# Economics 4905
# Financial Fragility and the Macroeconomy
# Fall 2015

Problem Set #1 Solution
Monday, September 14, 2015

Commodity, $l = 1$, chocolate. 5 consumers, $n = 5$:

$$\begin{aligned}\omega &= (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5) \\ &= (100, 90, 80, 10, 50)\end{aligned}$$

## 1 Money Taxation

1 Money. Chocolate price of money is $P^m \geq 0$.
In each of the following cases, solve for the set $\mathcal{P}^m$ of equilibrium prices $P^m$:

(a)

$$\begin{aligned}\tau &= (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5) \\ &= (1, 1, 0, -1, -1)\end{aligned}$$

(b)

$$\tau = (10, 5, 1, -5, -6)$$

(c)

$$\tau = (10, 8, 0, -8, -10)$$

**Solutions:**

(a) Mr 1: $100 - P^m > 0$ i.e. $P^m < 100$ and
Mr 2: $90 - P^m > 0$ i.e. $P^m < 90$.
Hence we have $\mathcal{P}^m = [0, 90)$

(b) $\sum_h \tau_h = 10 + 5 + 1 - 5 - 6 = 5 \neq 0$. Hence $\tau$ is not balanced. The equilibrium price of money must be $\mathcal{P}^m = \{0\}$.

(c) Mr 1: $100 - 10P^m > 0$, i.e. $P^m < 10$ and
Mr 2: $90 - 8P^m > 0$, i.e. $P^m < \frac{90}{8} = 11.25$.
Hence we have $\mathcal{P}^m = [0, 10)$.

# 2

2 Monies, red dollars $R$ and blue dollars $B$, with respective chocolate prices of money, $P^B \geq 0$ and $P^R \geq 0$.
In each of the following cases, solve for the equilibrium exchange rate between $B$ and $R$. Do these depend on $\omega$? Give the economic explanation.

(a)
$$\tau^R = (1, 1, 1, 0, 0), \;\; \tau^B = (0, 0, 0, -1, -1)$$

(b)
$$\tau^R = (1, 1, 1, -1, -1), \;\; \tau^B = (1, 0, 0, 0, 0)$$

(c)
$$\tau^R = (1, 0, 0, 0, -1), \;\; \tau^B = (2, 0, 0, -1, -1)$$

**Solutions:**

(a) We have $(1 + 1 + 1 + 0 + 0)R = (0 + 0 + 0 + 1 + 1)B$ or $3R = 2B$. Hence $\frac{R}{B} = \frac{2}{3}$ and $\frac{B}{R} = \frac{3}{2}$. The exchange rate is $\frac{2}{3}$ or $\frac{3}{2}$.

(b) $\sum_h \tau^R > 0$ and $\sum_h \tau^B > 0$. Hence $P^R = P^B = 0$. The exchange rate $= \frac{0}{0}$ is indeterminate.

(c) Both tax systems are balanced, $\sum_h \tau^R = \sum_h \tau^B = 0$, and the equilibrium exchange rate, $\frac{0}{0}$ is indeterminate.

These are independent of $\omega$. The supply and demand for currencies completely determines the exchange rate unless one or both currencies are worthless. If both tax policies are balanced, then the exchange rate is indeterminate since there are no currency trades.

## 3 Absence of Money Illusion

Explain the difference between "absence of money illusion" and the "quantity theory of money". Be precise (with symbols).

**Solution:** Taxes matter only through their real values. Only the term $P^m \tau_h$ matters to Mr. $h$.

(a) Absence of money illusion: Let $P^m$ be an equilibrium price of money if the tax vector is $\tau$. If the tax vector is $\lambda\tau$, where $\lambda$ is a positive scalar, then $\frac{P^m}{\lambda}$ is also an equilibrium price of money. In other words, if $\mathcal{P}^m = [0, \bar{P}^m)$ when the tax vector is $\tau$, then when the tax vector is $\lambda\tau$, $\mathcal{P}^m = [0, \frac{\bar{P}^m}{\lambda})$.

(b) Quantity Theory. If $P^m$ is an equilibrium price of money when the tax vector is $\tau$, then $\frac{P^m}{\lambda}$ is the money price when the tax vector is $\lambda\tau$.

The quantity theory is true if and only if people believe it to be true.

Absence of money illusion is a statement about sets.