

Handout on Risk Aversion

For an individual with a utility of consumption function denoted $U(C)$ that exhibits positive but diminishing marginal utility, a measure of risk aversion commonly used in Financial Economics is something called *Relative Risk Aversion (RRA)*, which is defined as follows:

$$RRA = -C \frac{U''(C)}{U'(C)}.$$

Note that because the second derivative in the numerator is negative in sign, RRA is a positive valued number. If we compare values of RRA for 2 individuals, the one with the higher RRA is deemed to be more averse to risk than the other. At this point the measure is quite abstract but we'll try to give it more substance in what follows. To do so it is useful to introduce a class of utility functions that exhibit *Constant Relative Risk Aversion (CRRA)* – which is to say that the risk aversion measure RRA has the same value irrespective of the level of consumption.

A $CRRA$ utility function is of the form

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma},$$

where γ is a parameter with any value $\gamma > 0$, except for $\gamma = 1$, in which case the function takes the form $U(C) = \ln(C)$.

For each member of this class of utility functions one can apply the preceding definition of *Relative Risk Aversion* to deduce that $RRA = \gamma$, irrespective of the level of consumption. (In the $\ln(C)$ case, $RRA = 1$). The parameter γ is often referred to as the coefficient of relative risk aversion.

If 2 individuals have different $CRRA$ utility functions, the one with the higher value of γ is deemed to be the more risk averse.

Now let's give this some substance by considering how individuals with $CRRA$ utility functions that have different values of γ would evaluate the following risky situation:

An individual's wealth will equal either 50,000 or 100,000 each with probability $\frac{1}{2}$ so that expected wealth is $E[W] = 75,000$.

The following table shows the certainty equivalent wealth W_{CE} associated with various values for the coefficient or relative risk aversion γ .

Value of γ	1	2	5	10	30
Value of W_{CE}	70,711	66,667	58,566	53,991	51,209

The first column of the table indicates that an individual with a logarithmic utility function ($\gamma = 1$) would value the risky wealth as equivalent to 70,711 with certainty. This individual would be willing to pay a risk premium of 4,289 ($= E[W] - W_{CE}$) in order to exchange the risky wealth for its expected value of 75,000. The second column in the table indicates that an individual with a value of $\gamma = 2$ values the risky wealth as equivalent to 66,667 with certainty. This individual would be willing to pay a risk premium of 8,333 in order to exchange the risky wealth for its expected value of 75,000. The risk premium is almost twice as large as that for an individual with $\gamma = 1$, confirming that the individual with the higher value for γ is, indeed, the more risk averse.

As we progress through the table, we can see that the certainty equivalent declines as the value of γ increases. Again, this indicates that risk aversion increases as γ increases. Observe the final column in the table. An individual with $\gamma = 30$ is incredibly risk averse. Such an individual would be willing to pay a risk premium of 23,791 to exchange the risky wealth for its expected value of 75,000. If the individual were to under take this transaction, he/she would be left with 51,209 with certainty – an amount that is barely more than the worst possible outcome of 50,000 for the risky wealth. This individual is so incredibly risk averse, he/she would probably be afraid to leave the house.

Empirical studies based on actual behavior of individuals have consistently yielded estimates in the range 1 to 4 for the coefficient of relative risk aversion, with a mean value for $\hat{\gamma} = 2$.