Question 1. Assume $\omega = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5) = (150, 80, 75, 25, 10)$. For each of the following, calculate the set of equilibrium money prices, P^m:

a) $\tau = (50, 25, 15, -15, -30)$
b) $\tau = (50, 10, 0, -20, -40)$
c) $\tau = (30, 20, -5, -10, -35)$
d) $\tau = (3, 0, 0, -1, -2)$
e) $\tau = (2, 2, 1, 1, -1)$

Solution: Mr. h's general problem is

$$\max_{x_h} \; u_h(x_h)$$

subject to

$$x_h + P^m \tau_h = \omega_h.$$

We want the equilibrium money prices, P^m, such that $x_h > 0$ for all h. Thus we are looking at

$$x_h = \omega_h - P^m \tau_h > 0 \text{ for all } h$$
$$\Rightarrow \omega_h > P^m \tau_h \text{ for all } h$$

Note that for $\tau_h \leq 0$ this obviously holds since $\omega_h > 0$. Thus we only need to check the condition for those h such that $\tau_h > 0$. Or we can write it as

$$P^m < \min_h \left[\frac{\omega_h}{max(0, \tau_h)}\right].$$

Noting that the price of money can be zero in equilibrium, we now calculate this value for each part in order to get the set of equilibrium money prices, which follow.

a) The tax system is not balanced ($\sum_h \tau_h = 45$), hence the set of equilibrium prices of money must be $P^m = \{0\}$.
b) $P^m \in [0, 3)$
c) $P^m \in [0, 4)$
d) $P^m \in [0, 50)$
e) As in part (a), the tax system is not balanced, so the set of equilibrium prices of money is $P^m = \{0\}$.

Question 2. There are two monies, Red(R), and Blue(B). The units are $R\$$ and $B\$$. Calculate the exchange rate between $R\$$ and $B\$$ for each of the following tax and transfer systems (giving units in your answers):

a) $\tau^R = (2, 1, 0)$, $\tau^B = (5, 3, -12)$

b) $\tau^R = (5, 4, -2)$, $\tau^B = (1, 0, 0)$

c) $\tau^R = (8, -2, -6)$, $\tau^B = (4, 1, -5)$

d) $\tau^R = (7, 2, -12)$, $\tau^B = (6, 5, -2)$

e) In (a) - (d), are your answers independent of endowments? Why?

Solution: The general problem of Mr. h now becomes

$$\max_{x_h} \; u_h(x_h)$$

subject to

$$px_h + P^{mR}\tau_h^R + P^{mB}\tau_h^B = p\omega_h,$$

where p^{mR} is the number of chocolates per $R\$$. We take the sum over all h and use the goods market clearing condition $\sum_h x_h = \sum_h \omega_h$, yielding

$$P^{mR}\sum_h \tau_h^R + P^{mB}\sum_h \tau_h^B = 0 \tag{1}$$

When well defined, we want to solve for the exchange rate between $R\$$ and $B\$$, $XR_{RB} = \frac{P^{mR}}{P^{mB}} = -\frac{\sum_h \tau_h^B}{\sum_h \tau_h^R}$. Measured as chocolates per $R\$$ relative to chocolates per $B\$$, $\frac{B\$}{R\$}$ are the units for the exchange rate.

a) $XR_{RB} = \frac{4}{3}\frac{B\$}{R\$}$

b) Neither tax system is balanced ($\sum_h \tau_h^R > 0$ and $\sum_h \tau_h^B > 0$), so we know that $P^{mR} = P^{mB} = 0$. Hence the equilibrium exchange rate is indeterminate ($XR_{RB} = \frac{0}{0}\frac{B\$}{R\$}$).

c) Now both tax systems are balanced, and the equilibrium exchange rate is again indeterminate ($XR_{RB} = \frac{0}{0}\frac{B\$}{R\$}$).

d) $XR_{RB} = 3\frac{B\$}{R\$}$

e)
  a) Yes, we can determine the equilibrium exchange rate independent of endowments. This is a purely financial market.

  b) We cannot determine the equilibrium exchange rate, irrespective of endowments.

  c) With endowments given, we can pin down say $P^{mR} \in [0, \bar{P}^{mR})$ and $P^{mB} \in [0, \bar{P}^{mB})$ such that consumption of all individuals remains positive. Then holding fixed a possible $P^{mB} > 0$ and taking P^{mR} to zero, the exchange rate goes to zero. Likewise, if $P^{mR} > 0$ is fixed, and we take P^{mB} to zero, we find the exchange rate goes to infinity. Thus we have an indeterminate exchange rate with balanced taxes, irrespective of endowments.

  d) Same as part (a).

Question 3. Let there be two consumers, Mr. 1 and Mr. 2.

a) In (τ_1, τ_2) space, graph the set of balanced taxes F_{bal}.

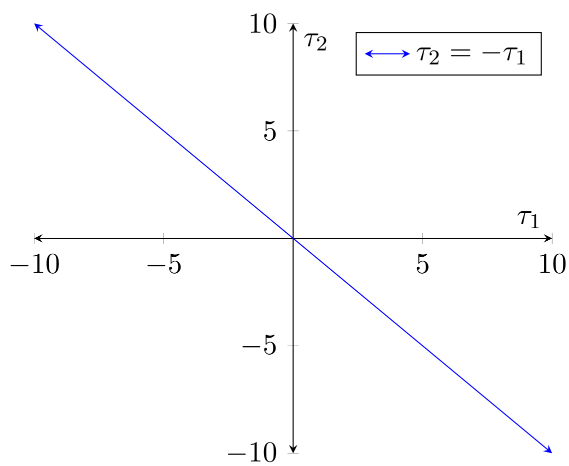
b) Graph the set of bonafide taxes, F_{bon}.

c) Let $\omega = (3, 7)$. Graph the set of normalized bonafide taxes, F^n_{bon}, i.e. the set of taxes consistent with $P^m = 2$.

d) Let $\omega = (3, 7)$. Use the diagram in (c) to calculate the set of equilibrium money prices. Hint: Use absence of money illusion.

Solution:

a) For taxes to be balanced we must have that $\sum_h \tau_h = 0$. In our case this means that we have $\tau_2 = -\tau_1$. So the graph of F_{bal} is as follows a -45° line.



b) We know that taxes are balanced if and only if they are bonafide. Hence the graph of F_{bon} is the same as the graph for F_{bal} from part (a).

c) We set $\omega = (3, 7)$ and now look for the taxes, (τ_1, τ_2), that are consistent with $P^m = 2$. We are thus looking for taxes such that $P^m = 1$ is a consistent price given that the equilibrium price interval is determined by $P^m < min[\frac{3}{max(0,\tau_1)}, \frac{7}{max(0,\tau_2)}]$. Plugging in $\tau_1 = -\tau_2$ and considering taxes consistent with $P^m = 2$ we therefore need to solve

$$2 < min\left[\frac{3}{max(0, \tau_1)}, \frac{7}{max(0, -\tau_1)}\right]$$

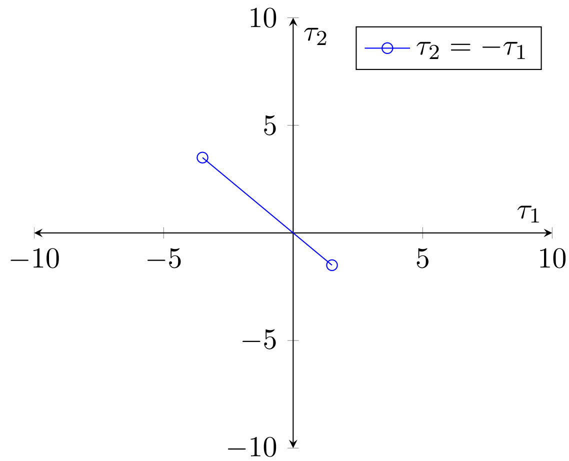
We proceed by cases.

Case 1: If $\tau_1 = 0$. Then the constraint becomes $P^m < \infty$, and hence $(0, 0)$ is consistent with $P^m = 2$.

Case 2: If $\tau_1 > 0$ then P^m is constrained by $\frac{3}{\tau_1}$. So this is consistent with $P^m = 2$ provided that $\tau_1 < \frac{3}{2}$.

Case 3: If $\tau_1 < 0$ this implies that P^m is constrained by $\frac{7}{-\tau_1} = \frac{7}{\tau_2}$. Thus for this section to be consistent with $P^m = 2$ it must be that $\tau_2 < \frac{7}{2}$.

We therefore depict the set F^n_{bon} as



d) We know that by the absence of money illusion in this model, taxes only matter through their real value. That is, only the term $P^m \tau_h$ matters to Mr. h. So if we scale τ by $\lambda > 0$ to become $\lambda\tau$, the price of money must adjust accordingly to $\frac{P^m}{\lambda}$. Then as λ approaches zero, we get that the equilibrium price of money can be $P^m \in [0, \infty)$. Likewise as λ approaches infinity, the equilibrium price of money goes to 0. This approach can therefore be used accordingly to define a mapping from a given τ vector to equilibrium prices of money.