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Karl Shell
402D Uris Hall
www.karlshell.com
karl.shell@cornell.edu

Lecture 2

- Time (Intertemporal economics)
  - Future markets
  - Money markets & spot markets
- Uncertainty
  - Contingent claims markets
    - “Arrow-Debreu”
  - Securities Markets & Spot Markets
    - Arrow
- Reference
  - Arrow Paper in RES
    - Translated from CNRS
    - Translated from mimeo

Future Markets

- Time, $t = 1, 2$
- Commodity $l = 1$ per period
- $x_h^t > 0\,,\, \omega_h^t > 0$ for $t = 1\,,\ 2\,;$
  $h = 1, \ldots, n$
- p^t price of commodity at time 1
  to be delivered at time t
- Present prices
  - $p^1 = 1$
  - $p^2 = \frac{1}{1+r} = \frac{1}{R}$

Futures Markets

$$p^1 x_h^1 + p^2 x_h^2 = p^1 \omega_h^1 + p^2 \omega_h^2$$

$$p^1 (x_h^1 - \omega_h^1) + p^2 (x_h^2 - \omega_h^2) \ = 0$$

$$z_h^1 + \left(\frac{1}{1+r}\right) z_h^2 = 0$$

$$h = 1, \ldots, n$$

$$\sum_{h=1}^{n} z_h^t = 0 \quad t = 1\,, 2$$

- Expectations are not up front because all trading is at time 1
- No re-trading at time t
  - By assumption

  or

  - By perfect foresight

Perfect Borrowing and Lending (Money Market)

- Futures market for commodities is closed
- Saving through lending money
- Dis-saving is through borrowing money
- Spot markets meet in $t = 1$ and

$$t = 2$$

- Reasons money market is not "perfect"
  - DSGE (RBC)
  - DSGE with borrowing constraints
- Remarks on RCK

Inside Money

- AX traveler checks
- Here extended
- Isomorphic to Arrow article
- Irving Fisher

Inside Money (continued)

- Holdings of inside money
  - Purchase of inside money

  money = $m_h^t \quad t = 1, 2 \,; h = 1, \ldots, n$
  - $\sum_{h=1}^{n} m_h^t = 0 \quad t = 1, 2$

- Outside money is created by the government and the banking system
- Present price of money

  $p^{mt} \geq 0 \quad t = 1, 2$

Monetary Equilibrium

- Consumer Problem

  max $\quad V_h(x_h^1, x_h^2)$

  subject to

  $x_h^1 + p^{m1} m_h^1 = \omega_h^1$

  $p^2 x_h^2 + p^{m2} m_h^2 = p^2 \omega_h^2$

  $m_h^1 + m_h^2 = 0$

  for $h = 1\,,2$

- Special Case
  - $V_h(x_h^1, x_h^2) = U(x_h^1) + \beta U(x_h^2)$

- Perfect Foresight about

  $p^2 \; > 0$ and $p^{m2} > 0$

- Materials Balance

  $\sum_h x_h^t = \sum_h \omega_h^t$,

  $\sum_{h=1}^{h} m_h^t = 0$ for $t = 1\,,2$

- Solve for p^2 , p^{m1} , and p^{m2}

Monetary Equilibrium (continued)

- Rewriting constraints
  - $z_h^1 = -p^{m1} m_h^1$
  - $p^2 z_h^1 = p^{m2} m_h^1$

- $z_h^1 + p^2 z_h^1 = (p^{m2} - p^{m1}) m_h^1$

- Hence, $p^{m2} = p^{m1} = p^m \geq 0$

- Equilibrium allocation $x_h \in \mathbb{R}_{++}^{2n}$

is the same as for Future Market if $p^m > 0$

Uncertainty (isomorphic to intertemporal)

- See Arrow article
- 2 states of nature $s = \alpha\ ,\ \beta$
- $h = 1, \dots, n$ consumers
- Contingent commodity $x_h(s) > 0$

delivered only in state s

- Contingent endowments $\omega_h(s) > 0$
- Preferences

$$V_h\big(x_h(\alpha), x_h(\beta)\big) \\ = \pi(\alpha) U_h\big(x_h(\alpha)\big) + \pi(\beta) U_h\big(x_h(\beta)\big)$$

Contingent Claims (continued)

- Consumer Problem

$\max \pi(\alpha)U_h\big(x_h(\alpha)\big) + \big(1 - \pi(\alpha)\big)U_h\big(x_h(\beta)\big)$

Subject to

$p(\alpha)x_h(\alpha) + p(\beta)x_h(\beta)$

$= p(\alpha)x_h(\alpha) + p(\beta)\omega_h(\beta)$

Or $\quad p(\alpha)z_h(\alpha) + p(\beta)z(\beta) = 0$

Find $\big(p(\alpha), p(\beta)\big)$ such that

- CP determines $x_h(\alpha), x_h(\beta)$

and materials balance

- $\sum_h x_h(s) = \sum_h \omega_h(s)$ for $s = \alpha, \beta$

Arrow Securities

- $b_h(s)$ is the quantity bought of security s
- Security s pays 1 unit of account in state s; otherwise, nothing
- $p_b(s)$is the price of security s
- $p_b(\alpha)b_h(\alpha) + p_b(\beta)b_h(\beta) = 0$
- Purchase of security is financed by sale of other security (not necessary)

Arrow Securities (continued)

- CP

max $\pi(\alpha)U_h\big(x_h(\alpha)\big) + \big(1 - \pi(\alpha)\big)U_h\big(x_h(\beta)\big)$ s.t.

1) $p(\alpha)x_h(\alpha) = p(\alpha)\omega_h(\alpha) + b_h(\alpha)$

2) $p(\beta)x_h(\beta) = p(\beta)\omega_h(\beta) + b_h(\beta)$

3) $p_b(\alpha)b_h(\alpha) + p_b(\beta)b_h(\beta) = 0$

- Multiply (1) by $p_b(\alpha)$ and (2) by $p_b(\beta)$

1) $p_b(\alpha)p(\alpha)z_h(\alpha) = p_b(\alpha)b_h(\alpha)$

2) $p_b(\beta)\, p(\beta)z_h(\beta)$=$p_b(\beta)b_h(\beta)$

But by *3)* we have
$\hat{p}(\alpha)z_h(\alpha) + \hat{p}(\beta)z_h(\beta) = 0$
Where $p_b(s)p(s) = \hat{p}(s)$ for $s = \alpha, \beta$

Arrow Securities (continued)

- CE is $(\hat{p}(\alpha), \hat{p}(\beta)) \in \mathbb{R}^{2n}_{++}$ in which

$(x_h(\alpha), x_h(\beta)) \in \mathbb{R}^2_{++}$ solves

PC for $h = 1, \dots, n$

and

$\sum_h z_h(s) = 0$ for $s = \alpha, \beta$

Conclusion

- Every contingent claims equilibrium allocation is also an Arrow securities equilibrium allocation
- Every AS equilibrium in which $p_b(s) > 0$ for $s = \alpha, \beta$ is also CC equilibrium allocation
- Every FM equilibrium allocation is also an MM equilibrium allocation
- Every MM equilibrium allocation in which $p^m > 0$ is also an FM equilibrium allocation
- But interpretations of MM differ widely from interpretations of FM