Inside Money (continued)

$$z_h^1 = -p^m m_h^1 = -L_h$$
$$p^2 z_h^2 = p^m m_h^1 = L_h$$

If $m_h^1 > 0$, h is a lender.
If $m_h^1 < 0$, h is a borrower.

Optimal lending $L_h = p^m m_h^1$ in period one good terms depends on P^m and m_h^1 solely through the product: $p^m m_h^1$

Let $\hat{p}^m = \lambda p^m$,
Then $\hat{L} = L$ if $\widehat{m}_h^1 = m_h^1/\lambda$

Combining equations:

$$z_h^1(p^2) + p^2 z_h^2(p^2) = 0$$

Outside Money

τ_h is the money tax on Mr. h
$\tau_h > 0$ is tax $\tau_h < 0$ is a subsidy

Consumer Problem

$$\max U_h\left(x_h^1, \dots, x_h^i, \dots x_h^l\right) \text{ subject to}$$
$$px_h = p\omega_h - p^m\tau_h \text{ or}$$
$$pz_h = -\,p^m\tau_h\,,\, h = 1\,, \dots, n$$

Equilibrium $(p, p^m) \in \mathbb{R}_{++}^{2l} \times \mathbb{R}_{+}$

$$\sum_1^n z_h = 0$$

$x_h \in \mathbb{R}_{++}^{2l}\,, \omega_h \in \mathbb{R}_{++}^{2l}\,, z_h \in \mathbb{R}^{2l}$
$\tau \in \mathbb{R}^n \quad \tau = (\tau_1, \dots, \tau_h, \dots, \tau_n)$
Summing budget constraints over h yeilds

$$p\sum_1^n z_h = p^m \sum_1^n \tau_h$$

Definitions

τ is said to be <u>balanced</u> if we have $\sum_1^n \tau_h = 0$. Ricardo for intertemporal interpretation.
τ is said to be <u>bonafide</u> if there is a competitive equilibrium.

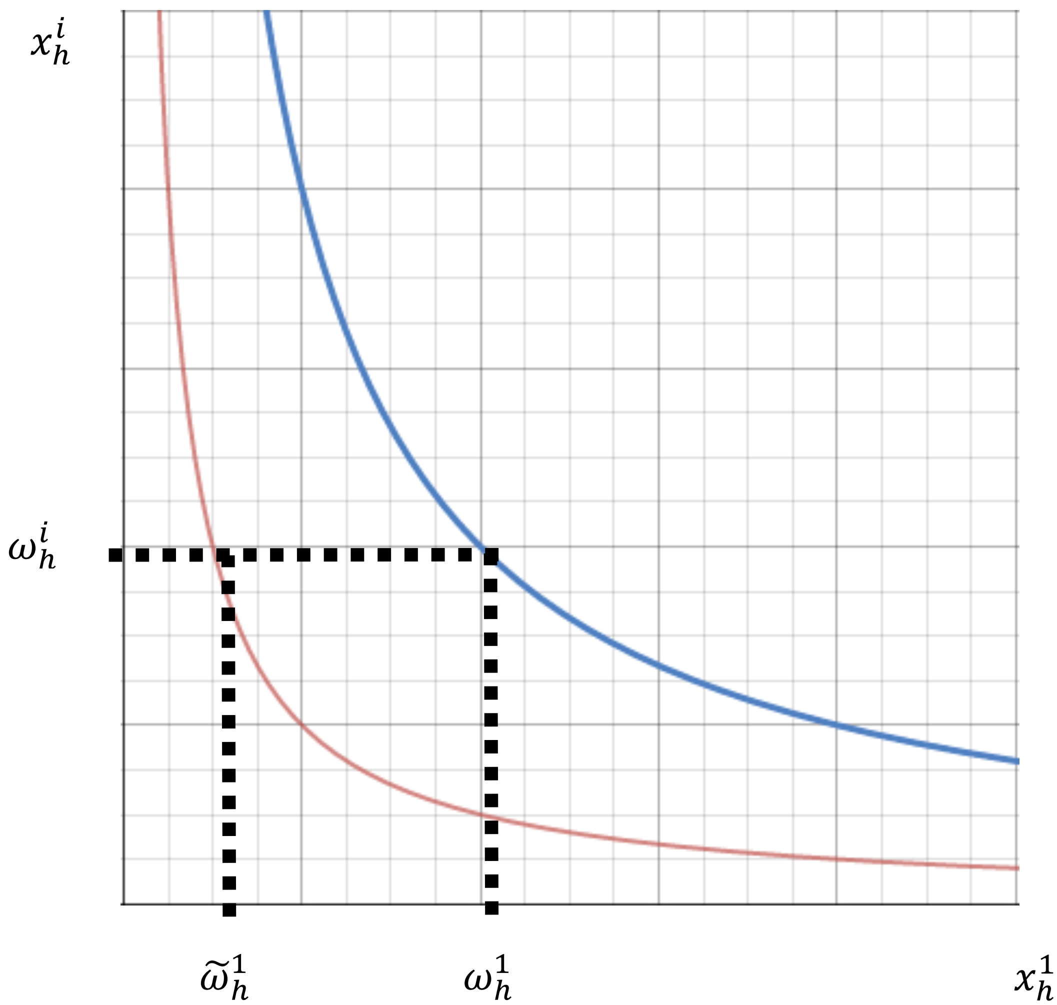
(p, p^m) with $p^m > 0$

From summed budget constraints we have

$$-p^m \sum_1^n \tau_h = 0$$

So either $p^m = 0$ or $\sum_1^n \tau_h = 0$ or both. An imbalanced τ is not bonafide
Bonafide ⇒ Balanced

Balanced ⇔ Bonafide?



Tax adjusted Endowments

$\widetilde{\omega}_h = (\widetilde{\omega}_h^1, \dots, \widetilde{\omega}_h^i, \dots, \widetilde{\omega}_h^l)$
$= (\omega_h^1 - P^m \tau_h, \widetilde{\omega}_h^2, \dots, \widetilde{\omega}_h^i, \dots, \widetilde{\omega}_h^l)$

For P^m sufficiently small, $\widetilde{\omega}_h \in \mathbb{R}_{++}^{2l}$

Special Case

$l = 1$

$x_h \in \mathbb{R}^{2l}_{++}, \omega_h \in \mathbb{R}^{2l}_{++}, p \in \mathbb{R}^{2l}_{++}, p^m \in \mathbb{R}_+$

$px_h = p\omega_h - p^m\tau_h\,,\ x_h = \omega_h - P^m\tau_h > 0$

let P^m = goods price of money
$P = 1/P^m$= general price level

Study the set of $\mathcal{P}^m$ of equilibrium P^m:
Assume $\tau_h > 0$

Then we have
$0 \leq P^m < (\omega_h/\tau_h)$

Examples

$\omega = (10,\ 8,\ 6,\ 4,\ 2)$

1. $\tau = (3,\ 3,\ 0,\ -3, -3)$

   $\sum \tau_h = 6 - 6 = 0$

   Balanced

   $3P^m < 10,\ \ P^m < \frac{10}{3}$

   $3P^m < 8,\ \ P^m < \frac{8}{3}$

   $P^m = \left[0, \frac{8}{3}\right)$

2. $\tau = (3,\ 2,\ 0,\ -3,\ -3)$

   $\sum \tau_h = 5 - 6 = -1 \quad \neq 0$

   $P^m = 0$

3. $\tau = (2,\ 0,\ 0,\ -1,\ -1)$

   $\sum \tau_h = 2 - 2 = 0$

   $2P^m < 10$

   $0 \leq P^m < 5, \quad P^m = [0,5)$

4. $\tau = (1,\ 0,\ 0,\ 0,\ 0)$

   $\sum \tau_h = 1 \neq 0$

   $P^m = 0$