Economics 4905 Spring 2016
Financial Fragility and the Macroeconomy
Cornell University

Clarification of Wednesday, April 6 Lecture

$$V_h = \pi(\alpha)u_h(x_h(\alpha)) + \pi(\beta)u_h(x_h(\beta)),$$

positive marginal utility, $u_h' > 0$,
declining marginal utility, $u_h'' < 0$,
so u_h is strictly increasing and strictly concave
h is risk-averse.

Assume that $x_h(\alpha) \neq x_h(\beta)$. To show that h prefers consumption smoothing. In particular, she prefers $\bar{x}_h = \pi(\alpha)x_h(\alpha) + \pi(\beta)x_h(\beta)$ in each state to $(x_h(\alpha), x_h(\beta))$.

$$\begin{aligned} V_h(\bar{x}_h, \bar{x}_h) &= \pi(\alpha)u_h(\bar{x}_h) + \pi(\beta)u_h(\bar{x}_h) \\ &= u_h(\bar{x}_h) \qquad \text{because } \pi(\alpha) + \pi(\beta) = 1 \\ &= u_h(\pi(\alpha)x_h(\alpha) + \pi(\beta)x_h(\beta)) \\ &> \pi(\alpha)u_h(x_h(\alpha)) + \pi(\beta)u_h(x_h(\beta)) \end{aligned}$$

because the concave function u_h is above its chords.

Remark:

$$\sum_h \bar{x}_h = \sum_h \bar{x}_h(s) \quad \text{for } s = \alpha, \beta.$$