Economics 6130-2
Cornell University, Fall 2016
Macroeconomics, I
Final Examination with Solutions

Uris Hall 202
Tuesday, December 13, 2016, 2PM-4:30PM

Answer each of the 4 questions. The exam is designed for 120 minutes, but you have 150 minutes. Please do not take any advice, animate or inanimate. Do not use computers or calculators.

Show your work, but you need not take time to simplify your arithmetic.

Please leave your belongings at a place designated by the proctor.

Please use separate sheets for each of your 4 answers. Please write in dark black ink or dark black pencil.

1. Overlapping Generations, I (30 minutes):

- 2-period lives,
- 1 commodity per period, $\ell = 1$,
- Stationary environment,
- 1 person per generation,

$$\omega_0^1 = B \text{ for } t = 0$$
$$(\omega_t^t, \omega_t^{t+1}) = (A, B) \text{ for } t = 1, 2, \cdots .$$
$$u_0(x_0^1) = D \log x_0^1 \text{ for } t = 0$$
$$u_t(x_t^t, x_t^{t+1}) = C \log x_t^t + D \log x_t^{t+1} \text{ for } t = 1, 2, \cdots .$$

Define

$$z^t = \omega_t^t - x_t^t \quad \text{and} \quad z^{t+1} = x_t^{t+1} - \omega_t^{t+1}$$

Solve for

a. The reflected, translated offer curve, ROC

b. The steady-states

c. The set of equilibrium money prices, $\mathscr{P}^m$

d. The full dynamic analysis, including the stability of steady states

e. The welfare analysis

For each of the following cases:

1. $A = B = 100$, $C = 1$, $D = 5$,
   and $m_0^1 = 10$ for $s = 0$ and $m_s^t = 0$ otherwise.
2. $A = B = 100$, $C = 1$, $D = 5$,
   and $m_0^1 = 5$, $m_1^2 = 8$, and $m_s^t = 0$ otherwise.
3. $A = B = 2$, $C = 4$, $D = 1$,
   and $m_0^1 = 1$, $m_s^t = 0$ otherwise
4. $A = 10, B = 1$, $C = 5$, $D = 1$,
   and $m_0^1 = 1$, $m_s^t = 0$ otherwise

Solutions:

By solving the optimization problem

$$\arg\max_{x_t^t, x_t^{t+1}} \{u_t(x_t^t, x_t^{t+1})\}$$

$$\text{s.t. } p^t x_t^t + p^{t+1} x_t^{t+1} \leq p^t \omega_t^t + p^{t+1} \omega_t^{t+1}$$

We may derive the reflected offer curve in general as

$$z^{t+1} = \frac{BCz^t}{AD - (C+D)z^t}$$

Where $z^{t+1} = x_t^{t+1} - \omega_t^{t+1}$ and $z^t = \omega_t^{t+1} - x_t^{t+1}$.

Case 1.

$A = B = 100$, $C = 1$, $D = 5$, and $m_0^1 = 10$ for $s = 0$ and $m_s^t = 0$ otherwise.

1.a. Plugging into the reflected offer curve equation above, we get

$$z^{t+1} = \frac{100z^t}{500 - 6z^t}$$

1.b. The steady states will be where $z = \frac{100z^t}{500-6z^t}$. Thus, $z = 0$ will be the non-monetary steady state, while the second steady-state may be found as

$$z = \frac{100z}{500 - 6z} \quad \Rightarrow \quad 1 = \frac{100}{500 - 6z} \quad \Rightarrow \quad 500 - 6z = 100 \quad 6z = 400 \quad \Rightarrow \quad z = \frac{400}{6} = \frac{200}{3}$$

The monetary steady state is thus $\bar{z} = \frac{200}{3}$.

1.c. The set of equilibrium money prices is thus

$$\mathscr{P}^m = \left[0, \frac{20}{3}\right]$$

1.d. If $0 < p^m < \frac{20}{3}$, then z^t is declining, and the bubble fades away through inflation. $z = 0$ is a stable steady state, in which money is worthless. $z = \bar{z}$ is a steady state. If $z > \bar{z}$, hyperinflation ensues and the bubble bursts in finite time. We may note that this is a Samuelson case.

1.e. $p^m = \frac{20}{3}$ is Pareto Optimal, while $p^m = 0$ is not. $p^m \in (0, \frac{20}{3})$ is not a steady state and is not Pareto Optimal, but it is Pareto superior to $p^m = 0$.

Case 2.

$A = B = 100$, $C = 1$, $D = 5$, and $m_0^1 = 5$, $m_1^2 = 8$, and $m_s^t = 0$ otherwise.

2.a. The offer curve remains the same as in Case 1.

$$z^{t+1} = \frac{100z^t}{500 - 6z^t}$$

2.b. The steady states will also remain the same. $z = 0$ will be the non-monetary steady state and $\bar{z} = \frac{200}{3}$ will be the monetary steady state.

2.c. Since $m_0^1 = 5$ and $m_1^2 = 8$, we may note that $m_0^1 + m_1^2 = 13$. Mr. 1. exchanges 13 dollars for chocolate in period 2. We may treat Mr. 1 as Mr. 0 in the previous example. All purchases made in excess of the endowment must be financed with money.

$$z^t = (m_0^1 + m_1^2)p^m \leq \omega_{t-1}^t$$

So

$$\bar{z} = 13\bar{p}^m$$

Since $\bar{z} = \frac{200}{3}$, we get that $\bar{p}^m = \frac{200}{39}$. The set of equilibrium money prices is thus

$$\mathscr{P}^m = \left[0, \frac{200}{39}\right]$$

2.d. If $0 < p^m < \frac{200}{39}$, then z^t is declining, and the bubble fades away through inflation. $z = 0$ is a stable steady state, in which money is worthless. $z = \bar{z}$ is a steady state. If $z > \bar{z}$, hyperinflation ensues and the bubble bursts in finite time. We may note that this is a Samuelson case.

2.e. $p^m = \frac{200}{39}$ is Pareto Optimal, while $p^m = 0$ is not. $p^m \in (0, \frac{200}{39})$ is not a steady state and is not Pareto Optimal, but it is Pareto superior to $p^m = 0$.

Case 3.

$A = B = 2$, $C = 4$, $D = 1$, and $m_0^1 = 1$, $m_s^t = 0$ otherwise

3.a. Plugging into the reflected offer curve equation above, we get

$$z^{t+1} = \frac{8z^t}{2 - 5z^t}$$

3.b. $z = 0$ will be the non-monetary steady state. The fixed point of the system, however, is going to be where $z = -1$. Since our proposed steady state is $z = -\frac{6}{5} \notin \mathbb{R}_+$, we do not have a monetary steady state.

3.c. The set of equilibrium money prices is thus

$$\mathscr{P}^m = \{0\}$$

3.d. Trajectories originating away from the non-monetary steady state with $p^m = 0$ will be deflationary.

3.e. The non-monetary steady state with $p^m =$ is going to be the unique steady state.

Case 4.

$A = 10, B = 1$, $C = 5$, $D = 1$, and $m_0^1 = 1$, $m_s^t = 0$ otherwise

4.a. Plugging into the reflected offer curve equation above, we get

$$z^{t+1} = \frac{5z^t}{10 - 6z^t}$$

4.b. $z = 0$ will be the non-monetary steady state. The second steady-state may be found as

$$z = \frac{5z}{10 - 6z} \quad \Rightarrow \quad 1 = \frac{5}{10 - 6z} \quad \Rightarrow \quad 10 - 6z = 5 \quad 6z = 5 \quad \Rightarrow \quad z = \frac{5}{6}$$

The monetary steady state is thus $\bar{z} = \frac{5}{6}$.

4.c. Since $m_0^1 = 1$, $\bar{z} = m_0^1 \bar{p}^m$, such that $\frac{5}{6} = \bar{p}^m$. Thus, $\mathscr{P}^m = \left[0, \frac{5}{6}\right]$.

4.d. If $\bar{p}^m \in (0, \frac{5}{6})$, then z^t declines asymptotically to zero. If $p^m > \frac{5}{6}$, then z^t increases until the bubble bursts in finite time.

4.e. We are once more in the Samuelson case, where the non-monetary equilibrium is stable but not Pareto Optimal. $p^m = \frac{5}{6}$ is Pareto optimal.

2. Diamond-Dybvig Bank (30 minutes):

The probability of being impatient is $\lambda = \frac{1}{2}$. The type (patient or impatient) is realized in period 1 and it is private information. The utility function is:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

where $\gamma = \frac{1}{2} > 0$. Each individual has one unit of endowment in period 0. There is costless storage. If the endowment $y = 1$ is invested in period 0 and is harvested in period 1, the rate of return is 0. If harvested late, the rate of return is 4. Assume that the banking industry is free-entry.

(a) What is the depositor's *ex-ante* expected utility W as a function of c_1, consumption in period 1, and c_2, consumption in period 2?

Solution:

$W = \lambda u(c_1) + (1-\lambda)u(c_2)$

$W = \frac{\lambda c_1^{1-\gamma}}{1-\gamma} + \frac{(1-\lambda)c_2^{1-\gamma}}{1-\gamma}$

(b) Let $\bar{c} = \frac{c_1+c_2}{2}$. Show that the depositor prefers this consumption smoothing.

Solution:

$u'(c) = \frac{(1-\gamma)c_1^{-\gamma}}{1-\gamma} = c^{-\gamma} > 0$

$u''(c) = -\gamma c^{-\gamma-1} < 0$

So $u(c)$ is strictly concave (And the consumer is risk-averse. Concave functions lie above their chords (Jensen's inequality):

$u(\lambda c_1 + (1-\lambda)c_2) > \lambda u(c_1) + (1-\lambda)u(c_2)$ when $c1 \neq c2$,

So $W(\bar{c}, \bar{c}) > W(c_1, c_2)$ where $\bar{c} = \lambda c_1 + (1-\lambda)c_2$.

(c) What is the bank's resource constraint RC? Write this down precisely and explain it in words.

Solution:

$(1-\lambda)d_2 \leq (1-\lambda d_1)R$

Where d_t is the withdrawal allowed in period $t = 1, 2$.

The LHS of the inequality is the funds to be withdrawn in period 2. The RHS is the resources available at the bank in period 2. If the inequality is violated, the bank is insolvent.

(d) What is the incentive problem? Write down the incentive constraint IC precisely, and explain it in words.

Solution:
$d_1 \leq d_2$ is the ICC.

If the inequality does not hold, every depositor will attempt to withdraw early: the depositors would not self-select correctly.

(e) Solve for the so-called "optimal deposit contract" for the post-deposit bank (assuming that there is no run). (That is: Write down the optimal first-period payment d_1^* as a function of λ, R and γ)

Solution:
The bank will act to maximize $W(d_1, d_2)$ while constrained by the resources such that $(1-\lambda)d_2 = (1-\lambda d_1)R$.

$$\arg\max_{d_1, d_2}\{W(d_1, d_2)\} \Rightarrow \arg\max_{d_1, d_2}\{\lambda u(c_1) + (1-\lambda)u(c_2)\}$$

$$\text{Subject to } (1-\lambda)d_2 - (1-\lambda d_1)R = 0$$

We may therefore write the Lagrangian

$$\mathscr{L}(d_1, d_2) = [\lambda u(d_1) + (1-\lambda)u(d_2)] - \delta[(1-\lambda)d_2 - (1-\lambda d_1)R]$$

So then the first order conditions are:

$$\frac{\partial \mathscr{L}}{\partial d_1} = \lambda u'(d_1) - \delta\lambda R = 0 \quad \Rightarrow \quad \lambda u'(d_1) = \delta\lambda R \Rightarrow u'(d_1) = \delta R$$

$$\frac{\partial \mathscr{L}}{\partial d_2} = (1-\lambda)u'(d_2) - \delta(1-\lambda) = 0 \quad \Rightarrow \quad (1-\lambda)u'(d_2) = \delta(1-\lambda) \Rightarrow u'(d_2) = \delta$$

Thus, it follows that, as in Problem 1:

$$\frac{u'(d_1)}{u'(d_2)} = \frac{\delta R}{\delta} = R$$

Recall that $u'(c) = c^{-\gamma}$,

$$\frac{d_1^{-\gamma}}{d_2^{-\gamma}} = \left(\frac{d_1}{d_2}\right)^{-\gamma} = \left(\frac{d_2}{d_1}\right)^{\gamma} = R$$

So then $\frac{d_2}{d_1} = R^{\frac{1}{\gamma}}$. We may then may recall the resource constraint to state that, if $(1-\lambda)d_2 = (1-\lambda d_1)R$ then $d_2 = \frac{(1-\lambda d_1)R}{(1-\lambda)}$. Therefore:

$$\frac{(1-\lambda d_1)R}{d_1(1-\lambda)} = R^{\frac{1}{\gamma}} \Rightarrow (1-\lambda d_1) = R^{\frac{1}{\gamma}-1} d_1 (1-\lambda)$$

Dividing through by d_1,

$$\frac{1}{d_1} - \lambda = R^{\frac{1}{\gamma}-1}(1-\lambda) \Rightarrow \frac{1}{d_1} = R^{\frac{1}{\gamma}-1}(1-\lambda) + \lambda$$

Finally, it becomes clear that

$$d_1^* = \frac{1}{R^{\frac{1}{\gamma}-1}(1-\lambda) + \lambda}$$

$$d_1^* = \frac{1}{\lambda + (1-\lambda)R^{1/\gamma - 1}} = \frac{1}{0.5 + 0.5 \cdot 5^{(1/\frac{1}{2}-1)}} = \frac{1}{3}$$

$$d_2^* = d_1^* \cdot R^{1/\gamma} = \frac{R^{1/\gamma}}{\lambda + (1-\lambda)R^{1/\gamma - 1}} = \frac{1}{3} \cdot 5^2 = \frac{25}{3}$$

(f) Is there a run equilibrium to this "optimal contract"?

Solution:
If $\gamma > 1$, we have $d_1^* > 1$, and a run would be possible. However, since $\gamma = 1/2 < 1$, it holds that $d_1^* < \omega = 1$.
The contract is dominant strategy incentive-compatible (DSIC). If patient depositors run, the bank will still have sufficient funds to pay off all depositors. Thus, there will not be a run equilibrium in this case.

3. Money Taxation (30 minutes):

Consider an economy with a single commodity, $\ell = 1$, chocolate. There are 5 consumers, so $n = 5$. The endowments are defined as

$$\omega = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5)$$
$$= (150, 120, 90, 60, 30)$$

3.1 A Single Currency

There is one money. The chocolate price of money is $P^m \geq 0$. In each of the following cases, solve for the set $\mathscr{P}^m$ of equilibrium prices P^m, given the following tax policies τ. Provide the units in which the variables are measured.

(a) $\tau = (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5) = (3, 3, 0, 0, -6)$

Solution:
In general,

$$x_h = \omega_h - \tau_h P^m > 0$$

Taxes τ are in money, but Mr. h's endowment ω_h is in chocolate. The price of money, P^m, is the rate of exchanging one unit of currency (like dollars) with a unit of real good, here chocolate; P^m is therefore in $chocolate/dollars$.

For Mr. 1, we have $150 - 3P^m > 0$. Therefore $P^m < 50$.

For Mr. 2, $120 - 3P^m > 0$. Therefore $P^m < 40$.

Thus, we have $\mathscr{P}^m = [0, 40)$. Note that a worthless currency, $P^m = 0$, is an equilibrium outcome.

(b) $\tau = (30, 15, 0, -24, -21)$

Solution:
For Mr. 1, $150 - 30P^m > 0$, so $P^m < 5$.
For Mr. 2, $120 - 15P^m > 0$. Thus, $P^m < 8$.
We have $\mathscr{P}^m = [0, 5)$.

(c) $\tau = (60, 6, 3, -6, -60)$

Solution:
We may immediately note that $\sum_h \tau_h = 60 + 6 + 3 - 6 - 60 = 3 \neq 0$. Thus, taxes are not balanced in this finite economy. The equilibrium price of money must therefore be $\mathscr{P}^m = \{0\}$, such that taxes fail to be bonafide as well. The result will be an economy in autarky, as money will be worthless.

3.2 Two Monies

Consider a scenario where there are 2 monies, red dollars R and blue dollars B, with respective chocolate prices of money, $P^R \geq 0$ and $P^B \geq 0$.

In each of the following cases, solve for the equilibrium exchange rate between B and R. Do these depend on the endowments ω? Give the economic explanation for your answer.

(a) $\tau^R = (3, 3, 3, 0, -6)$ and $\tau^B = (3, 0, 0, 0, -6)$

Solution:
Recalling that $x_h = \omega_h - P^R \tau_h^R - P^B \tau_h^B$, we may rearrange the equation to get

$$x_h - \omega_h = -P^R \tau_h^R - P^B \tau_h^B$$

If we sum over h consumers, we get

$$\sum_h (x_h - \omega_h) = -P^m \sum_h \tau_h^R - P^m \sum_h \tau_h^B$$

And since when markets clear, $\sum_h (x_h - \omega_h) = 0$,

$$P^R \sum_h \tau_h^R + P^B \sum_h \tau_h^B = 0 \quad \Rightarrow \quad P^R \sum_h \tau_h^R = -P^B \sum_h \tau_h^B$$

Rearranging further, we get the exchange rate as

$$\frac{P^R}{P^B} = -\frac{\sum_h \tau_h^B}{\sum_h \tau_h^R}$$

In this case, $\sum_h \tau_h^R = 3 + 3 + 3 - 6 = 3$, while $\sum_h \tau_h^B = 3 - 6 = -3$, so

$$\frac{P^R}{P^B} = -\left(\frac{-3}{3}\right) = 1$$

Of course, this is also equivalent to $\frac{P^B}{P^R} = 1$ as well.

(b) $\tau^R = (3, 3, 0, -3, -6)$ and $\tau^B = (3, 3, 3, 0, -6)$

Solution:
Here, $\sum_h \tau_h^R = 3 + 3 - 3 - 6 = -3$, while $\sum_h \tau_h^B = 3 + 3 + 3 - 6 = 3$. Thus, it again holds that $\frac{P^R}{P^B} = -\left(\frac{-3}{3}\right) = 1$ (and exchanging in the other direction, $\frac{P^B}{P^R} = 1$).

(c) $\tau^R = (9, 6, 3, 0, -18)$ and $\tau^B = (12, 0, -3, -3, -6)$

Solution:

Finally, we have $\sum_h \tau_h^R = 9 + 6 + 3 - 18 = 0$, while $\sum_h \tau_h^B = 12 - 3 - 3 - 6 = 0$. The exchange rate is therefore indeterminate, as $\frac{P^R}{P^B} = \frac{0}{0}$ is not well-defined.

These exchange rates are independent of the endowments ω; the supply and demand for the currencies completely determines the exchange rate between them unless one or both currencies are worthless. If both tax policies are balanced, then the exchange rate is indeterminate since there are no currency trades.

3.3 The Absence of Money Illusion

Explain the difference between the "absence of money illusion" and the "quantity theory of money". Be precise (with symbols).

Solution:

Taxes only matter through their real values. Only the term $P^m \tau_h$ matters to Mr. h.

Absence of money illusion: Let P^m be an equilibrium price of money given the tax vector τ. If the tax vector is multiplied by some scalar λ to become $\lambda\tau$, then $\frac{P^m}{\lambda}$ is *an* equilibrium price of money. In other words, if $\mathscr{P}^m = [0, \bar{P}^m]$ when the tax vector is τ, then when the tax vector is $\lambda\tau$, it follows that $\mathscr{P}^m = \left[0, \frac{\bar{P}^m}{\lambda}\right)$.

Quantity theory of money: If P^m is an equilibrium price of money when the tax vector is τ, then when taxes become $\lambda\tau$, the equilibrium price of money becomes $\frac{P^m}{\lambda}$.

The quantity theory of money is true if and only if people believe it to be true, while the absence of money illusion is a statement about sets.

In other words, if outside money is doubled, then under the quantity theory of money, the price of money will halve (and the price level for real goods, by extension, will double). In contrast, with an absence of money illusion, it is only a *possibility* that the same fiscal policy change will halve the price of money and double the price level. The actual price of money and price level after the tax regime change, however, will be indeterminate.

As noted in lecture, our models are consistent with the AMI, but not strictly with QTM.

4. Overlapping Generations, II (30 minutes):

$$u_0(x_0^1) = \beta x_0^1 \text{ for } t = 0,$$
$$u_t(x_t^t, x_t^{t+1}) = x_t^t + \beta x_t^{t+1} \text{ for } t = 1, 2, \cdots,$$
$$\omega_0^1 = 100 \text{ for } t = 0,$$
$$(\omega_t^t, \omega_t^{t+1}) = (100, 100) \text{ for } t = 1, 2, \cdots,$$
$$z^t = \omega_t^t - x_t^t \text{ and } z^{t+1} = x_t^{t+1} - \omega_t^{t+1} \text{ for } t = 1, 2, \cdots.$$

Derive and graph the offer curve OC. Decide whether Ricardo or Samuelson. Calculate the set of Money prices $\mathscr{P}^m$, do the full dynamic analysis, provide the full welfare analysis for each of the following:

Solution: If Mr. t must solve

$$\arg\max_{x_t^t, x_t^{t+1}} \{u_t(x_t^t, x_t^{t+1}) = x_t^t + \beta x_t^{t+1}\}$$

$$\text{such that } p^t x_t^t + p^{t+1} x_t^{t+1} = p^t \omega_t^t + p^{t+1} \omega_t^{t+1}$$

Then the Lagrangian will be

$$\mathscr{L} = x_t^t + \beta x_t^{t+1} - \lambda[p^t x_t^t + p^{t+1} x_t^{t+1} - p^t \omega_t^t + p^{t+1} \omega_t^{t+1}]$$

Such that the FOC becomes

$$\frac{\partial \mathscr{L}}{\partial x_t^t} = 1 - \lambda p^t = 0 \Rightarrow 1 = \lambda p^t$$

$$\frac{\partial \mathscr{L}}{\partial x_t^{t+1}} = \beta - \lambda p^{t+1} = 0 \Rightarrow \beta = \lambda p^{t+1}$$

Plugging this condition into the budget constraint gives:

$$p^t x_t^t + \beta p^t x_t^t = p^t \omega_t^t + \beta p^t \omega_t^{t+1}$$

Dividing both sides by p^t,

$$x_t^t - \omega_t^t = \beta(\omega_t^{t+1} - x_t^{t+1})$$

Then using the definition given, the offer curve may be derived as:

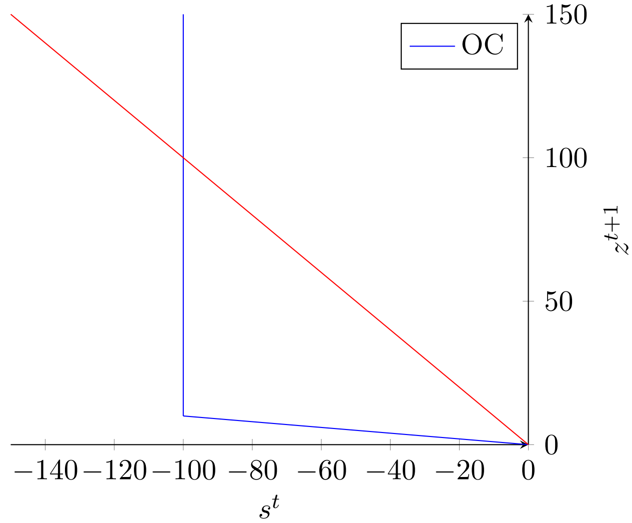
$$z^{t+1} = -\frac{1}{\beta}(-z^t)$$

This offer curve is going to be common for all cases.

(a) $\beta = 10, \quad m_0^1 = 10, \quad m_s^t = 0$ otherwise
Letting $s^t \equiv -z^t$, we have

$$OC : z^{t+1} = -\frac{1}{10} s^t$$

$$ROC : z^{t+1} = \frac{1}{10} z^t$$



The slope of reflected offer curve at the endowment point is $\frac{1}{10}$, implying that $r < 0$. This is Samuelson case.

We are given $m_0^1 = 10$. We need:

$$z^1 = p^m m_0^1 \leq \omega_1^1$$

$$10p^m \leq 100 \Rightarrow p^m \leq 10$$

The equilibrium set of money prices is: $\mathscr{P}^m = [0, 10]$.

This $\bar{p^m} = 10$ is Pareto optimal, but it is *not* stationary. $p^m \in (0, 10)$ is not optimal, but it is Pareto-Superior to the non-monetary autarky. If the economy starts in autarky, the money bubble will never from. If $p^m \in (0, 10]$, the money will inflate, price levels will rise, and the bubble will fade away asymptotically.

(b) $\beta = 10, \quad m_0^1 = m_1^1 = m_1^2 = 2, \quad m_s^t = 0$ otherwise

With same β, the offer curve will look the same as in (a). We are again in Samuelson case.

Now since we have $m_0^1 = 2$, $x_1^{1,m} = 2$, and we know that $x_1^{1,m} + x_1^{2,m} = 0$ for any positive price of money. Thus when Mr.1 is old,

$$z^2 = p^m(-x_1^{2,m} + m_1^1 + m_1^2) \leq \omega_2^2$$

$$6p^m \leq 100 \Rightarrow p^m \leq \frac{100}{6}$$

The equilibrium set of money prices is therefore $\mathscr{P}^m = [0, \frac{50}{3}]$.

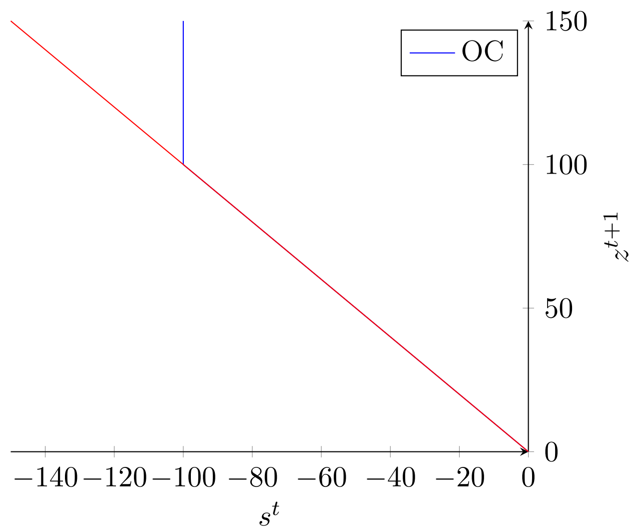
The dynamics and welfare remains the same as in part (a). The $\bar{p^m} = \frac{50}{3}$ is Pareto optimal, but it is *not* stationary. $p^m \in (0, \frac{50}{3})$ is not optimal, but it is Pareto-Superior to the non-monetary autarky. If the economy starts in autarky, the money bubble will never from. If $p^m \in (0, \frac{50}{3}]$, the money will inflate, price levels will rise, and the bubble will fade away asymptotically.

(c) $\beta = 1, \quad m_0^1 = 1, \quad m_1^1 = 2, \quad m_1^2 = 3 \quad m_s^t = 0$ otherwise

Using the offer curve equation above, we have:

$$OC : z^{t+1} = -s^t$$

$$ROC : z^{t+1} = z^t$$



This is a razor-edge case with $r = 0$, but with monetary steady state, we are in Samuelson case.

As in the case (b), we have positive money endowment for Mr. 1. Since $m_0^1 = 1$, $x_1^{1,m} = 1$. When Mr.1 is old,

$$z^2 = p^m(-x_1^{2,m} + m_1^1 + m_1^2) \leq \omega_2^2$$

$$6p^m \leq 100 \Rightarrow p^m \leq \frac{100}{6}$$

The equilibrium set of money prices is $\mathscr{P}^m = [0, \frac{50}{3}]$.

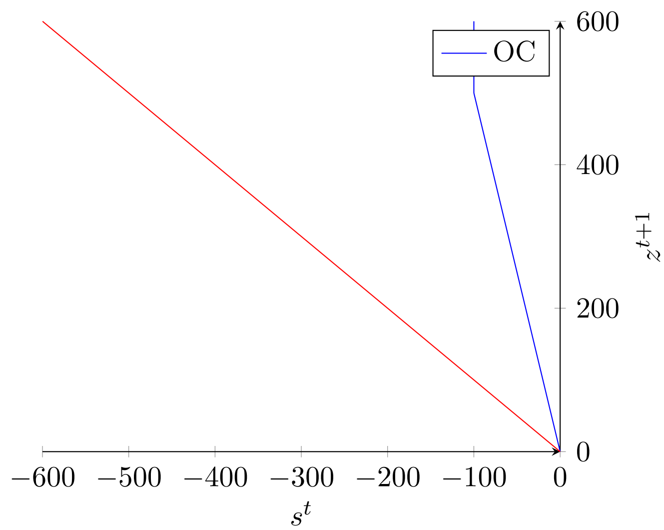
Because offer curve is exactly equal to resource constraints for feasible points, for any price of money in the equilibrium set will pick out a stationary point on the phase diagram. The Pareto Optimal allocation is associated with $p^m = \frac{50}{3}$ as the utility of initial old and Mr.1 is maximized and every other generation is made no worse off.

(d) $\beta = \frac{1}{5}$, $m_0^1 = 1$, $m_s^t = 0$ otherwise

Using the equation above gives:

$$OC : z^{t+1} = -5s^t$$

$$ROC : z^{t+1} = 5z^t$$



The slope of ROC at the endowment point is 5, implying that $r > 0$. We are in Ricardian case. There exists one stationary equilibrium, such that $z^t = z^{t+1} = 0$. Therefore, $\mathscr{P}^m = \{0\}$. For any $z^1 > 0$, the money bubble bursts in finite time. $p^m = 0$ is Pareto Optimal.

(e) $\beta = \frac{1}{5}$, $m_0^1 = 1$, $m_1^1 = -1$, $m_s^t = 0$ otherwise

The offer curve for this case remains exactly same as in part (d). With same β, the offer curve still has the same slope. Note that there will be no money holding in the economy after Mr.1 generation. Mr.0, however, must be able to sell his money to Mr.1, implying that $\mathscr{P}^m = \{0, 100\}$. Mr.0 consumes $100 + p^m$ and Mr.1 consumes $(100 - p^m, 100)$. All the other generations consume their endowment $(100, 100)$. In this case, more consumption by Mr.0 means less consumption by Mr.1. That is, there is no unique Pareto Optimal allocation.