Equilibrium Bank Runs
James Peck and Karl Shell (2003, JPE)

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Outline

Introduction

- In Diamond-Dybvig framework: bank runs can be avoided by suspending convertibility
- Green and Lin (2000): constrained-efficient allocation does not permit bank runs
- However, in reality, bank runs occur
- Can we construct optimal contracts with suspension schemes that allow for bank runs?
- If bank runs triggered by sunspots, the predeposit game allows for bank runs in the postdeposit game.

The Model

- 3 periods
- N ex-ante identical consumers, N finite, endowed with y units
- α impatient consumers, α random variable
- c^i: consumption at period i
- Utilities:
  - Patient: $u(c^1)$, $u'' < 0$, $\frac{xu''(x)}{u'(x)} < -1$
  - Impatient: $v(c^1 + c^2)$, $v'' < 0, \frac{xv''(x)}{v'(x)} < -1$
- $f(\alpha)$: probability of number of impatient consumers

Timing

1. Perfectly competitive bank designs a deposit contract, maximises ex-ante utility
2. Consumers deposit y at 0
3. Nature draws α from $f(\alpha)$ and randomly assigns the impatient consumers
4. Consumers learn privately their type and decide whether to arrive at bank at 1 or 2
5. At 1, consumers arrive at random order, z_j position of consumer j in the queue

Indirect Mechanism

- Sequential service constraint: consumption is allocated to individuals at the head of the queue
- Consumer's withdrawal is a function of the position z_j, not of her type
- Consumer's strategy is a function of her type
- Hence, we consider an indirect mechanism with withdrawal round as a function of type and withdrawal as a function of position
- Pay attention to contracts where incentive compatibility of the patient type is satisfied: consumption at 1 should be less

Banking Mechanism

- $c^1(z)$: consumption at 1 for a consumer at arrival position z
- $c^2(\alpha_1)$: consumption at 2 when the number of consumers choosing to withdraw at 1 is $\alpha_1 = 0, \dots, N-1$.
- Resource constraints:

$$c^2(\alpha_1) = \frac{[Ny - \sum_{z=1}^{\alpha_1} c^1(z)]R}{N - \alpha_1}, c^1(N) = Ny - \sum_{z=1}^{N-1} c^1(z)$$

- **Banking mechanism** m:

$$m = (c^1(1), \dots, c^1(z), \dots, c^1(N), c^2(0), \dots, c^2(N-1))$$

- The set of banking mechanism, M, includes all banking mechanisms that satisfy the resource constraints for $\alpha = 0, \dots, N-1$

Welfare

- Ex-ante welfare is the sum of expected utilities
- Welfare under a mechanism supporting symmetric constrained-efficient allocation (impatient consumers choose period 1, the patient period 2):

$$\begin{aligned}\hat{W}(m) &= \sum_{\alpha=0}^{N-1} f(\alpha)\left[u(c^1(z)) + (N-\alpha)v\left(\frac{[Ny - \sum_{z=1}^{\alpha}]R}{N-\alpha}\right)\right] \\ &+ f(N)\left[\sum_{z=1}^{N-1} u(c^1(z)) + u\left(Ny - \sum_{z=1}^{N-1} c^1(z)\right)\right] \qquad (1)\end{aligned}$$

Welfare

Definition

Given $m \in M$, the postdeposit game has a *run equilibrium*, if there is a Bayesian Nash equilibrium in which all consumers withdraw in period 1 independent of their types.

In the run equilibrium, welfare is given by

$$W^{\mathrm{run}}(m) = \sum_{\alpha=0}^{N} f(\alpha)\left[\frac{\alpha}{N}\sum_{z=1}^{N} u(c^1(z)) + \frac{N-\alpha}{N}\sum_{z=1}^{N} v(c^1(z))\right]$$

Incentive Compatibility

- Optimal contract must satisfy the incentive compatibility constraint
- Conditional on being patient, the probability that the number of impatient consumers is α is by Bayes' rule

$$f_p(\alpha) = \frac{\left[1 - \frac{\alpha}{N}\right] f(\alpha)}{\sum_{\alpha'=0}^{N-1} \left[1 - \frac{\alpha'}{N}\right] f(\alpha')}, \alpha = 0, 1, \ldots, N$$

- Incentive compatibility for patient consumers reads as

$$\sum_{\alpha=0}^{N-1} f_p(\alpha) \left[\frac{1}{\alpha+1} \sum_{z=1}^{\alpha+1} v(c^1(z))\right] \leq \sum_{\alpha=0}^{N-1} f_p(\alpha) v\left(\frac{[Ny - \sum_{z=1}^{\alpha} c^1(z)]R}{N - \alpha}\right)$$

Optimal Contract

- 'Optimal' contract solves

$$\max_{\{c^1(1),\ldots,c^1(N-1)\}} \hat{W}(m)$$

subject to IC

- Results in first-order conditions for $\hat{\alpha} = 0, \ldots, N-1$ for $c^1(\hat{\alpha})$
- However, incentive compatibility holds only when no other patient consumer withdraws in period 1.
- Instead, if patient consumer prefers to withdraw when other patient consumers choose 1, we have a run equilibrium
- m^* has a run equilibrium, if

$$\frac{1}{N}\sum_{z=1}^{N} v(c^1(z)) \geq v\left(\left[Ny - \sum_{z=1}^{N-1} c^1(z)\right] R\right)$$

Two-Consumer Economy

- Consider an example with two consumers, consumer is impatient with probability p
- Welfare:

$$\hat{W} = p^2[u(c)+u(2y-c)]+2p(1-p)[u(c)+v(2y-c)R]+2(1-p)^2 v(yR)$$

- Incentive compatibility:

$$p\left[\frac{v(c)}{2}+\frac{v(2y-c)}{2}\right]+(1-p)v(c)\leq pv((2y-c)R)+(1-p)v(yR)$$

- Run equilibrium exists, if

$$\frac{v(c)}{2}+\frac{v(2y-c)}{2}\geq v((2y-c)R)$$

Run Equilibrium

Proposition

For some economies, a run equilibrium exists at the optimal contract m_t^*.

- Let utility functions be $u(x) = \frac{Ax^{1-a}}{1-a}$, $v(x) = \frac{x^{1-b}}{1-b}$
- For certain parameter values, we can find a solution to the planner's problem
- Those sufficient conditions satisfy IC but also the condition for run equilibrium.
- In an optimal solution, there is partial suspension of convertibility, i.e. $c^1(1) > c^2(1)$
- One can show that a run equilibrium exists for larger dimensions as well
- Even if we allow the bank to ask the type of the agents in line, a run equilibrium is sustained by the implied direct mechanism.

Sunspots and the Propensity to Run

- Until now, we have restricted our attention to the postdeposit game.
- In the pre-deposit game, after the bank announces the mechanism, consumers decide whether to deposit or not
- Formalise now the notion of sunspots in a Diamond-Dybvig model
- Introduce sunspot variable $\sigma \sim U(0,1)$
- At period 1, each consumer learns her type and observes σ

Definition

Given a mechanism $m \in M$, the predeposit game has a *run equilibrium*, if there is a subgame-perfect equilibrium in which (i) consumers are willing to deposit and (ii) for a nonempty set of realisations of σ, all consumers withdraw in period 1.

Proposition

For a mechanism $m \in M$ yielding a post-deposit game where all patient consumers choose period 2 and welfare is strictly higher than under autarky, the predeposit game has a run equilibrium if and only if the postdeposit game has a run equilibrium

"$\Rightarrow$"

- Let mechanism m produce a run equilibrium
- As this is equilibrium also in the subgame, the post-deposit game must have a run equilibrium

"$\Leftarrow$"

- Construct a run equilibrium under mechanism m.
- Let cut-off strategies for patient consumers depend on threshold s over which they choose period 2.
- With small s, the ex-ante welfare is higher than under autarky and there are no positive deviations

Propensity to Run

- If the planner is unable to prevent bank runs, the optimal mechanism should depend on how consumers choose among the multiple equilibria, its *propensity to run*
- Interpret the threshold s as follows:
  - If $\sigma < s$, all consumers arrive at bank in period 1 as long as the postdeposit game has a run equilibrium
  - If $\sigma \geq s$, all patient consumers wait until period 2
  - Hence, the equilibrium can be characterised by the propensity to run s, and the optimal contract should be designed accordingly

Optimal Mechanism

- Ex-ante welfare for the predeposit game is given by

$$W(m,s) = \begin{cases} sW^{\text{run}}(m) + (1-s)\hat{W}(m), & \text{m has a run equilibrium} \\ \hat{W}(m), & \text{m has no run equilibrium} \end{cases}$$

Definition

s-optimal mechanism maximises $W(m,s)$ subject to (IC)

- Now, the idea of optimal contracts sustaining run equilibria, for sufficiently small s, can be formalised

Proposition

For some economies with sufficiently small propensity to run s, the optimal mechanism for the predeposit game has a run equilibrium.

s-Optimal Mechanism

- The proposition can be proven for the 2-consumer economy above
  - In the optimal mechanism of the postdeposit game, IC holds as equality
  - By continuity of welfare function, IC must bind also for sufficiently small s
  - Using this, c^1 can uniquely be solved, and welfare is higher than under autarky
- When s increases, the welfare in the equilibrium sustaining run equilibrium eventually becomes smaller than in the no-run equilibrium
- For more general set-up, finding an s-optimal mechanism is more difficult when IC does not bind

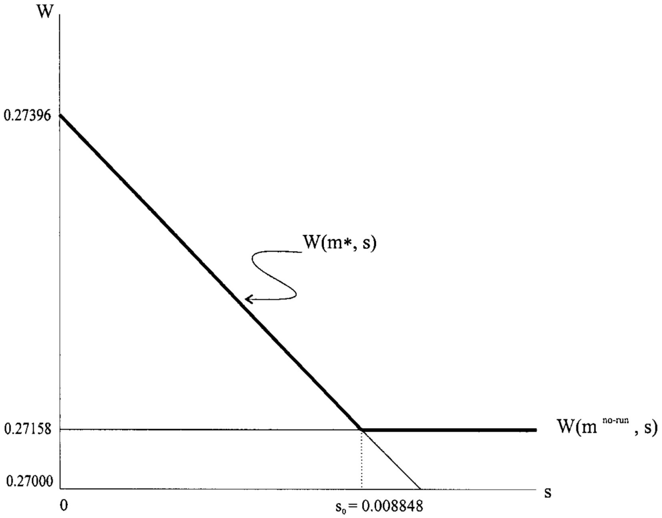
Example

TABLE 1

The "Optimal Contract" $\mathbf{m}^*$	
$c^1(1) = 3.1481$	$c^1(2) = 2.8519$
$c^2(0) = 3.1500$	$c^2(1) = 2.9945$
Best Mechanism Immune from Runs: $\mathbf{m}^{\text{no-run}}$	
$c^1(1) = 3.1463$	$c^1(2) = 2.8537$
$c^2(0) = 3.1500$	$c^2(1) = 2.9964$

- When no runs occur, $\hat{W}(m^*) = .27396$
- If run, $W^{\text{run}}(m^*) = .00519$
- With $m^{\text{no-run}}$, no-run condition holds with equality, $W(m^{\text{no-run}}) = .27158$
- If s is sufficiently small, $W(m^*, s) > \hat{W}(m^{\text{no-run}})$
- Cutoff value $s_0 = 0.00848$, where $W(m^*, s_0) = \hat{W}(m^{\text{no-run}})$

Welfare as a Function of s



Discussion

- Choosing between the run and no-run mechanisms is a tradeoff between efficiency and financial fragility
- Consumer beliefs were assumed based on the notion of sunspots
- Under other rational expectations, different equilibria are possible

Conclusions

- Possibility of a bank run does not depend on the design of the optimal deposit contract. Bank runs may occur even under suspension schemes.
- Welfare cost of preventing a run equilibrium
- Sunspots as triggering equilibria tolerating runs
- Equilibrium tolerates runs, if
  - Uncertainty about the number of impatient and patient consumers
  - Impatience of impatient consumers high
- In general, more complicated contracts do not necessarily prevent bank runs.