

Consider the limit economy where $m \to \infty$. Then $\pi(\alpha) = 1/5$, $\pi(\beta) = 4/5$, and

$$\frac{\dfrac{\partial u_1(8,5)}{\partial x_1^1}}{\dfrac{\partial u_1(8,5)}{\partial x_1^2}} = 5/2 \quad \text{and} \quad \frac{\dfrac{\partial u_1(2,5)}{\partial x_1^1}}{\dfrac{\partial u_1(2,5)}{\partial x_1^2}} = 5/8 .$$

The situation is summarized in Figure 1 below. Figure 1 represents the Edgeworth box, which is in 4-dimensional space, but since the dimensions of this box are independent of the state s (uncertainty is extrinsic), a 2-dimensional representation suffices. Let Mr. 1 be a consumer of type 1 and Mr. 2 be a consumer of type 2.

Figure 1 approximately here

By construction, Mr. 1's offer curve must pass through (2,5) and (8,5), since Mr. 1 knows the state of nature s in advance. The restrictions placed on Mr. 1's offer curve are few. Nothing prevents us from choosing it so that it will intersect Mr. 2's offer curve only once.

Mr. 2's offer curve passes above (2,5) and below (8,5), as the two indifference curves in the above figure demonstrate. In state α, Mr. 2 would want to bid less than 5, and in state β he would want to bid more than 5. But Mr. 2 cannot see which state has occurred, so when $\pi(\alpha) = 1/5$, a bid of 5 exactly balances the marginal gain (in expected utility) of bidding more in

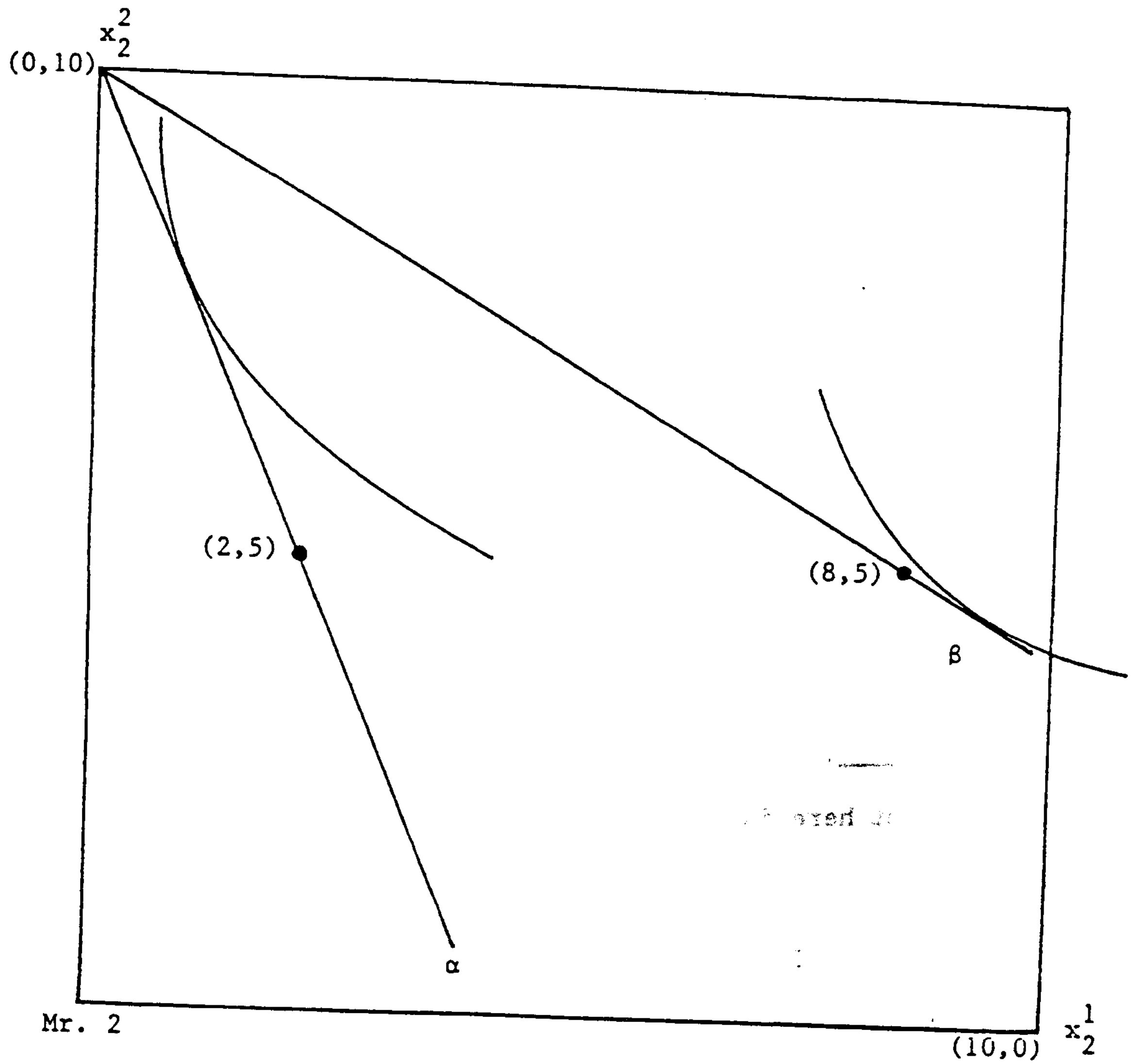

FIGURE 1

state β with the marginal loss of bidding more in state α. The utility functions of Mr. 1 and Mr. 2 are chosen to be strictly concave, strictly monotonic, smooth, and such that the closure (in $\mathbb{R}^2$) of each indifference curve is contained in $\mathbb{R}^2_{++}$.

We have then constructed a competitive economy in which the nonsunspot (Pareto-optimal) equilibrium is unique, but for which there is also a distinct correlated equilibrium. This correlated equilibrium is not based on perfectly correlated signals. Type 1 individuals base their strategies on the same signal, $s = \alpha, \beta$. Since there are at least 2 type 1 consumers, the correlated equilibrium is not a mixed-strategy equilibrium. Furthermore, the associated correlated-equilibrium allocation cannot be a mere randomization over pure-strategy equilibria; this would violate the assumption that type 1 consumers have strictly concave utility functions. Hence, the correlated equilibrium allocation is outside the convex hull of the set of nonsunspot Nash equilibrium allocations (here a singleton).

5. Concluding remarks.

What we present here is a simple family of examples of correlated equilibrium in market games. The examples are driven by asymmetric information. Income is not transferred across states of nature. The correlated equilibria are sunspot equilibria to the related securities games; see Peck and Shell (1985, 1988).[2] The limit economy is competitive. It posesses a correlated equilibrium driven by asymmetric information. This equilibrium is also sunspot equilibrium to the related securities economy.

We do not provide here an example which combines asymmetric-information effects with transfers of income across states of nature. To do this, we would have to specify fully the preferences of consumers of type 1, but this should not be difficult to do. If income is transferred across states, there must be money and credit available to the consumers. If consumers can be debtors, then bankruptcy rules are important. We tackle this problem in Peck and Shell (1988, Example 5.8), but the results seem to depend heavily on the specific form of the bankruptcy rule. Further progress in analyzing extrinsic uncertainty with asymmetric or incomplete information may be related to progress in analyzing bankruptcy in market economies subject to uncorrelated or imperfectly correlated random disturbances.

We have shown how economies of fully rational individuals generate sunspot equilibrium allocations. In the competitive economy this means that the perfect rationality of consumers does not ensure perfect rationality of social outcomes. It would be interesting to investigate the role of bounded individual rationality on sunspot equilibria, but this is not done here.[3]

FOOTNOTES

1 We suppress the subscript h in the remainder of the paper. There should be no confusion.

2 For a general discussion of the role of extrinsic uncertainty in games and market economies, see Shell (1988).

3 Some of this work is currently in progress.

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