LUMP-SUM TAXATION: THE STATIC ECONOMY

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1. INTRODUCTION

How does government fiscal policy affect the allocation of resources? This is a central question for the theory and practice of public finance. In a series of three papers (Balasko and Shell, 1981a, 1981b, 1986), we analyzed the effects of money lump-sum taxes and transfers in perfect-foresight, competitive, overlapping-generations economies. In the present chapter, we base our analysis on the finite (essentially static), pure-exchange, general-equilibrium, competitive model of Arrow, Debreu, and McKenzie. This should serve as a basis of comparison for similar analyses in more complicated economies. It also gives us the opportunity to present work in the tradition of Lionel McKenzie. In particular, we indicate that some of McKenzie's ideas for establishing the existence of nonmonetary equilibrium also turn out to play an important role in the analysis of monetary economies.

The government is assumed to be able to levy lump-sum taxes and distribute lump-sum transfers in terms of the monetary unit. The price of money (relative to commodities) is determined in the market.

The government's tax-transfer policy, or *fiscal policy*, is said to be *balanced* if the algebraic sum of taxes is zero. A fiscal policy is said to be *bonafide* (for a given economy) if it permits a competitive equilibrium in which the goods price of money is not zero; thus, if a fiscal policy is not

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bonafide, it can have no effect on resource allocation. We establish for the static economy that a fiscal policy is bonafide if and only if it is balanced. (In finite, dynamic economies, balanced fiscal policies are those in which the public debt is retired at the final date. In dynamic economies with infinite time horizons, there can be bonafide policies that are not balanced, and balanced policies that are not bonafide.)

The government seeks to control the economy through its fiscal policy. We show that the government is able to move continuously from one bonafide fiscal policy to another without leaving the set of bonafide policies. Lump-sum taxes and transfers are fully potent in the following sense: (1) each competitive equilibrium allocation (with or without taxation) is Pareto-optimal; and (2) each Pareto-optimal allocation is achievable as a competitive equilibrium with an appropriately chosen fiscal policy, without reallocating endowments.

There is, however, a major obstacle that the government faces in its attempt to implement a given Pareto-optimal allocation through the imposition of tax policy. Competitive equilibrium is not unique. Indeed, for each bonafide fiscal policy, there is a continuum of competitive equilibria. The set of equilibrium money prices contains zero and a neighborhood of zero. For some economies, the set of equilibrium money prices is an interval. For other economies, the set of equilibrium money prices is not connected.

In the traditional model, taxes are levied in terms of accounting units (rather than in monetary units). For this case, we establish that the set of bonafide policies in the Arrow–Debreu–McKenzie economy is a bounded subset of the balanced policies. Sufficiently small balanced fiscal policies are contained in the set of bonafide policies. The government can move continuously from one bonafide policy to another without leaving the set of bonafide policies.

Costless lump-sum taxation is an idealization, representing the unrealistic case in which fiscal policy is fully potent. We hope that the present study will serve as a springboard for future research that reflects the fact that actual taxes are limited in scope (see, e.g., Mirrlees, 1986) or, more generally, that actual taxes are costly to administer (see, Heller and Shell, 1974), and that competition is imperfect (see Peck, Shell, and Spear, 1992).

The concept of bonafide policies was introduced and extensively analyzed for overlapping-generations economies by Balasko and Shell (1981a, 1981b, 1986). See also Balasko and Shell (1985). This chapter is based on the static model developed in our unpublished working papers, Balasko and Shell (1980, 1983).

Our model is presented in Section 2. Bonafide fiscal policies are analyzed in Section 3. Section 4 contains the analysis of the set of equilibrium money prices. In Section 5, we discuss the robustness of the results. For finite economies, only our implicit assumption of the irreducibility of the economy (see McKenzie, 1959)—or, alternatively, of resource-relatedness (see Arrow and Hahn, 1971)—is critical. Our basic results do not, however, extend to dynamic economies in which the time horizon is infinite.

2. PRELIMINARIES

There are l commodities and n consumers. The preferences of each consumer are defined on the strictly positive orthant; the consumer's consumption set is $\mathbb{R}^l_{++}$. We make the following assumptions to simplify the analysis. Preferences are strictly increasing and strictly convex. Each indifference set is a smooth surface. The closure of each indifference surface in $\mathbb{R}^l$ is contained in $\mathbb{R}^l_{++}$. (This last assumption allows us to avoid boundary problems involving free commodities.)

Let p^j denote the price of commodity j ($j = 1, \ldots, l$), and let $p = (p^1, \ldots, p^j, \ldots, p^l)$ denote the vector of commodity prices. Choose commodity 1 as the numeraire and define the set of strictly positive, normalized commodity prices $\mathcal{P} = \{p \in \mathbb{R}^l_{++} | p^1 = 1\}$. We denote by $x_h = (x^1_h, \ldots, x^j_h, \ldots, x^l_h) \in \mathbb{R}^l_{++}$ the consumption plan for consumer h ($h = 1, \ldots, n$) and define $x = (x_1, \ldots, x_h, \ldots, x_n) \in (\mathbb{R}^l_{++})^n$, the commodity allocation vector. Demand for commodities by consumer h is obtained by maximizing that individual's preferences on the budget set $\{x_h \in \mathbb{R}^l_{++} | p \cdot x_h \leq w_h\}$, where w_h denotes the income of consumer h. For fixed $p \in \mathcal{P}$ and fixed $w_h \in \mathbb{R}_{++}$, this constrained maximization problem has a unique solution denoted by $f_h(p, w_h) \in \mathbb{R}^l_{++}$. Thus f_h: $\mathcal{P} \times \mathbb{R}_{++} \to \mathbb{R}^l_{++}$ is this consumer's demand function for the l commodities. Let $\omega_h = (\omega^1_h, \ldots, \omega^j_h, \ldots, \omega^l_h) \in \mathbb{R}^l_{++}$ denote the commodity endowments of consumer h, and define $\omega = (\omega_1, \ldots, \omega_h, \ldots, \omega_n) \in (\mathbb{R}^l_{++})^n$, the endowment allocation vector.

Next define the *fiscal policy* (or, a system of nominal (money) lump-sum taxes) by the vector $\tau = (\tau_1, \ldots, \tau_h, \ldots, \tau_n)$. The *set of feasible fiscal policies*, denoted by $\mathcal{T}$, is either $\mathbb{R}^n$ or a subset of $\mathbb{R}^n$. If τ_h is positive, consumer h is being taxed a positive amount, but if τ_h is negative, he is receiving a positive transfer (i.e., is "paying" a negative tax). If the government accepts only money in payment of taxes (i.e., it refuses to accept commodities of equal value), then the set of feasible fiscal policies $\mathcal{T}$ is given by $\{\tau \in \mathbb{R}^n | \Sigma_1^n \tau_h \leq 0\}$. That is, the government must distribute at least as much money as it collects. If, however, the government does accept commodities in lieu of money for payment of taxes, then τ_h ($h = 1, \ldots, n$) can be any real number and $\mathcal{T} = \mathbb{R}^n$. The former description of $\mathcal{T}$ is obviously more realistic than the latter one, but our formal results apply equally well for either description; only the interpretations differ.

We next define balanced fiscal policies. These policies play a central role in the analysis of the finite economy with money taxes and transfers.

DEFINITION 2.1. A fiscal policy $\tau = (\tau_1, \ldots, \tau_h, \ldots, \tau_n) \in \mathcal{T}$ is said to be *balanced* if $\Sigma_1^n \tau_h = 0$. The *set of balanced fiscal policies* $\mathcal{T}_{\text{bal}}$ is defined by $\mathcal{T}_{\text{bal}} = \{\tau \in \mathcal{T} | \Sigma_1^n \tau_h = 0\}$.

If τ is balanced in the static economy, then the supply of outside money is zero. In the perfect-foresight *finite-horizon* dynamic economy, balanced fiscal policies also play a central role, even though they allow for nonzero outside money supplies (on a period-by-period basis) (see Ricardo, 1817, Ch. 17; Lerner, 1947; Starr, 1974; Balasko and Shell, 1986). Neo-Ricardians such as Barro (1974) have incorrectly assigned the same central role to balanced fiscal policies in infinite-horizon models.

The price of money in terms of commodity 1 is p^m, a nonnegative scalar. We then have the income identities

$$w_h = p \cdot \omega_h - p^m \tau_h \tag{2.2}$$

for $h = 1, \ldots, n$. Income of consumer h is equal to the value of his commodity endowments minus the value of his tax obligation.

Let $q = (p, p^m)$ and $Q = \{q = (p, p^m) | p \in \mathcal{P} \text{ and } p^m \in \mathbb{R}_+\}$. We can now define competitive equilibrium.

DEFINITION 2.3. The price vector $q = (p, p^m) \in Q$ is said to be a *competitive equilibrium* associated with the endowments $\omega \in (\mathbb{R}_{++}^l)^n$ and the fiscal policy $\tau \in \mathcal{T}$ if it satisfies the equations

$$\sum_{h=1}^{n} f_h(p, w_h) = \sum_{h=1}^{n} \omega_h = r$$

and

$$w_h = p \cdot \omega_h - p^m \tau_h \qquad \text{for} \quad h = 1, \ldots, n,$$

where $r \in \mathbb{R}_{++}^l$ is the vector of aggregate resources. The competitive equilibrium $q = (p, p^m)$ is said to be *proper* if $p^m \neq 0$. The *set of competitive equilibria* associated with ω and τ is denoted by $Q(\omega, \tau) \subset Q = \mathcal{P} \times \mathbb{R}_+$.

The following proposition and its proof establish that there is always (at least the obvious) competitive equilibrium.

PROPOSITION 2.4. *The set $Q(\omega, \tau)$ is not empty. That is, there exists a competitive equilibrium.*

Proof. Set p^m equal to 0. Then $p^m\tau_h = 0$ and $w_h = p \cdot \omega_h$ for $h = 1, \ldots, n$. The economy then reduces to a standard exchange economy without taxes and transfers. Equilibrium is thus assured (see, e.g., Arrow and Hahn, 1971, Chapter 5). That is, there is $p \in \mathcal{P}$ such that $q = (p, 0) \in Q(\omega, \tau)$. ∎

If we change the monetary unit, then the set of competitive equilibria is changed in a simple way. The following proposition, which can be thought of as "absence of money illusion," spells this out.

PROPOSITION 2.5. *For each positive scalar λ, we have*

$$Q(\omega, \lambda\tau) = \{(p, (p^m/\lambda)) | (p, p^m) \in Q(\omega, \tau)\}.$$

Proof. From Definition 2.2, $w_h = p \cdot \omega_h - p^m\tau_h$ for $h = 1, \ldots, n$. Since the only effect of τ or p^m on the demand function f_h is through w_h, the result follows immediately from Definition 2.3. ∎

Proposition 2.5 is *not* a statement of the quantity theory of money. Doubling τ does double the outside money supply, but if τ is balanced,[1] the outside money supply is zero. Doubling τ is consistent with halving p^m, but that is not by any means the only possible outcome.

3. BONAFIDE FISCAL POLICIES

By Proposition 2.4, a competitive equilibrium always exists. However, for a given fiscal policy τ, all such equilibria might be trivial in the sense that the equilibrium price of money is zero. This reduces to the case without real taxes and transfers since then $p^m\tau = 0$ and $w_h = p \cdot \omega_h$ for $h = 1, \ldots, n$. If for given endowments ω, the government were to choose a fiscal policy τ with the property that $Q(\omega, \tau)$ contains no *proper* competitive equilibrium, then the government could not have a "*good faith*" expectation that its fiscal policy would affect the allocation of resources. This idea is formalized in the following definition.

DEFINITION 3.1. Fix endowments ω and the preferences of the n consumers. The fiscal policy $\tau \in \mathcal{T}$ is said to be *bonafide* if there is a *proper* competitive equilibrium $q = (p, p^m) \in \mathcal{P} \times \mathbb{R}_{++}$ associated with τ. The fiscal policy is said to be *normalized bonafide* if $q = (p, 1)$ is a (proper) competitive equilibrium associated with τ. Let $\mathcal{T}_{\text{bon}}$ denote the *set of bonafide fiscal policies* and let $\mathcal{T}_{\text{bon}}^{n}$ denote the set of *normalized bonafide fiscal policies*; $\mathcal{T}_{\text{bon}}^{n} \subset \mathcal{T}_{\text{bon}} \subset \mathcal{T}$.

[1] We show in the next section that if τ is not balanced (i.e., the outside money supply is not zero), then the equilibrium price of money must be zero.

From Proposition 2.4, the so-called "absence of money illusion" theorem, we know that the set of bonafide fiscal policies $\mathcal{F}_{\text{bon}}$ is the nonnegative cone in $\mathcal{F}$ generated by $\mathcal{F}^{\text{n}}_{\text{bon}}$, the set of normalized bonafide fiscal policies. The set $\mathcal{F}^{\text{n}}_{\text{bon}}$ is a "cross section" of the cone $\mathcal{F}_{\text{bon}}$.

The following proposition begins our program of establishing the relationship between bonafide fiscal policies and balanced fiscal policies.

PROPOSITION 3.2. *If the fiscal policy $\tau \in \mathcal{F}$ is bonafide, then τ is balanced. That is, the set $\mathcal{F}_{bon}$ is included in the set $\mathcal{F}_{bal}$.*

Proof. Since preferences are unsatiated, we have $p \cdot f_h(p, w_h) = w_h = p \cdot \omega_h - p^m \tau_h$ for $h = 1, \ldots, n$. Hence, using Definition 2.3 yields

$$p \cdot \sum_{h=1}^{n} f_h(p, w_h) = p \cdot \sum_{h=1}^{n} \omega_h - p^m \sum_{h=1}^{n} \tau_h = p \cdot \sum_{h=1}^{n} \omega_h,$$

and thus we have

$$p^m \sum_{h=1}^{n} \tau_h = 0.$$

We have shown that if the equilibrium (p, p^m) is proper, then the fiscal policy τ is balanced. ■

For any finite economy, Proposition 3.2 is a trivial consequence of Walras's law. In order to consider more interesting fiscal policies, one must go beyond the finite-horizon models to infinite economies such as the overlapping-generations model introduced by Samuelson (1958). That the balance of taxes and transfers permits a positive price of money in finite economies is neatly articulated in Starr (1974). Starr credits Lerner (1947) with the idea that the state through its taxing power can (at least temporarily) create fiat money that may bear a positive equilibrium price.

The next two propositions state that with lump-sum taxes as its only instrument the government's fiscal policy is fully potent, a well-known and important result in the public finance literature. We then use these propositions in analyzing the structure of the set of bonafide taxes.

PROPOSITION 3.3. *Fix resources $r \in \mathbb{R}^l_{++}$. Any Pareto-optimal allocation is a competitive equilibrium allocation associated with the fixed endowment vector $\omega = (\omega_1, \ldots, \omega_h, \ldots, \omega_n) \in (\mathbb{R}^l_{++})^n$ satisfying $\Sigma_1^n \omega_h = r$ and some fiscal policy $\tau = (\tau_1, \ldots, \tau_h, \ldots, \tau_n) \in \mathcal{F}$.*

Proof. We can restrict τ to the set $\mathcal{F}^{\text{n}}_{\text{bon}}$ and fix the value of p^m at 1.

A commodity price vector $p \in \mathcal{P}$ is said to *support* an allocation $x = (x_1, \ldots, x_h, \ldots, x_n)$ if for each consumer h ($h = 1, \ldots, n$) there is an income $w_h \in \mathbb{R}_{++}$ such that that individual's preferences are maximized at x_h given that person's budget set defined by prices p and income w_h.

Let $\mathfrak{P} \subset (\mathbb{R}^{l}_{++})^n$ be the set of Pareto-optimal allocations $x = (x_1, \ldots, x_h, \ldots, x_n)$ defined by the given consumer preferences and the given resources r. Let g: $\mathfrak{P} \to \mathscr{P}$ be the mapping that associates the Pareto-optimal allocation x with the unique price vector $p \in \mathscr{P}$ that supports x (see, e.g., Arrow and Hahn, 1971, Chapters 4 and 5). Define $\phi(x)$ by $\phi(x) = (g(x) \cdot (\omega_1 - x_1), \ldots, g(x) \cdot (\omega_h - x_h), \ldots, g(x) \cdot (\omega_n - x_n))$. We have $\phi(x) \in \mathbb{R}^n$. We also have $\Sigma_1^n x_h = r = \Sigma_1^n \omega_h$ and hence $g(x) \cdot \Sigma_1^n(\omega_h - x_h) = 0$, i.e., $\phi(x) \in \mathscr{F}_{\text{bal}} \subset \mathscr{F}$.

Next observe that if $q = (p, 1) \in Q(\omega, \tau)$ is a competitive equilibrium price system, associated with the fiscal policy $\tau = (\tau_1, \ldots, \tau_h, \ldots, \tau_n)$, then the competitive equilibrium allocation vector is $(f_1(p, p \cdot \omega_1 - \tau_1), \ldots, f_h(p, p \cdot \omega_h - \tau_h), \ldots, f_n(p, p \cdot \omega_n - \tau_n))$. Then $\tau = \phi(x)$ is the unique normalized fiscal policy that decentralizes the Pareto-optimal allocation x. ■

Proposition 3.3 is the lump-sum taxation version of the second welfare theorem. In the next proposition, provided for completeness, we give the lump-sum taxation version of the first welfare theorem.

PROPOSITION 3.4. *Let x be the allocation associated with the competitive equilibrium $q = (p, p^m) \in Q(\omega, \tau)$. Then x is Pareto-optimal with respect to resources $r = \Sigma_1^n \omega_h$ and the given consumer preferences.*

Proof. From Definition 2.3 we have $\Sigma_1^n x_h = r$ and hence $p \cdot \Sigma_1^n x_h = p \cdot r$. Assume that the allocation $y = (y_1, \ldots, y_h, \ldots, y_n)$ is Pareto-superior to $x = (x_1, \ldots, x_h, \ldots, x_n)$. Then $p \cdot y_h \geqq p \cdot x_h$ for $h = 1, \ldots, n$ with strict inequality for at least one h. Hence, we have $p \cdot \Sigma_1^n y_h > p \cdot r$, which contradicts $\Sigma_1^n y_h \leqq r$. The allocation y is not feasible if resources are fixed at r. ■

The next proposition is our basic result of this section. In it, we describe the fundamental properties of the set of bonafide fiscal policies. We work with a "cross section" of that set, the set of normalized bonafide fiscal policies. Among other things, we establish a continuity property of the set of bonafide policies. Our formal continuity property is stated in terms of arc-connectedness, a generalization of the familiar notion of convexity. Before stating our results, we provide the definition of arc-connectedness (see, e.g., Arrow and Hahn, 1971, pp. 400–401).

DEFINITION 3.5. The set S is said to be *arc-connected* if for every $s^0 \in S$ and $s^1 \in S$, there is a continuous function, $s(t)$, defined for $t \in [0, 1]$, with the property $s(0) = s^0$ and $s(1) = s^1$, and $s(t) \in S$ for each $t \in [0, 1]$.

That is, any two points in an arc-connected set can be joined by a continuous path lying entirely in the set.

PROPOSITION 3.6. *The set of normalized bonafide lump-sum taxes $\mathscr{T}^{n}_{bon}$ is bounded, arc-connected, and contains 0 in its relative interior.*

Proof.

(i) Boundedness. Let ϕ and g be the continuous mappings defined in the proof of Proposition 3.3. Let $\overline{\mathfrak{P}}$ be the closure of the set of Pareto-optimal allocations. The mappings g and ϕ have continuous extensions from $\mathfrak{P}$ to $\overline{\mathfrak{P}}$. The image $\phi(\overline{\mathfrak{P}})$ is compact, and hence bounded. Since we have $\phi(\mathfrak{P}) \subset \phi(\overline{\mathfrak{P}})$, $\phi(\mathfrak{P})$ is bounded; i.e., $\mathscr{T}^{n}_{\text{bon}}$ is bounded.

(ii) Arc-connectedness. It is well known that the set of Pareto-optimal allocations, $\mathfrak{P}$, is arc-connected (see, e.g., Balasko, 1979, Appendix 3, pp. 378–379). Hence, the set $\mathscr{T}^{n}_{\text{bon}}$ is arc-connected as the image by the continuous mapping ϕ of the set $\mathfrak{P}$.

(iii) Interiority of zero. Restrict attention to the subspace $\{\tau \in \mathscr{T} | \Sigma_1^n \tau_h = 0\} = \mathscr{T}_{\text{bal}}$. Let $q = (p, 1) \in Q(\omega, \tau)$. It results from the numeraire assumption, $p^1 = 1$, and the restriction $p^m = 1$, that $q = (p, 1)$ also belongs to the set $Q(\omega', 0)$ where $\omega' = (\omega'_1, \dots, \omega'_h, \dots, \omega'_n)$ is given by

$$(\omega_h^k)' = \omega_h^k \quad \text{for} \quad k = 2, \dots, l, \quad h = 1, \dots, n,$$

$$(\omega_h^1)' = \omega_h^1 - \tau_h \quad \text{for} \quad h = 1, \dots, n,$$

and

$$\omega'_h = \left((\omega_h^1)', \dots, (\omega_h^k)', \dots, (\omega_h^l)'\right) \quad \text{for} \quad h = 1, \dots, n,$$

provided that $\omega_h^1 - \tau_h > 0$ for $h = 1, \dots, n$. The set of vectors τ satisfying the above contains zero and is open in $\mathscr{T}_{\text{bal}}$. Since a (nonmonetary) competitive equilibrium obviously exists for these constructed no-taxation economies, the τ values thus constructed belong to $\mathscr{T}^{n}_{\text{bon}}$. ■

The following corollary states that set of bonafide fiscal policies is identical to the set of balanced fiscal policies.

COROLLARY 3.7. *The fiscal policy $\tau \in \mathscr{T}$ is bonafide if and only if it is balanced. That is, the set $\mathscr{T}_{bon}$ is identical to the set $\mathscr{T}_{bal}$.*

Proof. From Proposition 3.2, we have $\mathscr{T}_{\text{bon}} \subset \mathscr{T}_{\text{bal}}$. From Proposition 3.6, it follows that if τ belongs to $\mathscr{T}_{\text{bal}}$, then for each sufficiently large scalar θ, (τ/θ) belongs to $\mathscr{T}^{n}_{\text{bon}}$. Hence, from Proposition 2.5, we know that τ belongs to $\mathscr{T}_{\text{bon}}$. ■

The set of normalized bonafide fiscal policies has been useful in our analysis of bonafide policies (see, Corollary 3.7). It is also a useful construct for the analysis of equilibrium money prices (see, Section 4). But the set $\mathscr{T}^{n}_{\text{bon}}$ is interesting in its own right. In the traditional analysis of lump-sum fiscal policies, taxes and transfers are made in units of account rather than in monetary units. The traditional model is thus a special case of the present model, in which p^m is assumed to be identical to 1. Then the set of bonafide fiscal policies—policies consistent with equilibrium in which $p^m = p^1 = 1$—is the same as our set $\mathcal{T}^n_{bon}$ (Definition 3.1). If we accept the fiction of government taxes and transfers in accounting units (or in units of a given commodity), then we have from Proposition 3.6 that the set of bonafide fiscal policies is a bounded, arc-connected *subset* of the set balanced fiscal policies and that $\tau = 0$ lies in its relative interior. In the traditional model, not all balanced policies are bonafide, but sufficiently small balanced policies are bonafide (see Balasko and Shell, 1985).

We return in what follows to the case where taxes and transfers are denominated in money.

4. THE SET OF EQUILIBRIUM MONEY PRICES

Propositions 3.4 and 3.3 are respectively the lump-sum taxation versions of the first and second theorems of welfare economics. The second theorem (Proposition 3.3) is not quite as powerful as it might appear to be. It is true that each Pareto-optimal allocation is achievable as a competitive-equilibrium allocation for some properly chosen fiscal policy. If, however, the fiscal policy is bonafide, the competitive equilibrium allocation is not unique. This is easy to see. If τ is bonafide, there are at least two distinct equilibria: one with a zero money price (see, the proof of Proposition 2.4) and another with a positive price of money (see, Definition 3.1).

We shall establish that there is, in fact, at least a continuum of equilibria for each bonafide fiscal policy. (The vast multiplicity of equilibria is a common property of models with nominal assets.) The government imposes τ. The set of possible outcomes $Q(\omega, \tau)$ is infinite. A Pareto-optimal allocation will result, but it may very well not be the one the government is seeking.

Proposition 2.5 permits us to use Proposition 3.6 in analyzing the set of p^m consistent with competitive equilibrium for a given economy and a given fiscal policy. We begin with the basic definition.

DEFINITION 4.1. The *set of equilibrium money prices* $\mathcal{P}^m(\omega, \tau) \subset \mathbb{R}_+$ for a given economy with fiscal policy $\tau \in \mathcal{T}$ is defined by

$$\mathcal{P}^m(\omega, \tau) = \{p^m \mid (p, p^m) \in Q(\omega, \tau)\}.$$

Now we can derive some properties of the set $\mathcal{P}^m(\omega, \tau)$.

PROPOSITION 4.2. *Let $\tau \neq 0$ be a nontrivial, (not necessarily normalized) balanced fiscal policy. Then the set $\mathcal{P}^m(\omega, \tau)$ is bounded. Furthermore, 0 belongs to $\mathcal{P}^m(\omega, \tau)$ and there is an interval of sufficiently small positive $p^m \in \mathcal{P}^m(\omega, \tau)$ so that 0 is not isolated.*

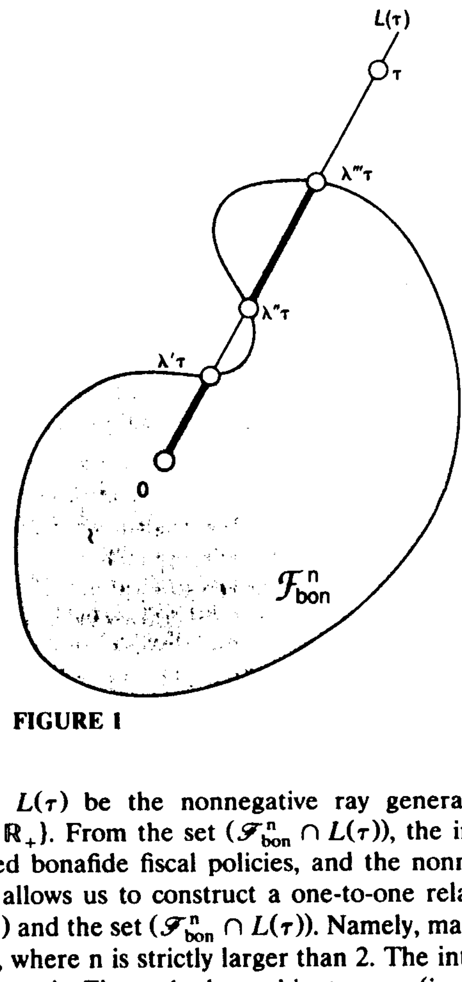

FIGURE 1

Proof. Let $L(\tau)$ be the nonnegative ray generated by $\tau \in \mathcal{F}_{bal}$, $L(\tau) = \{\lambda\tau | \lambda \in \mathbb{R}_+\}$. From the set $(\mathcal{F}^{n}_{bon} \cap L(\tau))$, the intersection of the set of normalized bonafide fiscal policies, and the nonnegative ray $L(\tau)$. Proposition 2.5 allows us to construct a one-to-one relationship between the set $\mathcal{P}^m(\omega, \tau)$ and the set $(\mathcal{F}^{n}_{bon} \cap L(\tau))$. Namely, map p^m in $\mathcal{P}^m(\omega, \tau)$ to $p^m\tau$ in $\mathcal{F}^{n}_{bon}$, where n is strictly larger than 2. The intersection of $\mathcal{F}^{n}_{bon}$ with $L(\tau)$ is shown in Figure 1; the ambient space (i.e., the space of the printed page) is $\mathcal{F}_{bal}$. The set $\mathcal{P}^m(\omega, \tau)$ is illustrated in Figure 2. Proposition 4.2 now can be shown to follow from Proposition 3.6. The boundedness of $\mathcal{F}^{n}_{bon}$ implies the boundedness of $\mathcal{P}^m(\omega, \tau)$. Since $\mathcal{F}^{n}_{bon}$ contains zero, $\mathcal{P}^m(\omega, \tau)$ contains zero. Since for p^m fixed at 1 there is a ball of $\tau \in \mathcal{F}_{bal}$ around the origin with the property that $\tau \in \mathcal{F}^{n}_{bon}$, we know that for fixed τ there is a positive scalar $\bar{p}^m$ such that $p^m \in \mathcal{P}^m(\omega, \tau)$ for each p^m in the interval $[0, \bar{p}^m)$. ∎

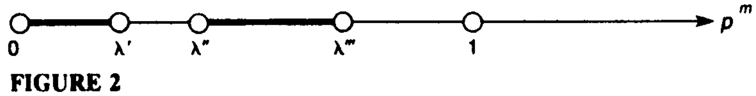

FIGURE 2

From Figures 1 and 2, we can see how the structure of the set $\mathscr{P}^m(\omega, \tau)$ is related to the structure of the set $\mathscr{T}^n_{bon}$. Figure 1 is an abstract representation. The space of the printed page is meant to represent the higher-dimensional object $\mathscr{T}^n_{bal} \subset \mathbb{R}^{n-1}$. The intersection of $\mathscr{T}^n_{bon}$ with the nonnegative ray generated by τ is indicated by two heavy line segments. The nonnegative ray intersects $\mathscr{T}^n_{bon}$ at $\lambda'\tau$, $\lambda''\tau$, and $\lambda'''\tau$. In Figure 2, $\mathscr{P}^m(\omega, \tau)$ is indicated by the two heavy line segments; $\mathscr{P}^m(\omega, \tau) = [0, \lambda') \cup (\lambda'', \lambda''')$.

If $\mathscr{T}^n_{bon}$ is convex (or merely star-shaped about the origin), then $\mathscr{P}^m(\omega, \tau)$ is a single interval. This is always the case in the two-consumer economy, $n = 2$, because $\mathscr{T}_{bal} \subset \mathbb{R}^2$ is the (one-dimensional) line $\{\tau | \tau_1 + \tau_2 = 0\}$ and $\mathscr{T}^n_{bon}$ is a (one-dimensional) connected subset of that line. Furthermore, if there is only one commodity, $l = 1$, then $\mathscr{P}^m(\omega, \tau)$ is obviously a single interval. Examples have been constructed in which the set $\mathscr{P}^m(\omega, \tau)$ is not connected.[2]

5. EXTENSIONS

There are three nearly equivalent formulations of our central result about lump-sum taxes and transfers in the competitive economy: (1) a fiscal policy is bonafide if and only if it is balanced; (2) the trivial fiscal policy $\tau = (0, \ldots, 0)$ is in the interior (relative to the set of balanced policies) of the set of normalized bona fide fiscal policies; and (3) for each balanced fiscal policy, there is a positive scalar $\bar{p}^m$ with the property that the interval $[0, \bar{p}^m)$ is contained in the set of equilibrium money prices.

How robust is our result to relaxation of our maintained hypotheses?

The strong regularity assumptions about consumer preferences (i.e., *strict* convexity, *strict* monotonicity, smoothness, and closure of indifference surfaces within the consumption set) are made only for expository convenience; they are not necessary for our basic result. These regularity assumptions are somewhat relaxed by Balasko and Shell (1986, Section 3), in which the present analysis is extended to encompass the perfect-foresight, finite-horizon, overlapping-generations economy. In an informal seminar presentation, David Cass showed that, as one would expect, the results of the present analysis hold in an exchange economy satisfying only the (weaker) assumptions (such as irreducibility or resource-relatedness)

[2] Jim Peck (1987a) has constructed an example of an economy with two commodities and three consumers, satisfying the regularity hypotheses of Section 2, in which the set of equilibrium money prices is not connected. Strong income effects drive Peck's example. See also Peck (1987b). Rod Garratt (1992) shows by example that this connectedness property can depend on the numeraire choice.

required for the standard proofs of existence of (nonmonetary) competitive equilibrium. Adding production to the model would not be difficult, nor would we expect it to change anything substantial.

On the other hand, our implicit assumption of irreducibility or resource-relatedness (cf., e.g., Arrow and Hahn, 1971, Chapter 5) plays an essential role in the analysis. To see this, consider an example with two consumers ($n = 2$) and two goods ($l = 2$). Consumer 1 likes only the first good and is endowed with only the first good. Consumer 2 likes only the second good and is endowed with only the second good. Neither consumer has endowment of the good that the other consumer desires. Hence the consumers are not resource related and the economy is reducible.

The commodity price vector is $p = (p^1, p^2) \in \mathbb{R}^2_+$. Restrict attention to the set of normalized prices $\bar{\mathcal{P}} = \{p \in \mathbb{R}^2_+ | p^1 + p^2 = 1\}$. Let $\tau = (\tau_1, \tau_2)$ be a balanced fiscal policy, so we have $\tau_1 + \tau_2 = 0$. Income of consumer h is given by $w_h = p^h \omega^h_h - p^m \tau_h$ for $h = 1, 2$, where p^m is the price of money. If $\tau_1 = \tau_2 = 0$, autarky is the unique competitive equilibrium allocation, i.e., $x^h_h = \omega^h_h$ for $h = 1, 2$. When $\tau \neq 0$, autarky (with $p^m = 0$) remains the unique competitive equilibrium allocation. The consumer with the subsidy (negative tax) desires to sell money in exchange for the commodity she desires, but the other consumer states that he cannot supply that commodity. Only the trivial fiscal policy $\tau = (0, 0)$ is bonafide. All other fiscal policies (balanced or not) are not bonafide.

Our basic result applies only to finite economies. That the number of commodities l and the number of consumers n are each finite is necessary to the linkage between bonafide and balanced policies. For the infinite-horizon overlapping-generations model, Samuelson (1958, esp. pp. 474–475) provides a nonpathological example in which bonafide policies are not necessarily balanced. See also Balasko and Shell (1981a). Balasko and Shell (1986) establish for the infinite-horizon overlapping-generations model that only *strongly* balanced fiscal policies, i.e., balanced policies with no more than a finite number of nonzero taxes, are necessarily bonafide. Balasko and Shell (1981a) provide a complete characterization of bonfide fiscal policies in the overlapping-generations economy.

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