

## Overlapping Generations II

### Question 1. Basic OLG

Consider a first economy (Economy I) that consists of overlapping generations of two-period lived people. A constant number  $N$  young people are born at each date  $t \geq 0$ , each with utility function  $u(c_t^t, c_t^{t+1}) = \log(c_t^t) + \log(c_t^{t+1})$ , where  $c_s^t$  is period  $t$  consumption for an agent born in period  $s$ . For all dates  $t \geq 1$ , young people are endowed with  $y > 0$  units of a non-storable consumption good when they are young and no endowment when they are old. In addition, there are  $N$  old people at time  $t = 0$ , who are collectively endowed with  $H$  units of unbacked fiat currency. Let  $p_t$  be the nominal price level at period  $t$ , denominated in dollars per period  $t$  goods.

- a) Define and compute an equilibrium with valued fiat currency for this economy. Argue that it exists and is unique.
- b) Could the government implement this CE allocation via tax-and-transfer policy if the initial old *were not* endowed with any fiat currency?

### Question 2. Social Security

Now consider a second economy (Economy II) that is identical to Economy I except that Economy II has a Social Security system (i.e., a pay-as-you-go pension system run by the government). The Social Security system works as follows: At each date  $t \geq 0$ , the government taxes  $\tau > 0$  units of period  $t$  consumption goods away from each young person and redistributes  $\tau$  units of the time  $t$  consumption good to each old person alive.

- a) Does Economy II possess a stationary equilibrium with valued fiat currency? Is it unique?
- b) Describe the restrictions on the parameter  $\tau$ , if any, that are needed to ensure the existence of an equilibrium with valued fiat currency.
- c) Describe how the value of currency or price level would vary across economies with differences in the size of the Social Security system (as measured by  $\tau$ ).