Lecture Notes on Bank Runs

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Bank Runs:

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Bank Runs

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$$U_I(C_I^1, C_I^2) = \begin{cases} \bar{u} + u(C_I^1 + C_I^2 - 1) & \text{if } C_I^1 \geq 1 \\ \\ \beta\bar{u} + u(C_I^1 + C_I^2 - 1) & \text{if } C_I^1 < 1 \end{cases} \tag{1}$$

and

$$U_P(C_P^1, C_P^2) = \bar{u} + u(C_P^1 + C_P^2 - 1),$$

Here: I and P denote **actually** impatient and **actually** patient.
Assuming that it is always best to take a consumption opportunity if one can, the above equations give utility as a function of withdrawals.
Feasibility is not relevant here.
Assume that the left-over cash balance (in or out of bank) is the argument of $u(\cdot)$. (Alternatively: at the end of period 2, left-over cash is deposited)

continuum of **ex-ante** identical consumers

y units of consumption

fraction α are impatient

α is uncertain, α_1 is actually withdrawal

$\overline{u}$ for best consumption opportunity

$\beta\overline{u}$ for next-best $\beta < 1$

utility of "left-over" bank balance, $u(\cdot)$

$f(\alpha)$ defined on $[0, \overline{\alpha}]$, where $0 < \overline{\alpha} < 1$, *e.g.* $\overline{\alpha} = 1/2$

$$f_P(\alpha) = \frac{(1-\alpha)f(\alpha)}{\int_0^{\bar{\alpha}}(1-a)f(a)da}.$$

Important for ICC.
Bayes Rule:

$$Prob(A|B) = \frac{Prob(B|A)Prob(A)}{Prob(B)}$$

$\mathrm{A} = \alpha$, $\mathrm{Prob(A)} = f(\alpha)$

$\mathrm{B} =$ depositor is patient

$\mathrm{Prob(B)} = \int_0^{\bar{\alpha}}(1-a)f(a)da$

$Prob(B|A) = 1-\alpha$

Now I stands for "withdraw in Period 1"

P Stands for "did not (or could not) withdraw in Period 1"

α_1 measures actually withdraw in period 1

2 Technologies (Wallace)

liquid: 1 unit yields 1 unit in period 1

OR $R_l > 1$ units in period 2

illiquid: 1 unit yields 0 in period 1

but R_i units $> R_l > 1$ in period 2

Sequential Service: z "order" in queue

Wallace:

$c^1(z)$: withdrawal in period 1 (by impatient and possibly by runners)

Lower case $c's$ introduced for study of Glass-Steagall bank

$c^1(z)$: period-1 withdrawal

$c_I^2(\alpha_1)$: period-2 withdrawal "from liquid asset" by period-1 withdrawer

$c_P^2(\alpha_1)$: period-2 withdrawal "from liquid asset" by period-1 non-withdrawer

γ : fraction of y invested in I

$$C_I^2(\alpha_1) = c_I^2(\alpha_1) + (1-\gamma)R_i y.$$

$$C_P^2(\alpha_1) = c_P^2(\alpha_1) + (1-\gamma)R_i y.$$

$C_I^2(\alpha_1) \geq 0$ and $C_P^2(\alpha_1) \geq 0$

$$\alpha_1 c_I^2(\alpha_1) + (1-\alpha_1) c_P^2(\alpha_1) = [\gamma y - \int_0^{\alpha_1} c^1(z) dz] R_\ell. \qquad \text{(RC)}$$

$$M = \{\gamma, c^1(z), c_I^2(\alpha_1), c_P^2(\alpha_1) | \text{ Equation (RC) holds for all } \alpha_1\}$$

2 Financial Systems:

- Unrestricted Bank, Unified System
- Restricted Bank, Glass-Steagall Bank, Separated System

(1) In the *separated financial system*, consumers place a fraction $(1-\gamma)$ of their wealth in technology i, whose return cannot be touched by the bank. In terms of resource constraint (RC), this is equivalent to imposing the additional constraints: $c_P^2(\alpha_1) \geq 0$ and, more importantly, $c_I^2(\alpha_1) \geq 0$. Combined with incentive compatibility, these additional constraints give rise to overinvestment in technology ℓ and the possibility of bank runs

.

(2) In the *unified financial system*, the bank is able to invest in both technologies. This allows the bank more flexibility in smoothing consumption and preventing runs. For example, when $\bar{\alpha}$ consumers arrive in period 1 (the worst case scenario), the bank can liquidate all of its technology ℓ holdings, but differentially reward consumers from technology i in period 2. Consumers who arrive in period 1 might receive less than $(1-\gamma)R_i y$ in period 2, while consumers who wait might receive more than $(1-\gamma)R_i y$. In terms of resource constraint (RC) this is equivalent to allowing $c_P^2(\alpha_1)$ or $c_I^2(\alpha_1)$ to be negative.

Sequential Service Result

$$c^1(z) = 1 \text{ for } z \leq \gamma y.$$

$$c^1(z) = 0 \text{ otherwise} \tag{3}$$

Welfare under Unrestricted Banking

$$
\begin{aligned}
W = {} & \int_0^{\gamma y} [\bar{u} + (1-\alpha)u((1-\gamma)yR_A + c_P^2(\alpha) - 1) \\
& + \alpha u((1-\gamma)yR_A + c_I^2(\alpha))] f(\alpha) d\alpha \\
& + \int_{\gamma y}^{\bar{\alpha}} [(1-\alpha+\gamma y)\bar{u} + (\alpha - \gamma y)\beta \bar{u} \\
& + (1-\alpha)u((1-\gamma)yR_A + c_P^2(\alpha) - 1) \\
& + (\alpha - \gamma y)u((1-\gamma)yR_A + c_P^2(\alpha) - 1) \\
& + \gamma y u((1-\gamma)yR_A + c_I^2(\alpha))] \; f(\alpha) d\alpha
\end{aligned} \tag{4}
$$

Incentive Compatibility (ICC):
I denotes early withdrawal (running), P denotes early non-withdrawal

$$\begin{aligned}
&\int_0^{\bar{\alpha}} u(c_P^2(\alpha) + (1-\gamma)yR_A - 1) f_P(\alpha) d\alpha \\
&\geq \int_0^{\gamma y} u(c_I^2(\alpha) + (1-\gamma)yR_A) f_P(\alpha) d\alpha \\
&+ \int_{\gamma y}^{\bar{\alpha}} (\gamma y/\alpha) u(c_I^2(\alpha) + (1-\gamma)yR_A) \\
&+ (1-\gamma y/\alpha) u(c_P^2(\alpha) + (1-\gamma)yR_A - 1) f_P(\alpha) d\alpha.
\end{aligned} \tag{5}$$

Resource constraint (RC):

$$\begin{aligned}
&\alpha_1 c_I^2(\alpha_1) + (1-\alpha_1) c_P^2(\alpha_1) = (\gamma y - \alpha_1) R_B \quad \text{if } \alpha_1 \leq \gamma y \\
&\gamma y c_I^2(\alpha_1) + (1-\gamma y) c_P^2(\alpha_1) = 0 \quad \text{if } \alpha_1 > \gamma y.
\end{aligned} \tag{6}$$

Profit-maximizing perfectly-competitive bank chooses the contract so as to:

$$\begin{aligned} &\max W \\ &\text{wrt } \gamma, c_I^2(\alpha_1), c_P^2(\alpha_1) \\ &\text{subject to ICC(5) and RC (6).} \end{aligned} \tag{7}$$

The so-called "optimal contract" for the unified system satisfies $\gamma y < \bar{\alpha}$. The “first” γy impatient consumers to arrive are fully served by the bank in period 1. There is a positive probability that $\alpha > \gamma y$ holds, in which case $(\alpha - \gamma y)$ impatient consumers are rationed. Patient consumers do not withdraw in period 1, and we have full consumption smoothing, i.e.,

$$c_I^2(\alpha_1) = c_P^2(\alpha_1) - 1 \qquad \text{for all } \alpha_1 \leq \gamma y. \tag{8}$$

Proof:

$$\left(\frac{\partial W}{\partial \gamma}\right)_{\gamma=\bar{\alpha}/y} = \int_0^{\bar{\alpha}} y(R_B - R_A)u'[c_P^2(\alpha) - 1 + (y - \bar{\alpha})R_A]f(\alpha)d\alpha < 0.$$

Consumption Smoothing:

$$\begin{aligned} c_P^2(\alpha_1) &= c_I^2(\alpha_1) + 1 && \text{for all } \alpha_1, \\ \text{So } C_P^2(\alpha_1) &= C_I^2(\alpha_1) + 1 && \text{for all } \alpha_1 \end{aligned}$$

$c_I^2(\alpha_1)$ could be negative, even though
$C_P^2(\alpha_1)$ and $C_I^2(\alpha_1)$ must be non-negative

Central Result for Unified System:

Under the optimal contract, there is no run equilibrium.

The Separated System (Restricted or Glass-Steagall Bank) holds only I

γy deposited in bank

$(1-\gamma)y$ deposited in mutual fund with gross return R_i, which

is perfectly illiquid

Glass-Steagall Constraint (GSC):

$$c_I^2(\alpha_1) \geq 0 \text{ and } c_P^2(\alpha_1) \geq 0 \text{ for all } \alpha_1. \tag{10}$$

Bank's Problem

$$\max W \quad \text{wrt } \gamma, c_I^2(\alpha_1), c_P^2(\alpha_1) \quad \text{subject to RC, ICC, and GSC.} \tag{11}$$

$$\max W(\text{ rest. bank}) \leq \max W(\text{ unrest. bank})$$

Glass-Steagall Bank

$c_P^2(1) < c_P^2(\bar{\alpha}) < 1$
Hence: always a run equilibrium

2 Systems
Unrestricted:

- Higher Welfare. Pro-growth.
- More stable: never has panic-based runs
- Can run out of cash in period 1 (not a bad thing)

Restricted

- Lower welfare. Anti-growth.
- Less stable: always subject to runs
- Only runs out of cash during panic-based runs

Sunspot-driven runs on the Glass-Steagall Bank

Run equilibrium is not an equilibrium to the pre-deposit game

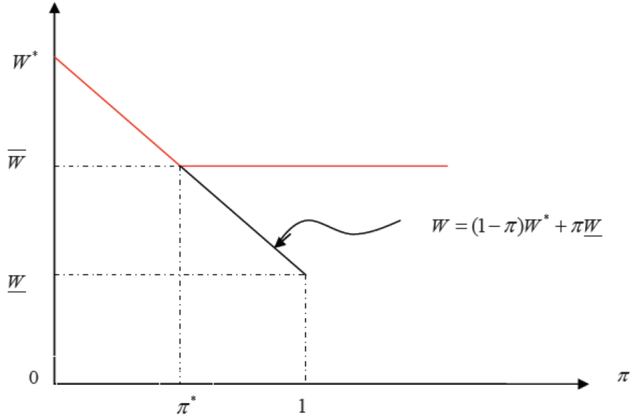
Hence introduce sunspot-triggerd runs, which occur with probability π

Let W^* be welfare under the so-called optimal contract without a run.

Let $\overline{W}$ be welfare under the best contract that is immune to runs.

Let $\underline{W}$ be welfare during a run under the so-called "optimal contract".

Optimal welfare is achieved by risking run is π is small.



Red indicates best W as function of π. If run risk is less than π^*, employ so-called "optimal contract". Otherwise, choose best contract immune to runs.

Numerical Example: The unified system

$$y = 10,\ u(c) = 100\log(c) - 249,\ \bar{u} = 20,\ R_A = 1.1,\ \beta = 0.7,$$

$$\text{uniform distribution with } \bar{\alpha} = 0.5\text{: } f(\alpha) = \begin{cases} 2 \text{ for } \alpha \in [0, 0.5] \\ 0 \text{ otherwise.} \end{cases} \tag{9}$$

- $R_B = 1.05$, we have $\gamma = 0.04544$, $\gamma y = 0.4544$ and $W = 0.8942$
- $R_B = 1.08$, we have $\gamma = 0.04807$, $\gamma y = 0.4807$ and $W = 0.9599$.

Numerical Example: The separated system (The Glass-Steagall Bank)

- $R_B = 1.08$. $\gamma = 0.09445 > 0.04807$, $W = 0.8688 < 0.9599$
- γ in GS is about twice γ in unrestricted bank.
- high γ is anti-growth

Sunspots and Glass-Steagall Example

- $R_B = 1.08$.
- best γ to avoid runs is $\overline{\gamma} = 0.09630$, $\pi^* = .5521\%$
- "Runs" back in the bank runs literature.