The Transactions Demand for Cash: An Inventory Theoretic Approach

William J Baumol, 1952

Key Assumptions

- Rational behavior
- Transactions are perfectly foreseen
- Transactions occur in a steady stream
- Withdraws cash at even intervals
- Cash is obtained by borrowing money or withdrawing money from investments.
- Non-coincidence of cash receipts and expenditures

Variables

- T – total value of the transactions in a time period
- i – interest opportunity cost, or interest cost in dollars per dollar per period
- C – value of each individual withdrawal
- b – broker's fee per withdrawal

Where $0 < C \leq T$

i, b are constant

Derivation

- T/C = total number of withdrawals over the time period
- bT/C = total cost of broker's fees over the time period
- C/2 = average cash holding at any given time
- iC/2 = interest cost of holding cash for the time period

Thus, the total cost the individual must pay for the use of cash is…

1) $$\frac{bT}{C} + \frac{iC}{2}.$$

Derivation

- Find the minimum C by setting the first derivative equal to zero

$$-\frac{bT}{C^2}+\frac{i}{2}=0,$$

2) $$C=\sqrt{\frac{2bT}{i}}.$$

Another Case…

- What happens when receipts precede expenditures?
  - The individual can choose not to invest and simply hold cash until needed
  - The individual can choose to invest and withdraw cash from investments when needed
- Any receipts exceeding anticipated disbursements will be invested

Variables

- T – total value of the transactions in a time period
- i – interest opportunity cost, or interest cost in dollars per dollar per period
- C – value of each individual withdrawal
- I – dollars invested
- $R = T - I$ – dollars withheld as cash
- $b_w + k_w C$ – broker's fee for withdrawal
- $b_d + k_d I$ – broker's fee for depositing

Where $R = T - I \geq 0$; $I \geq 0$

i, b, k_w, k_d are constant

Derivation

- R/T or (T-I)/T = dollars withheld from investment serve to meet payments for a fraction of the period between consecutive receipts
- (T-I)/2 = average cash holding for that fraction of time
- [(T-I)/T] i [(T-I)/2] = interest cost of withholding money
- Cost of withholding R and investing I…

$$\frac{T-I}{2} i \frac{T-I}{T} + b_d + k_d I.$$

Derivation

- I/T = dollars invested and then withdrawn serve to meet payments for the remainder of the period
- C/2 = average cash holding for the remainder of the period
- (C/2)i(I/T) = interest cost of obtaining cash from investments
- Cost of obtaining cash for the remainder of the period…

$$\frac{C}{2} i \frac{I}{T} + (b_w + k_w C) \frac{I}{C}$$

Derivation

- Total cost of cash operations

$$\frac{T-I}{2} i \frac{T-I}{T} + b_d + k_d I + \frac{C}{2} i \frac{I}{T} + (b_w + k_w C) \frac{I}{C}$$

- Differentiating w.r.t C and setting equal to zero yields

2)

$$C = \sqrt{\frac{2bT}{i}}$$

With b = b_w

Derivation

- Can also find R/2, the optimum average cash balance before drawing on invested receipts by differentiating the cost equation w.r.t. I and setting equal to zero

$$-\frac{T-I}{T}i + k_d + \frac{Ci}{2T} + \frac{b_w}{C} + k_w = 0$$

- Solving for T – I…

$$R = T - I = \frac{C}{2} + \frac{b_w T}{C\,i} + \frac{T(k_d + k_w)}{i}$$

- From equation 2 we know $C^2 = 2Tb_W/i$ thus substituting it into the second term of the equation we get

$$R = C + T\left(\frac{k_w + k_d}{i}\right)$$

Key Takeaway

- C varies as the square root of T, or that demand for cash rises less than in proportion with the volume of transactions

2) $$C = \sqrt{\frac{2bT}{i}}.$$

- Since the equation requires that average transactions velocity of circulation vary exactly in proportion with the quantity of cash, a doubling of the stocks of cash will, *ceteris paribus*, double velocity

Implications for Monetary Theory

- It generally pays to keep some cash, even in a stationary economy where there is a meaningful (finite) price level.
- There are economies of large scale in the use of cash
- The effect of real income of an injection of cash into the system was previously underestimated. In this case, people will want to get rid of cash and demand more goods and services, forcing the volume of transactions to rise and increasing employment.
  - ΔC = quantity of cash injected
  - $k \Delta C$ proportionality or constant velocity assumption
  - $\Delta T = 2k\Delta C + \frac{k}{C}\Delta C^2$.
  - Rewriting equation 2, $T = C^2 i/2b$.

Implications for Monetary Theory

- Wage cuts can help increase employment because the Pigou effect is stronger with the square root relationship between C and T than a constant transactions velocity relationship.
- In this case, the increase in purchasing power of the stock of cash resulting from fallen prices is equivalent to an injection of cash with constant prices.
- Large cash users should be expected to learn when it is profitable to reduce cash balances relative to transactions.
- With variable b, i, and non-zero rational transactions demand for cash, the brokerage fee should have both a lower and upper bound, at least when C is small.

Implications for Monetary Theory

- If payments are lumpy but foreseen, cash may be employed even more economically.
- The rise in demand for brokerage services resulting from an increase in transactions can increase the brokerage fee, and thus encourage people to hold more cash. If cash supplies are sticky, interest rates will rise.
- Widespread cash economizing may require an increase in precautionary cash holdings.
- For banks, whose precautionary cash requirements might also grow with the square root of the volume of its transactions, and whose cash demands are normally distributed, to maintain a fixed probability of not running out of funds, precautionary cash requirements will be met by keeping on hand a constant multiple of the standard deviation.

Baumol's Theory Today

- Due to advances in technology, such as ATMs and P2P payment systems like Paypal and Venmo, as well as fierce competition within the banking industry to reduce retail banking fees, the frequency of cash withdrawals has increased while the cost of withdrawals has decreased
  - Interest rate elasticity of demand has increased
  - Expenditure elasticity of demand has increased
  - Higher precautionary demand of cash in individuals with ATM cards
  - Positive benefit (15-30 euros a year) for individuals with ATM cards
  - Benefit increases with amount of cash reserves on hand

Sources

- Baumol, William J. "The Transactions Demand for Cash: An Inventory Theoretic Approach." *The Quarterly Journal of Economics* 66.4 (1952): 545-56. *JSTOR*. Web. 13 Apr. 2016.

- Alvarez, Fernando & Lippi, Francesco. "Financial Innovation and the Transactions Demand for Cash." *Econometrica, Econometric Society 77.2* (2009): 363-402. National Bureau of Economic Research. Web. 13 Apr. 2016.