Cornell University
Spring 2016
Economics 4905
Financial Fragility and the Macroeconomy

Answers to Problem Set 1

(1ai) The futures market:

$x_h = (x_h^1, x_h^2) \in \mathbb{R}^2_{++}$

$p = (p^1, p^2) \in \mathbb{R}^2_{++}$

$p = (1, p^2)$ present prices

$\omega_h = (\omega_h^1, \omega_h^2) \in \mathbb{R}^2_{++}$ endowments

$z_h^t = x_h^t - \omega_h^t$ excess demand

CP: $\max \ U_h(x_h^1, x_h^2)$

s.t. $x_h^1 + p^2 x_h^2 = \omega_h^1 + p^2 \omega_h^2,$

or $z_h^1 + p^2 z_h^2 = 0.$

x is a competitive equilibrium allocation if x_h solves CP for $h = 1, \dots, n$ and materials balance

$$\sum_h x_h = \sum_h \omega_h,$$

or $\sum_1^n z_h = 0.$

(1aii) The money market:

Perfect foresight. Inside money, perfect borrowing and lending.

$p^{m,t} \geq 0$ price if money in period $t = 1, 2$. m_h^t is money added in t.

Cannot die in debt and no reason to die as creditor:

$$m_h^1 + m_h^2 = 0.$$

$p^1 = 1$ spot market price of commodity 1

$p^2 = 1/R$ present price of commodity 1 delivered in period 2

CP: $\max U_h(x_h^1, x_h^2)$

s.t. $x_h^1 = \omega_h^1 - p^{m1} m_h^1$

$p^2 x_h^2 = p^2 \omega_h^2 + p^{m2} m_h^1$

x is a competitive equilibrium if CP is satisfied for $h = 1, \ldots, n$ and

$$\sum_h x_h^t = \sum_h \omega_h^t \quad \text{for } t = 1,2$$

and $\sum_h m_h^t = 0$ for $t = 1,2$.

If $p^{m1} \neq p^{m2}$, no equilibrium exists. So set

$$p^{m1} = p^{m2} = p^m \geq 0.$$

(1aiii) If $p^{m1} = 0$, equilibrium is autarky because the only imaginable trades are intertemporal.

(1a) Let $p^m > 0$, then

$$z_h^1 = -p^m m_h^1$$
$$p^2 z_h^2 = p^m m_h^1$$

The RHS are slack variables so we have $z_h^1 + p^2 z_h^2 = 0$.
Hence, every futures market equilibrium allocation x is also a money-market equilibrium allocation. If $p^m > 0$ (i.e. if borrowing–lending markets are open), then every money-market competitive equilibrium is also a futures market equilibrium allocation.

(1b) With perfect inside money, Mr. h can choose real borrowing $-p^m m_h^1$ independent of the value of $p^m > 0$. So if $\hat{p}^m = \lambda p^m$, then $\widehat{m}_h^1 = m_h^1/\lambda$ yields the same real intertemporal trade.

So for pure inside money, the (strong) quantity theory holds. (For outside money, only the weaker "absence of money illusion" applies.)

(1c) $p^m = 0$ is the extreme case in which borrowing and lending is impossible. This simple economy is then in autarky because intertemporal trade is the only possible trading since $l = 1$.

(2a) $\sum_h \tau_h = 0$

1 , 2 , 3 are taxed

$x_1 = 100 - 50P^m > 0$, $\quad P^m < 20$

$x_2 = 100 - 4P^m > 0$, $\quad P^m < \frac{90}{4}$

$x_3 = 80 - P^m > 0$, $\quad P^m < 80$

$P^m < 20 < 90/4 < 80$

$\mathcal{P}^m = [0,20)$

(2b) $\sum_h \tau_h = 2 \neq 0$

$\mathcal{P}^m = \{0\}$

(2c) $\sum_h \tau_h = 0$

1 , 2 , 3 are taxed

$x_1 = 100 - P^m > 0$, $\quad P^m < 100$

$x_2 = 90 - P^m > 0$, $\quad P^m < 90 < 100$

$x_3 = 80 - P^m > 0,$ $\quad P^m < 80 < 90 < 100$

$\mathcal{P}^m = [0,80)$

(2d) $\sum_h \tau_h = 2 \neq 0$

$\mathcal{P}^m = \{0\}$

(3a) $\sum_h \tau_h^R = -1$

$\sum_h \tau_h^B = 3$

$p^R \sum_h \tau_h^R + p^B \sum_h \tau_h^B = 0$

$-p^R + 3p^B = 0$

$p^R = 3p^B$

$\frac{\text{red dollars}}{\text{blue dollars}} = \frac{3}{1}$

Exchange rate: 1 red dollar = 3 blue dollars

(3b) $\sum_h \tau_h^R = 5$

$\sum_h \tau_h^B = -1$

$p^R \sum_h \tau_h^R + p^B \sum_h \tau_h^B = 0$

$5p^R - 1p^B = 0$

$5p^R = p^B$

$\frac{\text{red dollars}}{\text{blue dollar}} = \frac{1}{5}$

Exchange rate: 5 red dollars = 1 blue dollar

(3c) $\sum_h \tau_h^R = -2$

$\sum_h \tau_h^B = 7$

$p^R \sum_h \tau_h^R + p^B \sum_h \tau_h^B = 0$

$-2p^R + 7p^B = 0$

$2p^R = 7p^B$

$\frac{\text{red dollars}}{\text{blue dollar}} = \frac{7}{2}$

Exchange rate: 7 red dollars = 2 blue dollar

(3d) $\sum_h \tau_h^R = 0$

$\sum_h \tau_h^B = 0$

$p^R 0 + p^B 0 = 0$

Exchange rate is indeterminate unless one money is worthless and the other is not. In this case, the exchange rate would be zero or infinite.

(3e) The exchange rate in this model is independent of the non-financial economy. So the exchange rate is independent of ω.